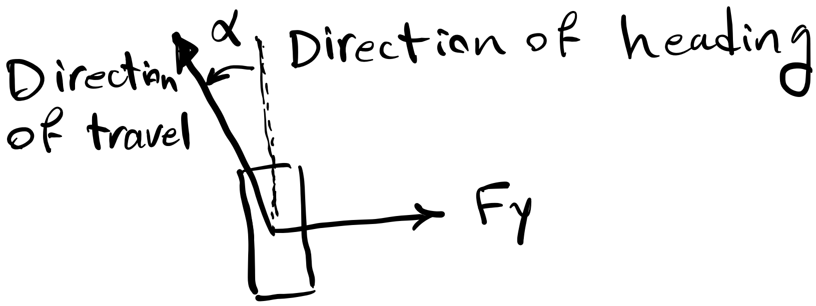
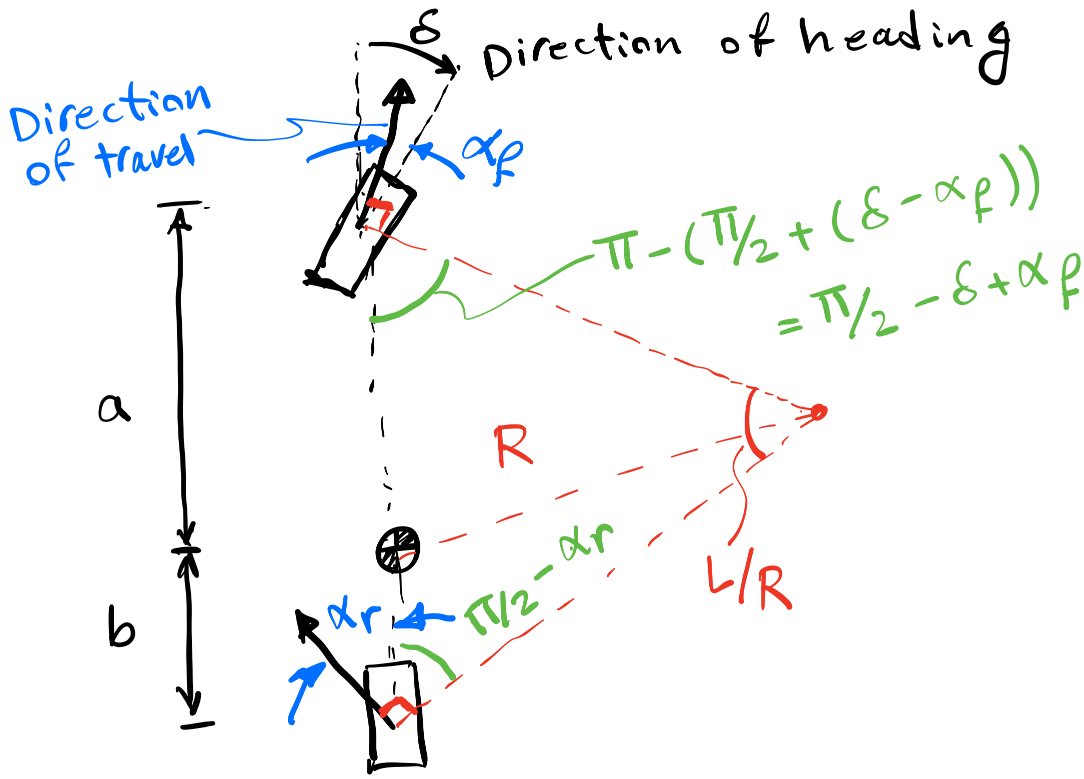
Module 6 - High speed cornering

At high speed, lateral acceleration will be present, and hence lateral forces and slip angles will be present at each wheel.

slip angle α = the angle between the direction of heading and its direction of travel.

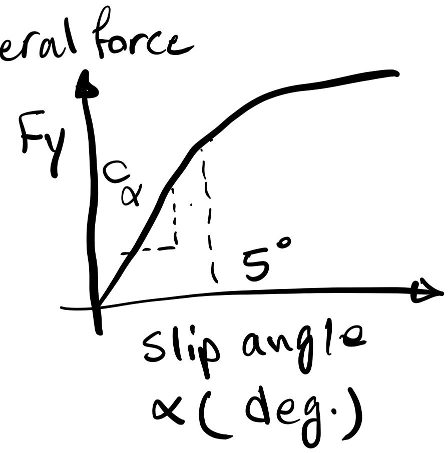


For analysis, it is convenient to represent thevehicle by the bicycle model as shown below.



Cornering force (for low slip angles (5 degrees or less) is (the relation is linear)

$$F_y = C_\alpha \alpha$$



C_α = cornering stifness
is a function of tire characteristics

$$F_{yf} = C_{\alpha f} \alpha_f$$

$$F_{yr} = C_{\alpha r} \alpha_r$$

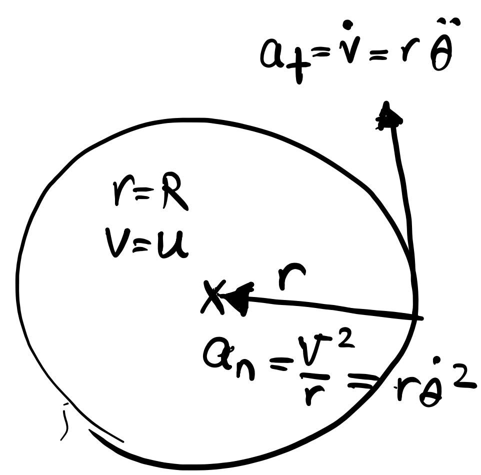
$$\pi = (\pi/2 + \alpha_f - \delta) + (\pi/2 - \alpha_r) + \frac{L}{R}$$

$$\Rightarrow \quad \delta = (57.3)\frac{L}{R} + \alpha_f - \alpha_r$$

$$F_{yf} = C_{\alpha f}\alpha_f \quad \& \quad F_{yr} = C_{\alpha r}\alpha_r$$

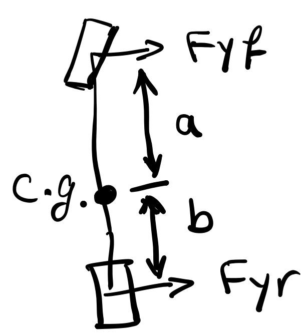
$$\Rightarrow \quad \delta = \frac{L}{R} + \frac{F_{yf}}{C_{\alpha f}} - \frac{F_{yr}}{C_{\alpha r}}$$

Now, Performing a force balance for maitaining a circle:



$$\boxed{\sum F_y = F_{yf} + F_{yr} = \frac{mu^2}{R}}$$

Taking moment about c.g. :



$$\sum M_{cg} = F_{yf}\,a - F_{yr}\,b = 0$$

$$\Rightarrow F_{yf} = b/a \, F_{yr}$$

$$F_{yr} \times \left(1 + \frac{b}{a}\right) = \frac{mu^2}{R}$$

$$F_{yr} = \frac{a}{L} \frac{mu^2}{R}$$

$$\alpha_r = \frac{mau^2}{LRC\alpha r} \qquad \alpha_f = \frac{mbu^2}{LRC\alpha f}$$

For $\delta = \frac{L}{R} + \alpha_f - \alpha_r$

$$\delta = \frac{L}{R} + \frac{mbu^2}{LRC\alpha f} - \frac{mau^2}{LRC\alpha r}$$

$$\delta = (57.3)\frac{L}{R} + \left(\frac{b}{C\alpha f} - \frac{a}{C\alpha r}\right)\frac{mu^2}{LR}$$

Now, we know:

$$W_f = mg\frac{b}{L}$$ Front static weight

$$W_r = mg\frac{a}{L}$$ Rear static weight

$$\delta = \frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)\frac{u^2}{gR}$$

$$\delta = (57.3)\frac{L}{R} + K \cdot a_y$$

K = understeer gradient (degrees/g)

a_y = lateral acceleration

(a) Neutral steer

If $\frac{b}{C_{\alpha f}} = \frac{a}{C_{\alpha f}}$ or $\frac{W_f}{C_{\alpha f}} = \frac{W_r}{C_{\alpha r}}$

$$\alpha_r = \alpha_f \Rightarrow \delta = \frac{L}{R}$$

Therefore, in a constant radius manuver turn, the steer angle will be constant regardless of the vehicle velocity. This is equilibrium condition, i.e. force at the c.g. causes identical increase in slip angle at both front and rear wheels.

(b) understeer

$$\frac{W_f}{C_{cf}} > \frac{W_r}{C_{\alpha r}} \quad \text{or} \quad \frac{b}{C_{\alpha f}} > \frac{a}{C_{\alpha r}}$$

$$\alpha_f > \alpha_r \quad \text{and} \quad K > 0$$

$$\delta = \frac{L}{R} + K\frac{u^2}{gR}$$

Therefore, steer angle must increase parabolically (u^2) as the vehicle forward speed increases in order to hold a constant radius.

(c) oversteer

$$\frac{W_f}{C_{\alpha f}} < \frac{W_r}{C_{\alpha r}} \quad \text{or} \quad \frac{b}{C_{\alpha f}} < \frac{a}{C_{\alpha r}}$$

$\Rightarrow \alpha_f < \alpha_r$ and $K < 0$

Here, the rear slip angle exceeds the front, and hence spin condition (outward drift at the rear of the vehicle turns the front wheels inward).

Therefor, the steer angle must decrease as the speed (i.e. lateral acceleration) is increased.