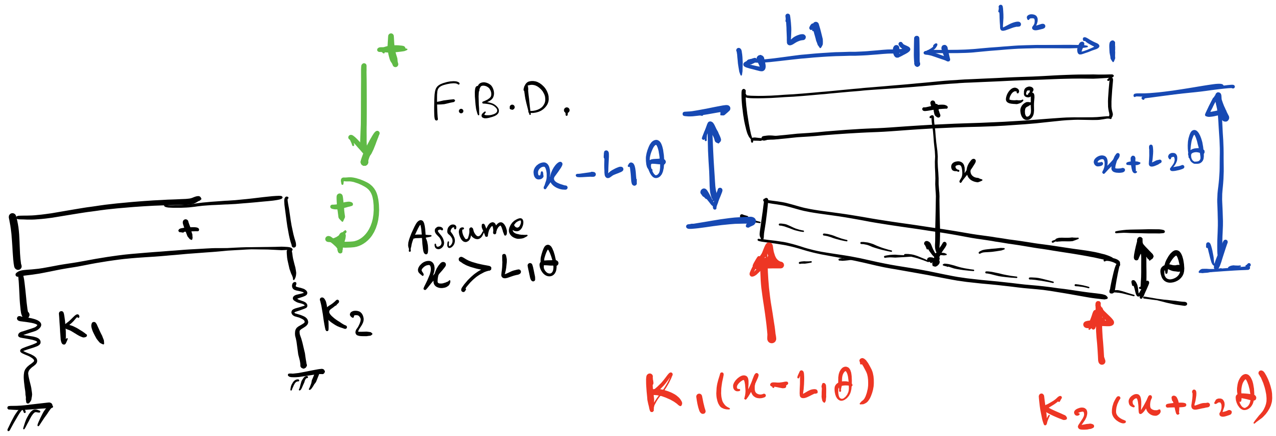
Module 4 - Half Vehicle Model - Rotational DOF



$$\Sigma F = m\ddot{x}$$
$$\Sigma M = J\ddot{\theta}$$

$$\begin{cases} m\ddot{x} = -K_1(x - L_1\theta) - K_2(x + L_2\theta) \\ J\ddot{\theta} = K_1(x - L_1\theta)L_1 - K_2(x + L_2\theta)L_2 \end{cases}$$

$$\begin{cases} m\ddot{x} = (-K_1 - K_2)x + (K_1L_1 - K_2L_2)\theta \\ J\ddot{\theta} = (K_1L_1 - K_2L_2)x + (-KL_1^2 - K_2L_2^2)\theta \end{cases}$$

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} K_1 + K_2 & -(K_1L_1 - K_2L_2) \\ K_2L_2 - K_1L_1 & K_1L_1^2 + K_2L_2^2 \end{bmatrix}\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x = Xe^{j\omega t} \Rightarrow \ddot{x} = -\omega^2 m x$$

$$\theta = \Theta e^{j\omega t} \Rightarrow \ddot{\theta} = -\omega^2 J \theta$$

$$\Delta(\omega) = \begin{vmatrix} K_1+K_2-\omega^2 m & K_2L_2-K_1L_1 \\ K_2L_2-K_1L_1 & K_1L_1^2+K_2L_2^2-\omega^2 J \end{vmatrix} = 0$$

$$\omega_{1,2}^2 = \left[\frac{K_1+K_2}{m} + \frac{K_1L_1^2+K_2L_2^2}{J} \pm \right.$$

$$\left. \sqrt{\left[\frac{K_1+K_2}{m} + \frac{K_1L_1^2+K_2L_2^2}{J} - 4K_1K_2(L_1+L_2)^2/mJ \right]} \right]$$

$$f_1 = \frac{\omega_1}{2\pi} \qquad f_2 = \frac{\omega_2}{2\pi}$$

Example:

Determine a) Natural frequency b) principal modes of vibration (mode shapes) c) the motion $x(t)$ and $\theta(t)$

mass = 1800 kg

whelbase = 3.6 m

c.g. = 1.6 m from the front

Radius of gyration about c.g. $(\rho) = 1.4$ m

$$J = m\rho^2 = (1800)(1.4)^2 = 3528$$

$$K_{f_1} = 42 \ \frac{KN}{m}$$

$$K_{r_2} = 48 \ \frac{KN}{m}$$

$$\frac{K_1 + K_2}{m} = \frac{(42+48)\times 10^3}{1.8\times 10^3} = 50$$

$$\frac{(42)(1.6)\,10^3 - (48)(2)\times 10^3}{1.8\times 10^3} = -16$$

$$\frac{K_1 L_1^2 + K_2 L_2^2}{J} = \frac{42\times 10^3 \times (1.6)^2 + (48)\times 10^3 \times (2)^2}{3521.8} = 84.9$$

$$\frac{4K_1K_2(L_1+L_2)^2}{mJ} = 16486.1$$

$$\omega_{1,2}^2 = \frac{1}{2}\left[50 + 84.9 \pm \sqrt{(50+84.9)^2 - 16486.1}\right.$$

$$\omega_{1,2}^2 = \begin{cases} 46.6 \\ 88.3 \end{cases} \Rightarrow \omega_{1,2} = \begin{cases} 6.83 \text{ rad/s} = 1.09 \text{ Hz} \\ 9.4 \text{ rad/s} = 1.5 \text{ Hz} \end{cases}$$

(b) Amplitude ratio fo the two modes of vibration are:

$$\frac{X}{\theta} = \frac{(K_1L_1 - K_2L_2)/m}{(K_1+K_2)/m - \omega_{1,2}^2} = \frac{-16}{50-\omega_{1,2}^2} = \begin{cases} -4.69 \\ 0.42 \end{cases}$$

$$\text{when} \begin{cases} x=1 \\ \theta = -1/4.69 \end{cases} \xRightarrow{\text{mode shape}} \begin{Bmatrix} 1 \\ -1/4.69 \end{Bmatrix}$$

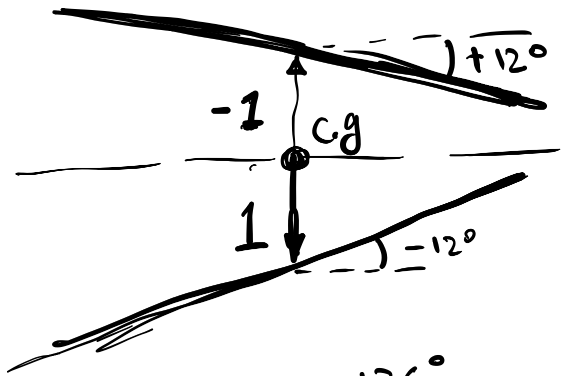
$\rightarrow -12^\circ$

$$\text{when} \begin{cases} x=1 \\ \theta = 1/0.42 \end{cases} \xRightarrow{\text{mode shape}} \begin{Bmatrix} 1 \\ 1/0.42 \end{Bmatrix}$$

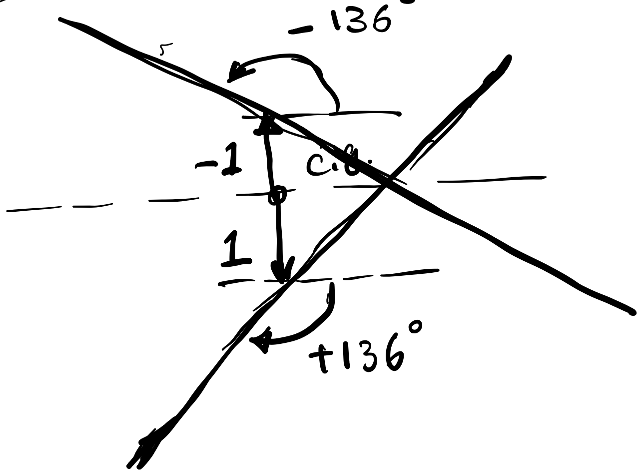
$\rightarrow +136^\circ$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 \\ -1/4.69 \end{bmatrix} A_{11} \sin(6.83t + \varphi_1) + \begin{Bmatrix} 1 \\ 1/0.42 \end{Bmatrix} A_{12} \sin(9.4t + \varphi_2)$$

Mode 1:



Mode 2:



Example

If m and I are the mass and the moment of inertia of the sprung mass, develop the equations of motion.

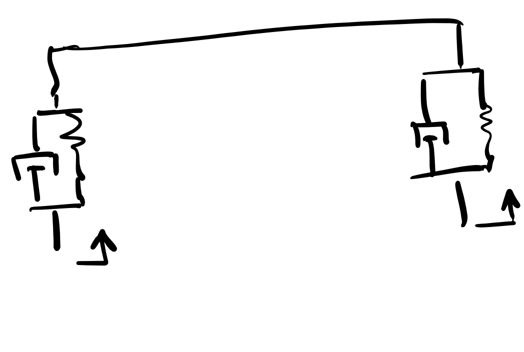
- The natural frequency of vertical motion of the unsprung mass is much higher than the natural frequency of the sprung mass.

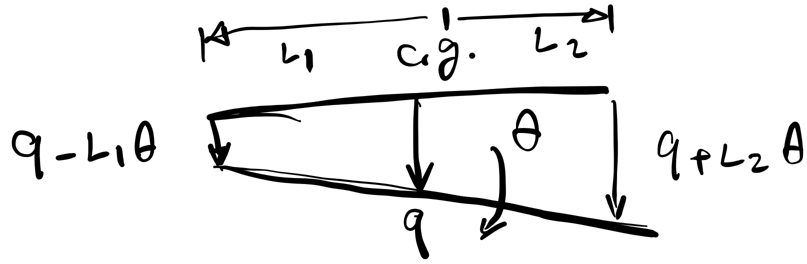
- At sufficiently small frequencies, the disturbances are transmitted directly to the sprung mass and the degrees of freedom of the unsprung mass (tire) can be neglected.

Assuming the sprung mass (body) to be rigid, two degress of freedom are assumed:

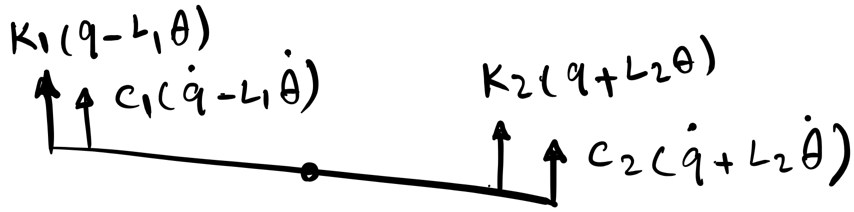
(a) vertical bounce q

(b) pitch θ





F.B.D.



$$m\ddot{q} + c_1(\dot{q} + L_1\dot{\theta}) + c_2(\dot{q} + L_2\dot{\theta}) + K_1(q - L_1\theta)$$

$$+ K_2(q + L_2\theta) = 0$$

$$I\ddot{\theta} + c_2(\dot{q} + L_2\dot{\theta})L_2 - c_1(\dot{q} - L_1\dot{\theta}_1)L_1$$

$$+ K_2(q + L_2\theta)L_2 - K_1(q - L_1\theta)L_1 = 0$$

$$[M]\begin{Bmatrix}\ddot{q}\\ \ddot{\theta}\end{Bmatrix} + [K]\begin{Bmatrix}q\\ \theta\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$