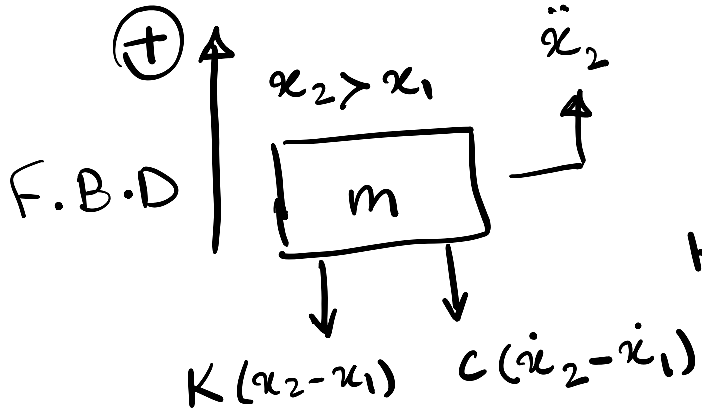
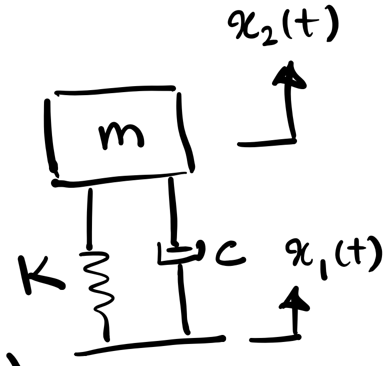
Quarter car model - one degree of freedom

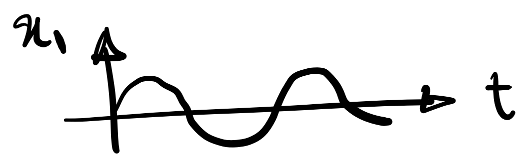




$$\sum F = m\ddot{x}_2$$

$$-K(x_2-x_1) - c(\dot{x}_2-\dot{x}_1) = m\ddot{x}_2$$

$$m\ddot{x}_2 + c\dot{x}_2 + Kx_2 = Kx_1 + c\dot{x}_1$$



$$x_1 = X e^{j\omega t - j\phi}$$

$$x_1 = \bar{X}_1 e^{j\omega t} \qquad x_2 = \bar{X}_2 e^{j\omega t}$$

$$\bar{X}_1 = X_1 e^{-j\phi} \qquad \bar{X}_2 = X_2 e^{j\gamma}$$

$$\omega_n^2 = \frac{K}{m} \qquad \frac{c}{K} = \frac{2\zeta}{\omega_n} \qquad r = \frac{\omega}{\omega_n} \qquad \zeta = \frac{c}{2\sqrt{Km}}$$

$$(K - \omega^2 + j\omega c)\bar{X}_2 = (K + j\omega c)\bar{X}_1$$

$$\frac{X_2}{X_1} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\gamma - \phi = \tan^{-1} 2\xi r - \tan^{-1} \frac{2\xi r}{1-r^2}$$

Frequency: $\omega = \frac{2\pi}{\tau}$ → period

spacial frequency $f_t = f_s \times \text{speed}$

freq. $= 2\pi\omega$

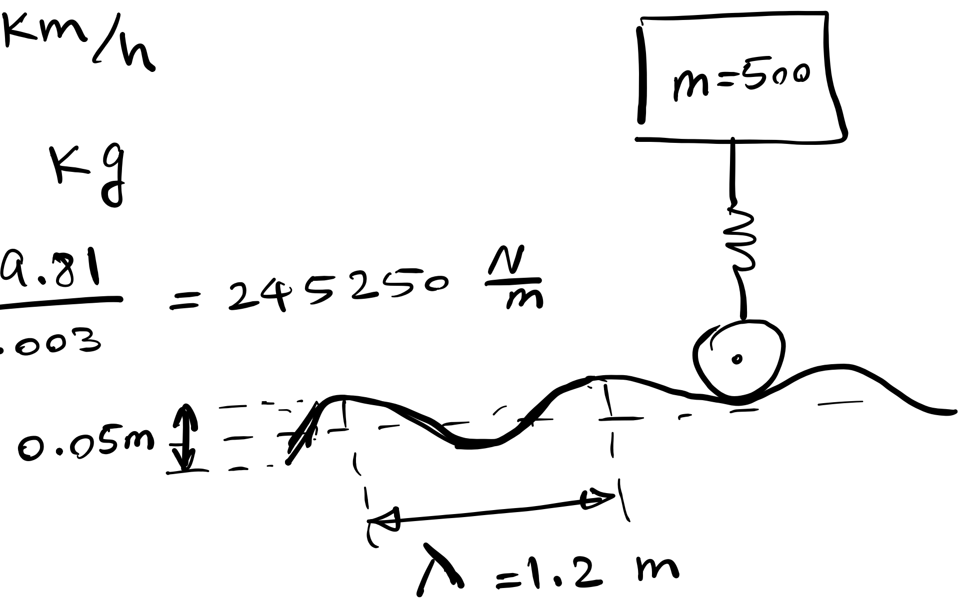
Example

Determine the amplitude of virtical vibrations.

$V = 25 \ Km/h$

$m = 500 \ Kg$

$K = \frac{75 \times 9.81}{0.003} = 245250 \ N/m$



$w_n^2 = \frac{K}{m} = \frac{245250}{500} = 490$

$\frac{X_2}{X_1} = \frac{1}{1-r^2}$

f_t = Excitation frequency =

$\left(\frac{25,000}{3600} \right) \times \frac{1}{1.2} = 5.78 \ Hz = 36.36 \ rad/s$

$$r^2 = \left(\frac{\omega}{\omega_n}\right)^2 = \frac{(36.36)^2}{490} = 2.7$$

$$X_1 = \frac{0.05}{2} = 0.025 \text{ m}$$

$$X_2 = \frac{X_1}{1-r^2} = \frac{0.025}{1-2.7} = 0.0147 \text{ m}$$

$$= 14.75 \text{ mm}$$

critical speed is when excitation frequency $\omega = \omega_n$

$$V_c = \lambda \left(\frac{\omega_n}{2\pi}\right) = 1.2 \sqrt{490}/2\pi = 4.23 \ \frac{m}{s}$$

$$= 15.23 \ \frac{Km}{h}$$

Example 2:

$m = 1000$ Kg fully loaded

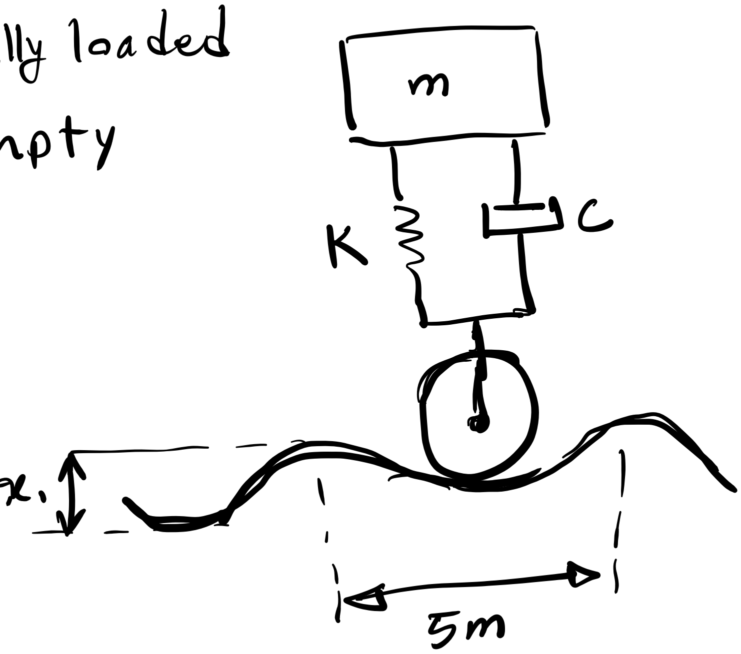
$m = 250$ Kg empty

$K = 350$ KN/m

$\zeta = 0.5$

$V = 100$ Km/h

$f_s = \frac{1}{5}$ cycles/meter



Determine the amplitude ratio of the vehicle when fully loaded and empty.

$$\text{Excitation freq.} = \frac{100,000}{3600} \times \frac{1}{5} = 5.56 \text{ Hz}$$

C and K are constants.

$$c = 2\zeta\sqrt{Km}$$

$$\zeta_1 = \frac{c}{2\sqrt{km}} = 0.5$$

$$\zeta_2 = \frac{c}{2\sqrt{\frac{km}{4}}} = \frac{c}{\sqrt{km}} = 1$$

$$\Rightarrow \zeta_2 = 2\zeta_1$$

	Fully loaded vehicle	Empty vehicle
ω_n	$\omega_n = 2.98$ Hz	$\omega_n = 5.96$
$r = \frac{\omega}{\omega_n}$	$5.56/2.98 = 1.87$	$r = \frac{5.56}{5.96} = 0.93$
$\frac{X_2}{X_1} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$	$\frac{X_2}{X_1} = 0.68$	$\frac{X_2}{X_1} = 1.13$