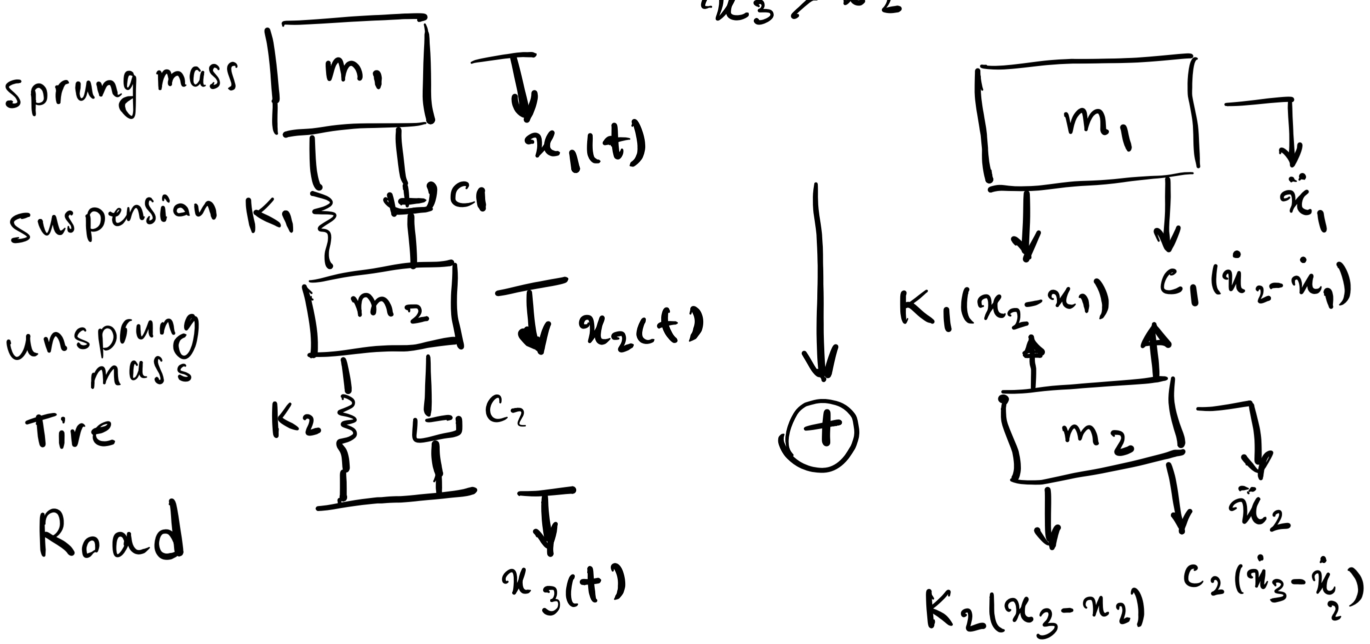
Module 3 - Quarter Car Model - 2DOF

2DOF suspension

$x_2 > x_1$

$x_3 > x_2$

F.B.D.



$$\Sigma F = m\ddot{x}$$

$$\begin{cases} K_1(x_2-x_1) + c_1(\dot{x}_2-\dot{x}_1) = m_1\ddot{x}_1 \\ K_2(x_3-x_2) + c_2(\dot{x}_3-\dot{x}_2) - K_1(x_2-x_1) \\ \quad - c_1(\dot{x}_2-\dot{x}_1) = m\ddot{x}_2 \end{cases}$$

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + K_1x_1 - K_1x_2 - c_2\dot{x}_2 = 0 \\ m_2\ddot{x}_2 + (c_1+c_2)\dot{x}_2 + (K_1+K_2)x_2 + c_1\dot{x}_1 + K_1x_1 \\ \qquad = c_2\dot{x}_3 + K_2x_3 \end{cases}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1+c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1+K_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

$$= \begin{bmatrix} 0 \\ c_2 \dot{x}_3 + K_2 x_3 \end{bmatrix}$$

$$\boxed{(j\omega)(j\omega) = -\omega^2}$$

$$\{x\} = \{\bar{X} e^{j\omega t}\} = \{\bar{X}\} e^{j\omega t}$$

$$\{\dot{x}\} = \{\bar{X}\} j\omega e^{j\omega t} \rightarrow \{\ddot{x}\} = -\{\bar{X}\} \omega^2 e^{j\omega t}$$

$$-\omega^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{Bmatrix} + j\omega \begin{bmatrix} c_1 & -c_2 \\ -c_2 & c_1+c_2 \end{bmatrix} \begin{Bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1+K_2 \end{bmatrix} \begin{Bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ (K_2 - j\omega c_2) \bar{X}_3 \end{Bmatrix}$$

$$\begin{bmatrix} K_1 - m^2 m_1 + j w c_1 & -k_1 - j w c_1 \\ -k_1 - j w c_1 & (k_1 + k_2) - \omega^2 m_2 + j\omega(c_1 + c_2) \end{bmatrix} \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ (k_2 - j\omega c_2)\bar{X}_3 \end{bmatrix}$$

$$\boxed{c_2 = 0}$$

$$[z_{ij}]\{\bar{X}\} = \{\bar{F}\}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = [z_{ij}(\omega)]^{-1}\{\bar{F}\}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1} \begin{bmatrix} \bar{F}_1 \\ \bar{F}_2 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{1}{z_{22} z_{11} - z_{21} z_{12}} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix} \begin{bmatrix} \bar{F}_1 \\ \bar{F}_2 \end{bmatrix}$$

$$\bar{X}_1 = \frac{Z_{22}\bar{F}_1 - Z_{12}\bar{F}_2}{|Z(w)|}$$

$$\Rightarrow \frac{(K_1 + jwc_1)K_2}{|Z(w)|} = \frac{\bar{X}_1}{\bar{X}_3}$$

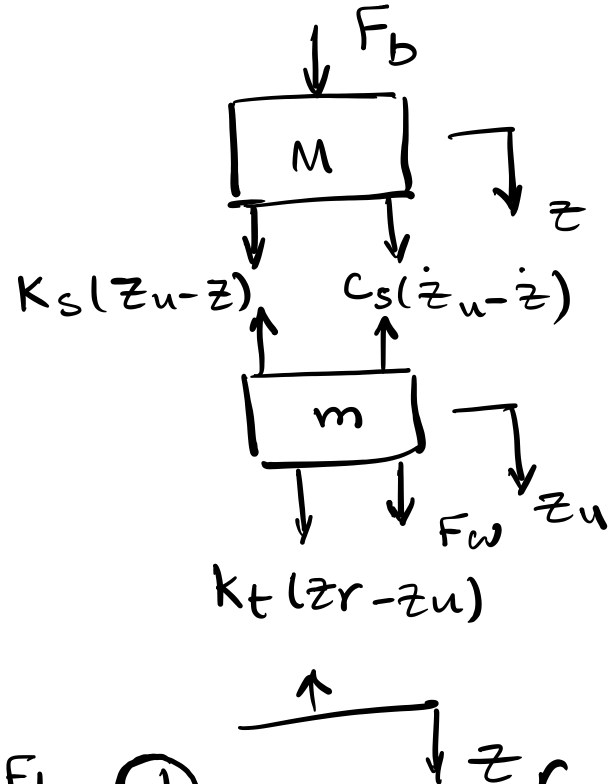
Response of the vehicle body

$$\bar{X}_2 = \frac{-Z_{21}\bar{F}_1 + Z_{11}\bar{F}_2}{|Z(w)|}$$

$$\Rightarrow \frac{\bar{X}_2}{\bar{X}_3} = \frac{\{(K_1 - w^2 m_1) - jwc_1\}K_2}{|Z(w)|}$$

Response of suspension

Example 2DOF:



$$M\ddot{z} = k_s(z_u - z)$$
$$+ c_s(\dot{z}_u - \dot{z}) + F_b$$

$$M\ddot{z} = c_s\dot{z}_u - c_s\dot{z}$$
$$+ k_s z_u - k_s z + F_b$$

$$M\ddot{z} + c_s\dot{z} + k_s z$$
$$= c_s\dot{z}_u + k_s z_u + F_b \quad (1)$$

$$m\ddot{z}_u = k_t(z_r - z_u) - k_s(z_u - z) - c_s(\dot{z}_u - \dot{z}) + F_w$$
$$= k_t z_r - k_t z_u - k_s z_u + k_s z - c_s\dot{z}_u$$
$$+ c_s\dot{z} + F_w$$

$$m\ddot{z}_u + c_s\dot{z}_u + (k_t + k_s)z_u = c_s\dot{z} + k_s z + k_t z_r + F_w \quad (2)$$

① & ②

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{z} \\ \ddot{z}_u \end{Bmatrix} + \begin{bmatrix} c_s & 0 \\ 0 & c_s \end{bmatrix} \begin{Bmatrix} \dot{z} \\ \dot{z}_u \end{Bmatrix}$$

$$+ \begin{bmatrix} k_s & 0 \\ 0 & k_t + k_s \end{bmatrix} \begin{Bmatrix} z \\ z_u \end{Bmatrix}$$

$$= \begin{Bmatrix} c_s \dot{z}_u + k_s z_u + F_b \\ c_s \dot{z} + k_s z + k_t z_r + F_w \end{Bmatrix}$$

$$\{z\} = \{\bar{z}\} e^{j\omega t}$$

$$\{\dot{z}\} = j\omega \{\bar{z}\} e^{j\omega t}$$

$$\{\ddot{z}\} = -\omega^2 \{\bar{z}\} e^{j\omega t}$$

$$\begin{bmatrix} K_s + j\omega c_s - \omega^2 M & 0 \\ 0 & K_t + K_s + j\omega c_s - \omega^2 m \end{bmatrix} \begin{Bmatrix} \bar{Z} \\ \bar{Z}_u \end{Bmatrix} e^{j\omega t}$$

$$= \begin{Bmatrix} \bar{F}_1 \\ \bar{F}_2 \end{Bmatrix}$$

Example:

$$\begin{bmatrix} K_1 - \omega^2 m_1 & -K_1 \\ -K_1 & (K_1 + K_2) - \omega^2 m_2 \end{bmatrix} \begin{pmatrix} \bar{X}_1 \\ \bar{X}_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\omega^2 = \lambda$$

Assume the neumerical problem as follows:

$$\begin{vmatrix} 2-\lambda & 3 \\ 4 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$\lambda = -2 \quad \text{or} \quad 5$$

$$\lambda = -2 \Rightarrow \begin{bmatrix} 4 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$4x_1 + 3x_2 = 0 \Rightarrow x_2 = -\tfrac{4}{3} x_1$$

$$\{x_1\} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix} \text{ or } \begin{bmatrix} 6 \\ -8 \end{bmatrix} \text{ or } \begin{bmatrix} 9 \\ -12 \end{bmatrix}$$

$$\{x_1\} = \alpha \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\lambda = 5 \Rightarrow x = \beta \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$[A]\{x\} = \lambda\{x\}$$

$$[A - \lambda I]\{x\} = \{0\}$$

$$|A - \lambda I| = 0 \Rightarrow \lambda = \lambda_1, \lambda_2$$