Mechatronics
ETM 4931
Lecture V

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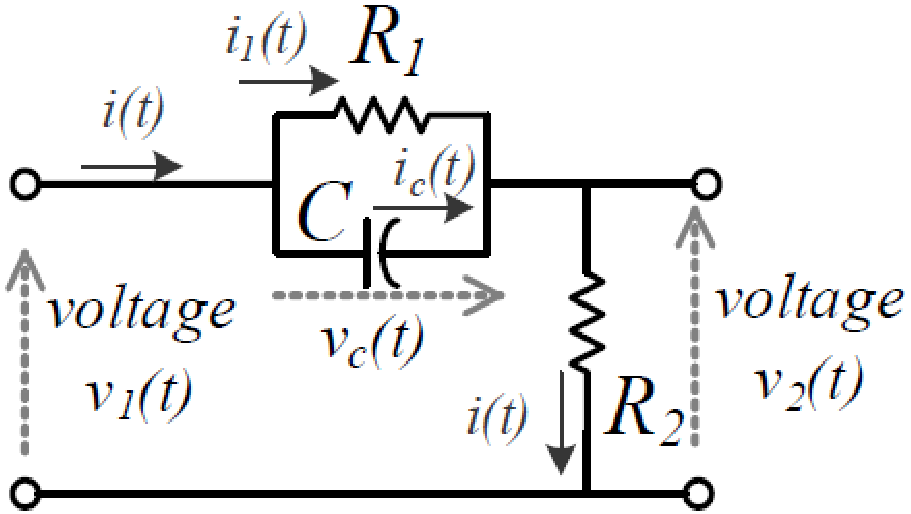
Phase Lead Compensator Modelling



$$v_c(t) = i_1(t)R_1$$

$$v_1(t) = v_c(t) + v_2(t)$$

$$v_c(t) = v_1(t) - v_2(t)$$

$$i_1(t) = i(t) - i_c(t)$$

$$i(t) = \frac{v_2(t)}{R_2}$$

$$i_c(t) = C\dot{v}_c(t) = C\dot{v}_1(t) - C\dot{v}_2(t)$$

Phase Lead Compensator Modelling

$$v_1(t) = R_1\left(\frac{v_2(t)}{R_2} - (C\dot{v}_1(t) - C\dot{v}_2(t))\right) + v_2(t)$$

$$R_2 v_1(t) = R_1 v_2(t) - R_2 C\dot{v}_1(t) + R_2 C\dot{v}_2(t) + R_2 v_2(t)$$

$$R_2 v_1(t) + R_1 R_2 C\dot{v}_1(t) = R_1 v_2(t) + R_2 v_2(t) + R_1 R_2 C\dot{v}_2(t)$$

Apply Laplace Transform and assume zero initial conditions.

$$\begin{aligned} R_2 V_1(s) + R_1 R_2 C s V_1(s) \\ = R_1 V_2(s) + R_2 V_2(s) + R_1 R_2 C s V_2(s) \end{aligned}$$

Phase Lead Compensator Modelling

$$\left(R_2 + R_1 R_2 Cs\right) V_1(s) = \left(R_1 + R_2 + R_1 R_2 Cs\right) V_2(s)$$

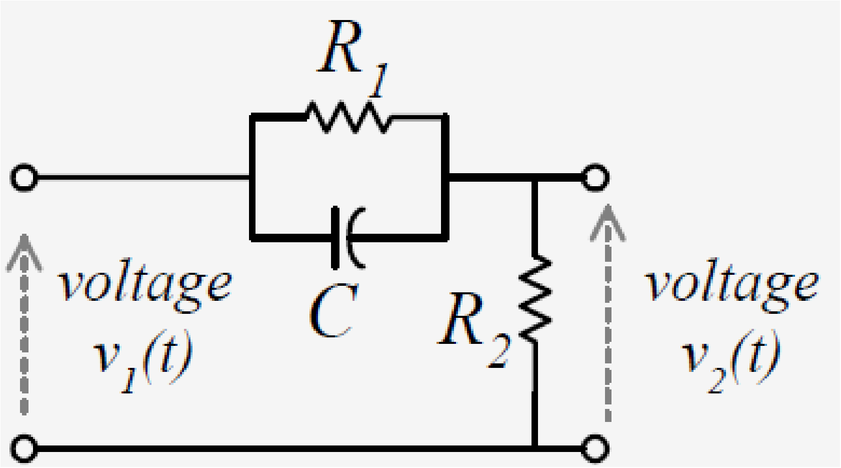
$$G_N(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2 + R_1 R_2 Cs}{R_1 + R_2 + R_1 R_2 Cs}$$

$$G_N(s) = \frac{R_2}{R_1 + R_2} \frac{(1 + R_1 Cs)}{1 + \frac{R_1 R_2 C}{R_1 + R_2} s}$$

$$\alpha \equiv \frac{R_1 + R_2}{R_2} \qquad T \equiv \frac{R_1 R_2 C}{R_1 + R_2}$$

$$G_N(s) = \left(\frac{1}{\alpha}\right)\left(\frac{1 + \alpha Ts}{1 + Ts}\right)$$

Phase Lead Compensator Modelling



$$G_N(s) = \frac{V_2}{V_1} = \frac{R_2}{R_2 + \frac{R_1 \frac{1}{CS}}{R_1 + \frac{1}{CS}}}$$

$$= \frac{R_2}{R_1 + R_2} \frac{(1 + R_1 Cs)}{1 + \frac{R_1 R_2 C}{R_1 + R_2} s}$$

Resistor:

$$v(t) = Ri(t)$$

$$\Rightarrow V(s) = \boxed{R}\ I(s)$$

Capacitor:

$$i(t) = C\frac{dv(t)}{dt}$$

$$\Rightarrow I(s) = CsV(s)$$

$$\Rightarrow V(s) = \boxed{\frac{1}{Cs}} I(s)$$

Phase Lead Compensator Modelling

$$G_N(s) = \frac{R_2}{R_1 + R_2}\frac{(1 + R_1 C s)}{1 + \frac{R_1 R_2 C}{R_1 + R_2} s} \quad \alpha \equiv \frac{R_1 + R_2}{R_2} \quad T \equiv \frac{R_1 R_2 C}{R_1 + R_2}$$

$$G_N(s) = \left(\frac{1}{\alpha}\right)\left(\frac{1 + \alpha T s}{1 + T s}\right)$$

$$R_1 = 395\Omega,\ R_2 = 30\Omega,\ C = 78.9\mu F$$

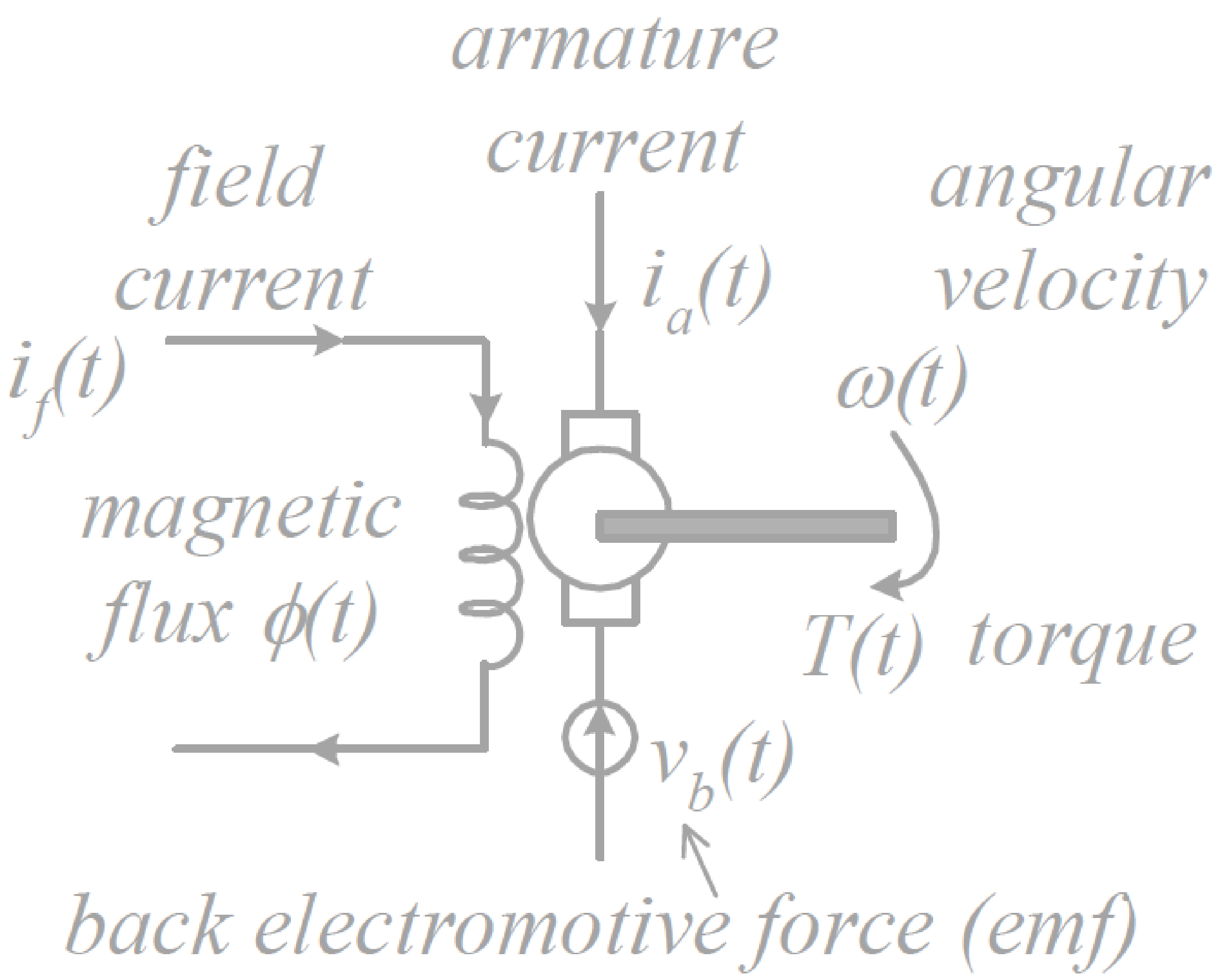
$$G_N(s) = \left(\frac{1}{14.2}\right)\left(\frac{1 + 0.0312s}{1 + 0.0022s}\right)$$

The RC network connected to a amplifier of gain *14.2* will give us the required compensator transfer function.

DC Motor, 1

- DC Motor:

The main function of a DC motor is to generate the Torque, so shaft can be rotated.



Videos – servo motors and stepper motors

- **What is a Servo Motor and How it Works?**
- https://www.youtube.com/watch?v=ditS0a28Sko
- **Servo Motor Advantages And Disadvantages**
- https://www.youtube.com/watch?v=zFe2JwP2NLw
- **What is a Stepper Motor and How it Works?**
- https://www.youtube.com/watch?v=VfqYN1eG9Zk
- **Stepper Motors Advantages and Disadvantages**
- https://www.youtube.com/watch?v=SlN2Hmsvxu4

DC Motor, 2

The torque developed by the motor is proportional to the *flux* and *armature current*:

$$T_m(t) = K_1 \phi i_a(t)$$

The flux is proportional to the *field current* $i_f(t)$:

$$\phi = K_f i_f(t)$$

Finally, the torque is:

$$T_m(t) = K_1 K_f i_a(t) i_f(t)$$

DC Motor, 3

$$T_m(t) = K_1 K_f \, i_a(t) i_f(t)$$

- Field current controlled DC Motor:
  - Fix the armature current $i_a(t)$, we have the ***field current controlled*** DC motor.

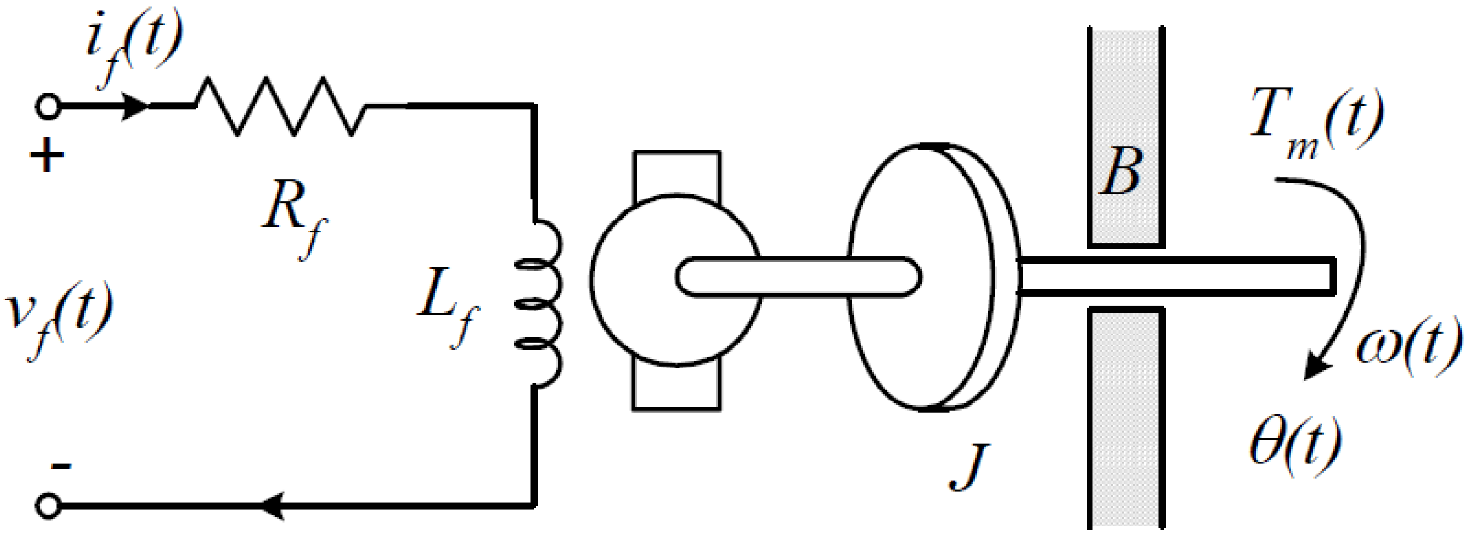
$$T_m(t) = \left[K_1 K_f \, i_a(t) \right] i_f(t) = K_m i_f(t)$$

- Armature current controlled DC Motor:
  - Fix the field current $i_f(t)$, we have the ***armature current controlled*** DC motor.

$$T_m(t) = \left[K_1 K_f \, i_f(t) \right] i_a(t) = K_a i_a(t)$$

System with field current controlled DC motor, 1

$$T_m(t) = K_m i_f(t)$$



Input: field voltage $v_f(t)$

Output: angular displacement $\theta(t)$

The field circuit: $$v_f(t) = R_f i_f(t) + L_f \frac{di_f(t)}{dt}$$

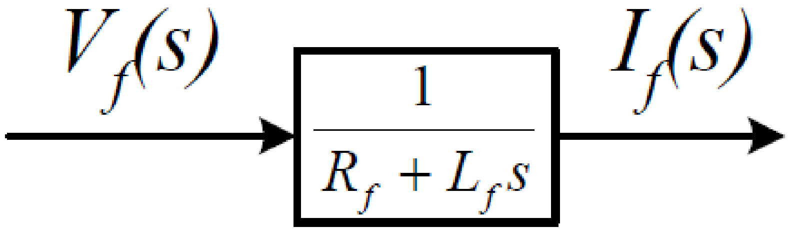
System with field current controlled DC motor, 2

The field circuit: $v_f(t) = R_f i_f(t) + L_f \dfrac{di_f(t)}{dt}$

$\mathcal{L}$

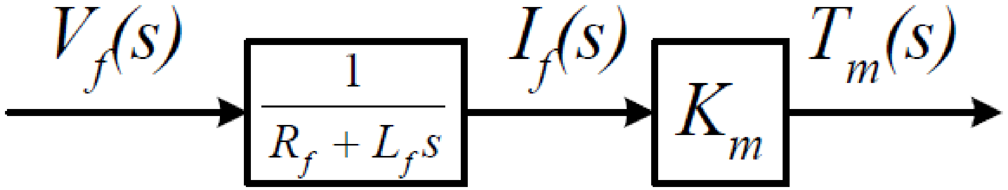
$$V_f(s) = R_f I_f(s) + L_f s I_f(s) \quad \Rightarrow I_f(s) = \frac{1}{R_f + L_f s} V_f(s)$$

Transfer function (TF): $\dfrac{I_f(s)}{V_f(s)} = \dfrac{1}{R_f + L_f s}$



System with field current controlled DC motor, 3

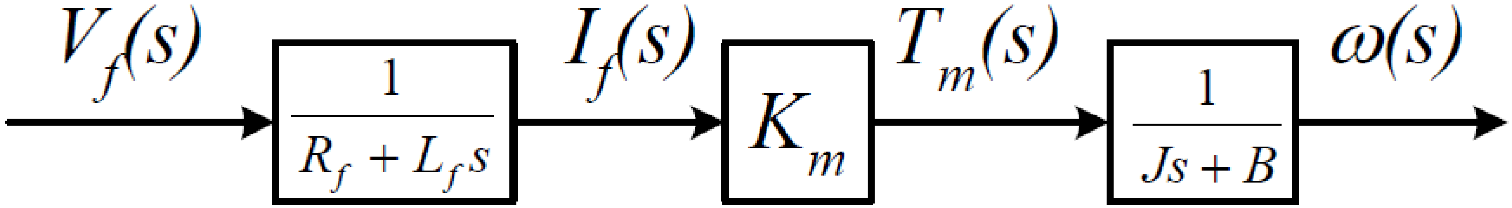
Torque: $T_m(t) = K_m i_f(t) \xrightarrow{\mathcal{L}} T_m(s) = K_m I_f(s)$



The load: $T_m(t) - B\omega(t) = J\dfrac{d\omega(t)}{dt} \xrightarrow{\mathcal{L}} T_m(s) - B\omega(s) = Js\omega(s)$

$$\omega(s) = \frac{1}{Js+B} T_m(s)$$

$$\frac{\omega(s)}{T_m(s)} = \frac{1}{Js+B}$$ TF



System with field current controlled DC motor, 4

The angular displacement:

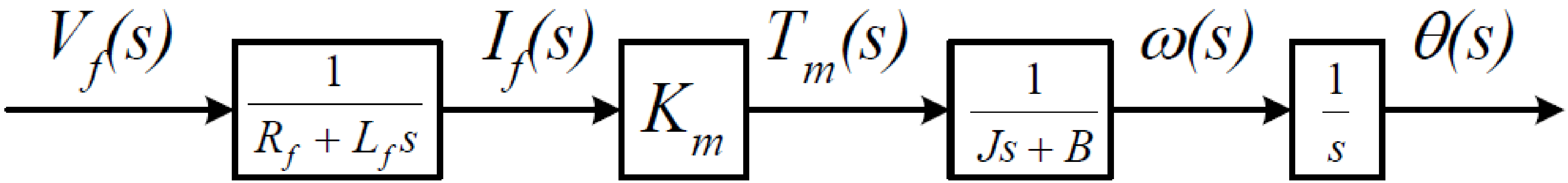
$$\omega(t) = \frac{d\theta(t)}{dt}$$

$\mathcal{L}$

$$\omega(s) = s\theta(s)$$

$$\theta(s) = \frac{1}{s}\omega(s) \longrightarrow \frac{\theta(s)}{\omega(s)} = \frac{1}{s}$$

TF

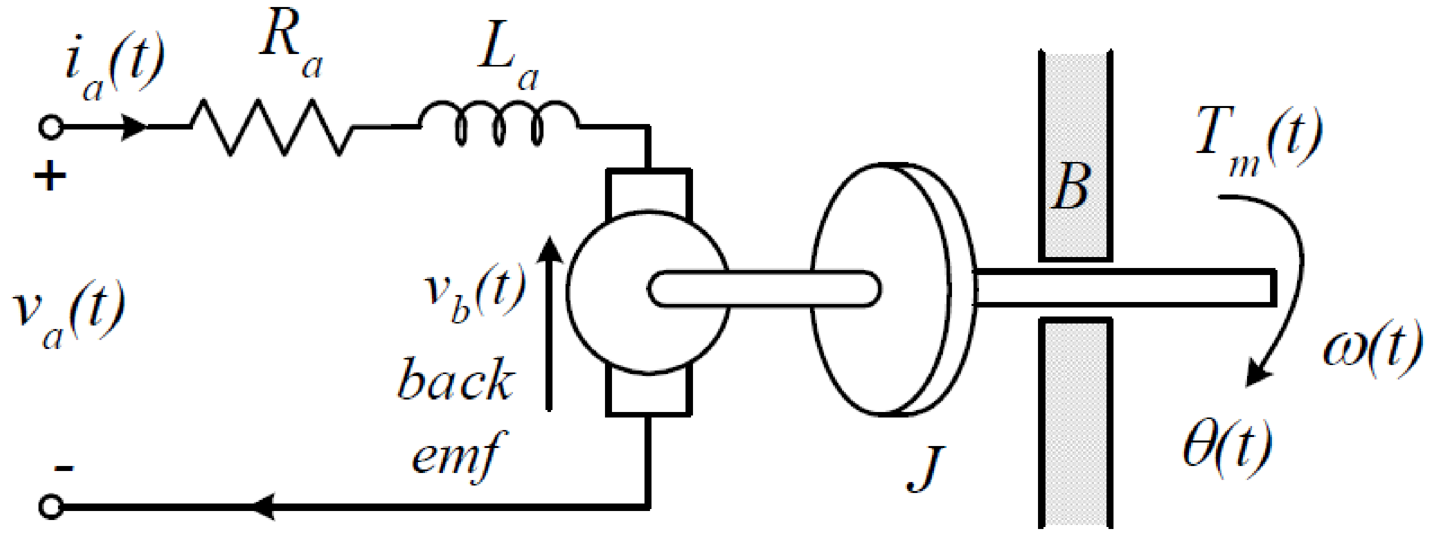


Linking all components together, the overall TF:

$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s(Js + B)(L_f s + R_f)}$$

Armature current controlled DC motor system, 1

$$T_m(t) = K_a i_a(t)$$



Input: voltage $v_a(t)$

Output: angular displacement $\theta(t)$

The armature circuit:

$$v_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t)$$

Armature current controlled DC motor system, 2

The armature circuit:

$$v_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t)$$

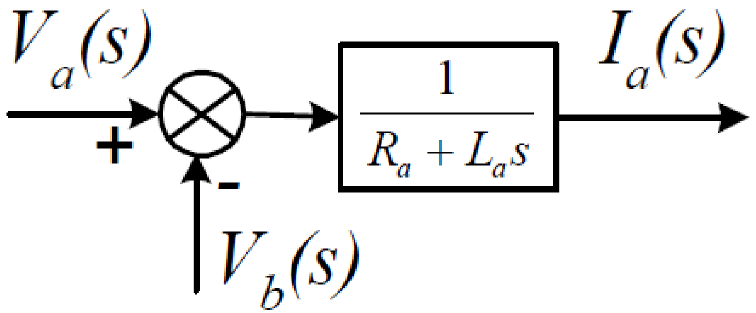
$\mathcal{L}$

$$V_a(s) = R_a I_a(s) + L_a s I_f(s) + V_b(s)$$

$$\Rightarrow I_a(s) = \frac{1}{R_a + L_a s}\left(V_a(s) - V_b(s)\right)$$

Transfer function:

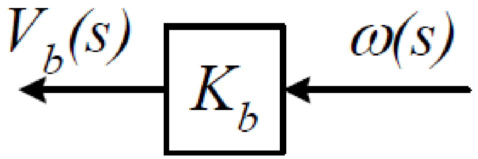
$$\frac{I_a(s)}{V_a(s) - V_b(s)} = \frac{1}{R_a + L_a s}$$



Back *emf*: $v_b(t) = K_b \omega(t)$

$\mathcal{L}$

$$V_b(s) = K_b \omega(s)$$



DC Motors Datasheet Example

gn tools chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://www.moog.com/content/dam/moog/literatur...

moog.com / moc23series

Share AI Assistant

Brush Motors

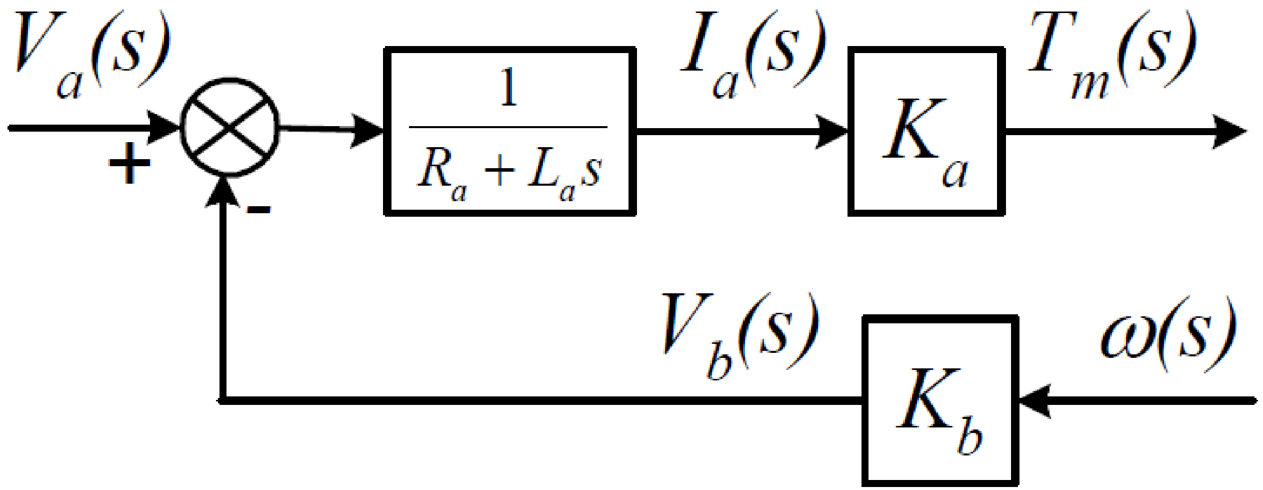
C23 SERIES SPECIFICATIONS – *Continuous Stall Torque 16.5 - 27 oz-in (0.117 -* *Peak Torque 125 - 250 oz-in (0.883 - 1.765 Nm,*

Part Number*		C23-L33					C23-L40				
Winding Code**		10	20	30	40	50	10	20	30	40	50
L = Length	inches	3.33					4				
	millimeters	84.6					101.6				
Peak Torque	oz-in	125.0	125.0	125.0	125.0	125.0	250.0	250.0	250.0	250.0	250.0
	Nm	0.883	0.883	0.883	0.883	0.883	1.765	1.765	1.765	1.765	1.765
Continuous Stall Torque	oz-in	16.5	16.5	16.5	16.5	16.5	27.0	27.0	27.0	27.0	27.0
	Nm	0.117	0.117	0.117	0.117	0.117	0.191	0.191	0.191	0.191	0.191
Rated Terminal Voltage	volts DC	12 -24	12 - 24	12 -36	12 - 60	12 - 60	12 - 24	12 - 48	12 - 60	12 - 60	12 - 60
Terminal Voltage	volts DC	12	12	24	36	48	12	24	36	48	60
Rated Speed	RPM	4700	2150	4200	3750	3000	2300	3600	3500	2850	2250
	rad/sec	492	225	440	393	314	241	377	367	298	236
Rated Torque	oz-in	7.5	12.6	12.7	14.4	15.8	17.3	25.5	25.3	25.6	24.2
	Nm	0.05	0.09	0.09	0.10	0.11	0.12	0.18	0.18	0.18	0.17
Rated Current	Amps	4.75	4.3	3	2	1.4	4.9	4.3	2.75	1.8	1.1
Rated Power	Watts	26.1	20.0	39.5	40.0	35.1	29.4	67.9	65.5	54.0	40.3
	Horsepower	0.03	0.03	0.05	0.05	0.05	0.04	0.09	0.09	0.07	0.05
Torque Senstivity	oz-in/amp	2.65	4.25	6.2	10.25	15.75	4.84	7.74	12	18.5	28.75
	Nm/amp	0.0187	0.0300	0.0438	0.0724	0.1112	0.0342	0.0547	0.0847	0.1306	0.2030
Back EMF	volts/KRPM	2	3.15	4.6	7.6	11.5	3.58	5.72	8.82	13.82	21.22
	volts/rad/sec	0.0191	0.0301	0.0439	0.0726	0.1098	0.0342	0.0546	0.0842	0.1320	0.2026
Terminal Resistance	ohms	0.60	1.00	1.70	4.00	9.00	0.70	0.96	2.30	5.50	12.00

https://www.moog.com/content/dam/moog/literature/MCG/moc23series.pdf

Armature current controlled DC motor system, 3

Torque: $T_m(t) = K_a i_a(t) \xrightarrow{\mathcal{L}} T_m(s) = K_a I_a(s)$



Armature current controlled DC motor system, 4

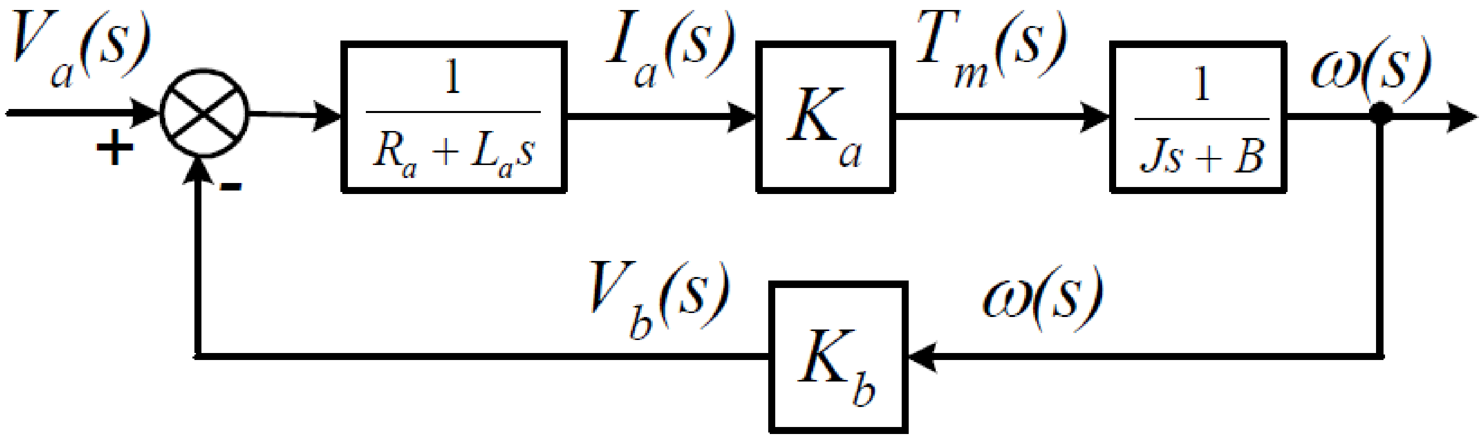
The load: $$T_m(t) - B\omega(t) = J\frac{d\omega(t)}{dt}$$

$\mathcal{L}$

$$T_m(s) - B\omega(s) = Js\omega(s)$$

$$\omega(s) = \frac{1}{Js+B}T_m(s)$$

$$\frac{\omega(s)}{T_m(s)} = \frac{1}{Js+B}$$ TF



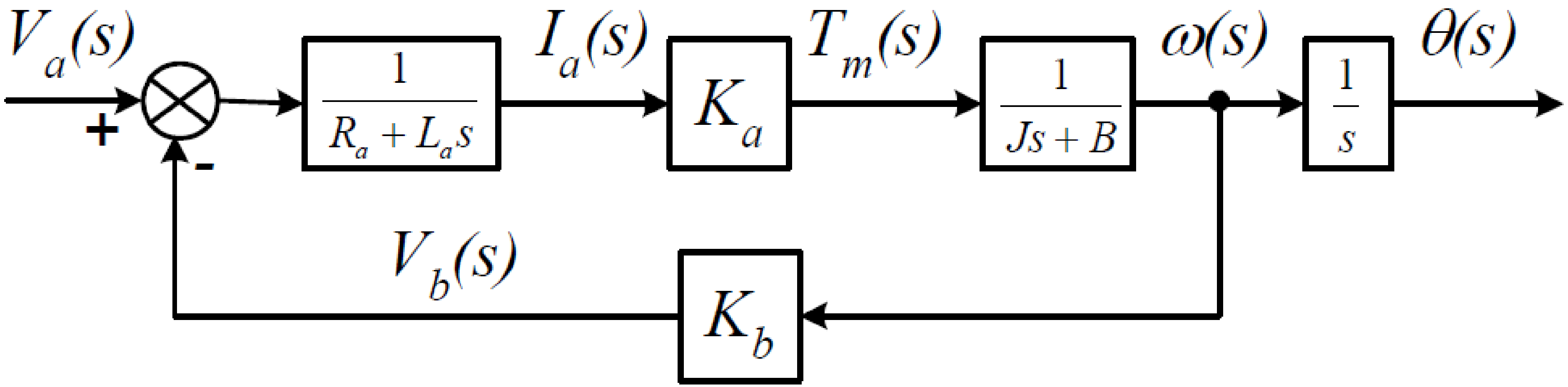
Armature current controlled DC motor system, 5

The angular displacement:

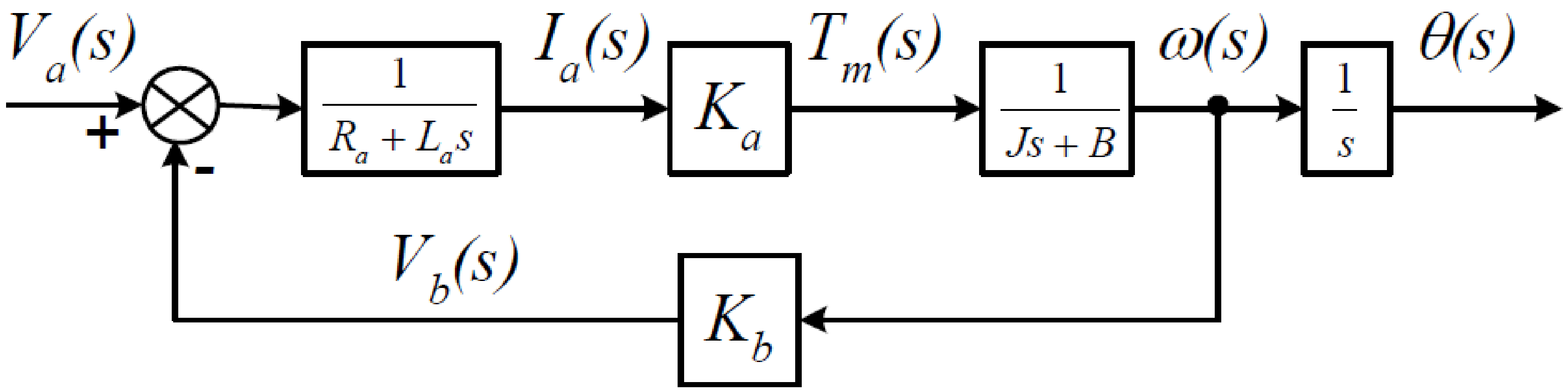
$$\omega(t) = \frac{d\theta(t)}{dt} \xrightarrow{\mathcal{L}} \omega(s) = s\theta(s)$$

TF

$$\frac{\theta(s)}{\omega(s)} = \frac{1}{s} \longleftarrow \theta(s) = \frac{1}{s}\omega(s)$$



Armature current controlled DC motor system, 6

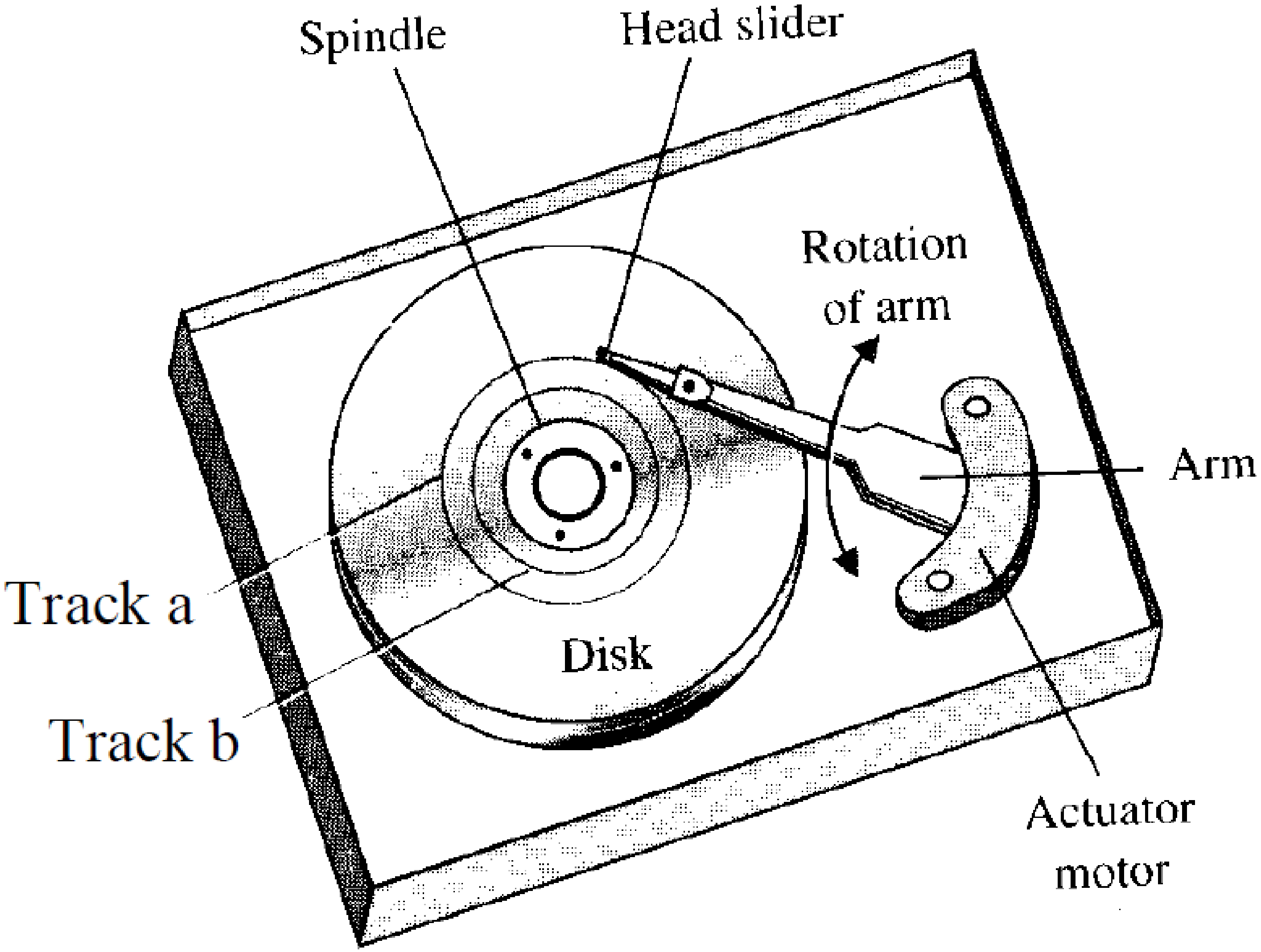


Linking all components together, the overall TF:

$$\frac{\theta(s)}{V_a(s)} = \frac{\dfrac{K_a}{(R_a + L_a s)(Js + B)}}{1 + \dfrac{K_a}{(R_a + L_a s)(Js + B)} \cdot K_b} \cdot \frac{1}{s}$$

$$= \frac{K_a}{s\left[(R_a + L_a s)(Js + B) + K_a K_b\right]}$$

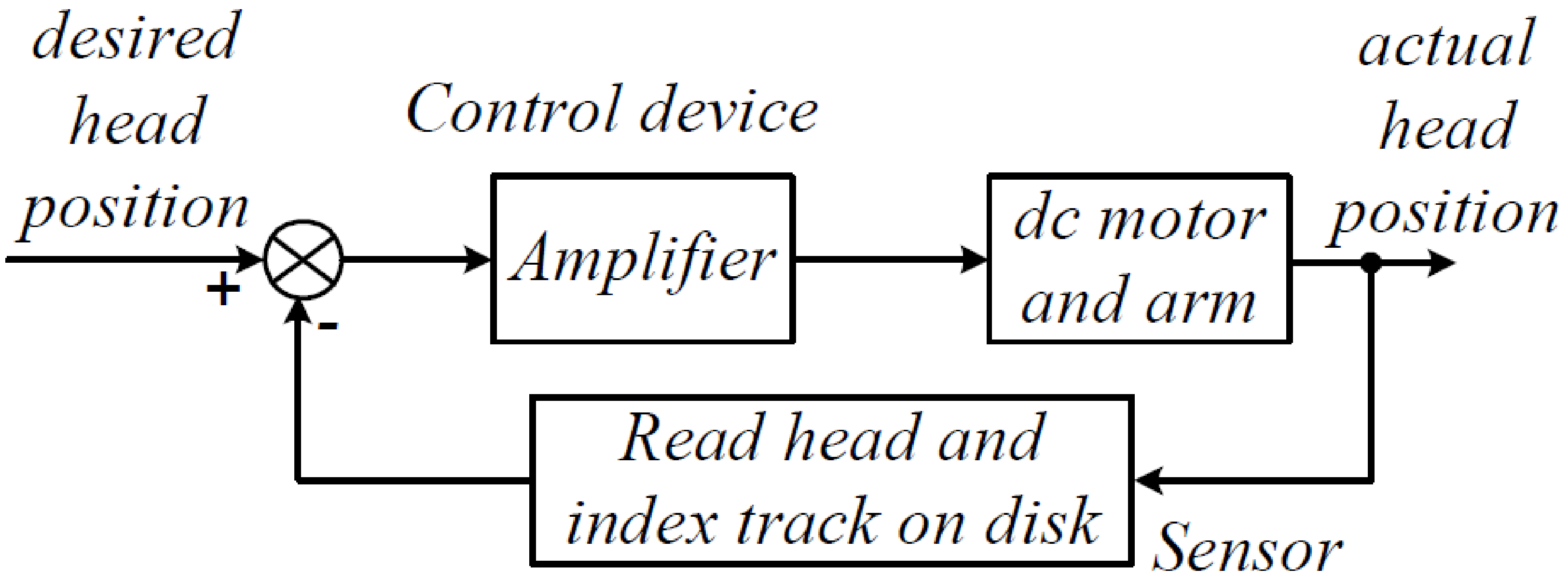
A Real World Example: Hard disk (or CD player) drive control system



PID videos

- What is a PID Controller?
- https://www.youtube.com/watch?v=sFqFrmMJ-sg
- What are PID Tuning Parameters?
- https://www.youtube.com/watch?v=1ImhKwpSmuc
- How to Tune a PID Controller
- https://www.youtube.com/watch?v=IB1Ir4oCP5k
- DC motor PID speed control
- https://www.youtube.com/watch?v=HRaZLCBFVDE
- How to control a DC motor with an encoder
- https://www.youtube.com/watch?v=dTGITLnYAY0

Disk drive control system: schematic diagram



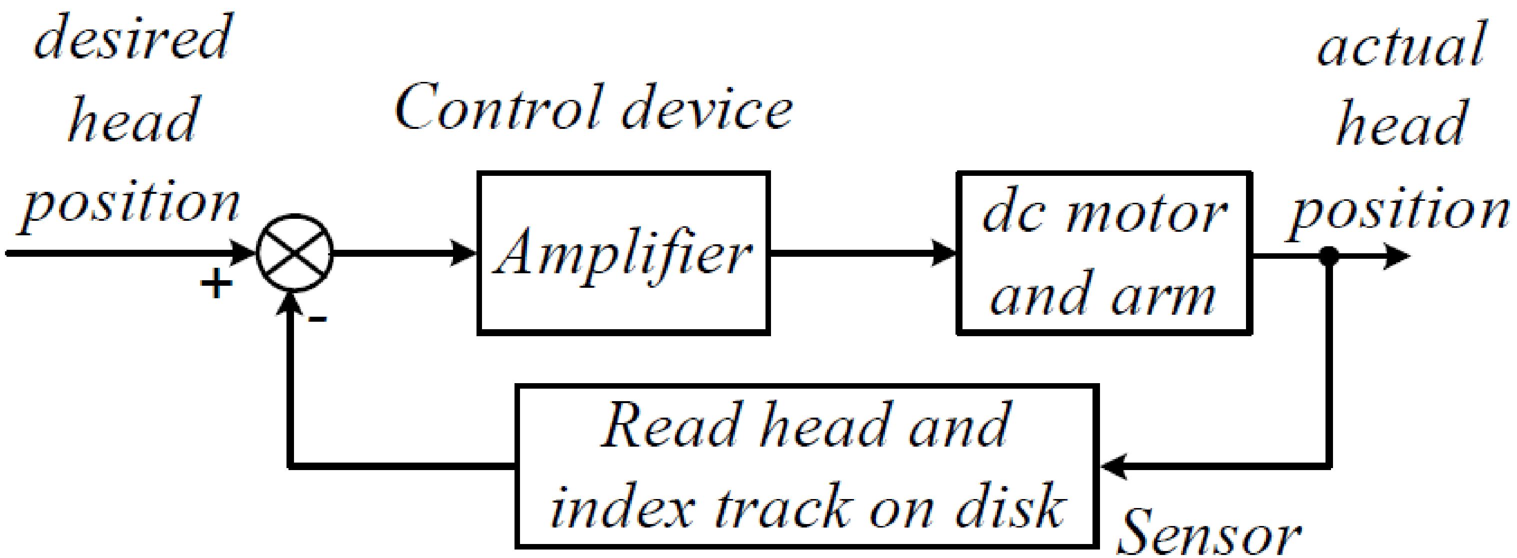
Modern hard drive: seek time *8.9ms*

When we put all components together, how to check the required performance is achieved? If not, how to improve it?

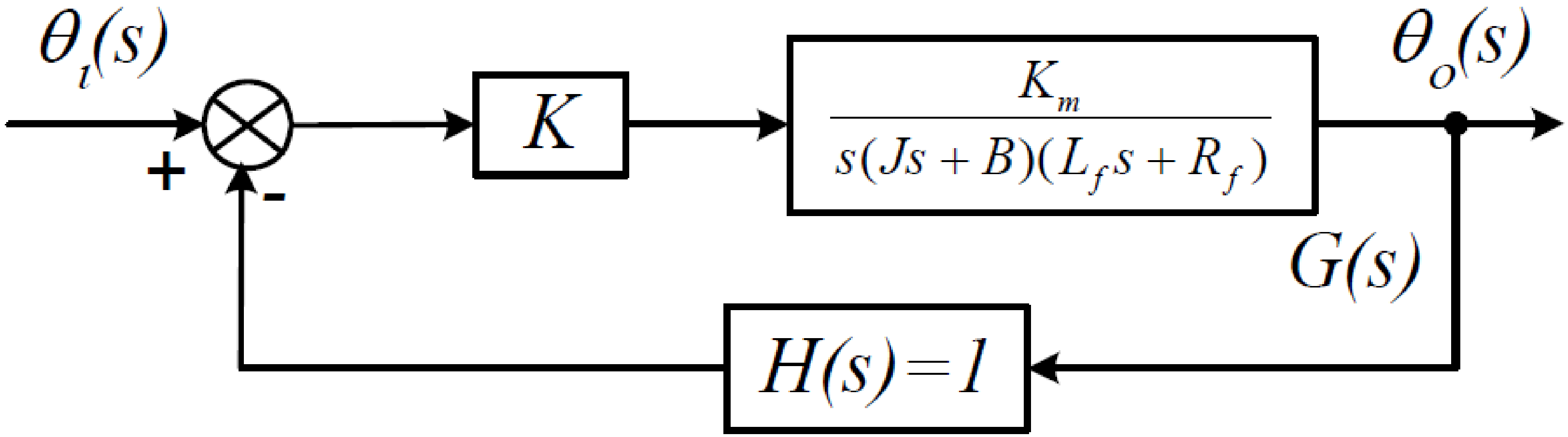
If we have a transfer function, we can find the settling time and the way of improving it.

PO also should be small, less vibration, less noise.

Modelling of disk drive control system, 1



Use the field current controlled DC motor.



Modelling of disk drive control system, 2

Typical parameters for disk driver reader:

J	$1\ Nms^2/rad$	B	$20kg/m/s$	K	$10\text{-}1000$
R_f	1Ω	K_m	$5Nm/A$	L_f	$0.001\ H$

Motor and arm:

$$G(s) = \frac{K_m}{s(Js+B)(L_f s + R_f)}$$

$$= \frac{5}{s(s+20)(0.001s+1)}$$

Modelling of disk drive control system, 3

$$G(s) = \frac{5}{s(s+20)(0.001s+1)}$$

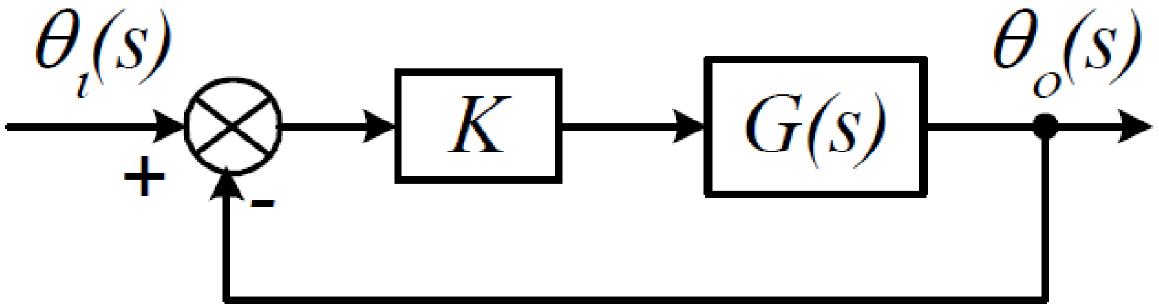
$$= \frac{0.25}{s} \cdot \frac{1}{(0.05s+1)} \cdot \frac{1}{(0.001s+1)}$$

$$\approx = \frac{0.25}{s} \cdot \frac{1}{(0.05s+1)} = \frac{0.25}{s(0.05s+1)} = \frac{5}{s(s+20)}$$

$$\frac{1}{(0.05s+1)}$$ Dynamic model of mechanical part (slow): *transient response develop slowly.*

$$\frac{1}{(0.001s+1)}$$ Dynamic model of electrical part (fast): *transient response disappear quickly.*

Modelling of disk drive control system, 4



$$T(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{KG(s)}{1 + KG(s)}$$

$$= \begin{cases} \dfrac{5K}{s^2 + 20s + 5K} & \textit{approximation} \\[2ex] \dfrac{5K}{0.0001s^3 + 1.002s^2 + 20s + 5K} & \textit{no approximation} \end{cases}$$

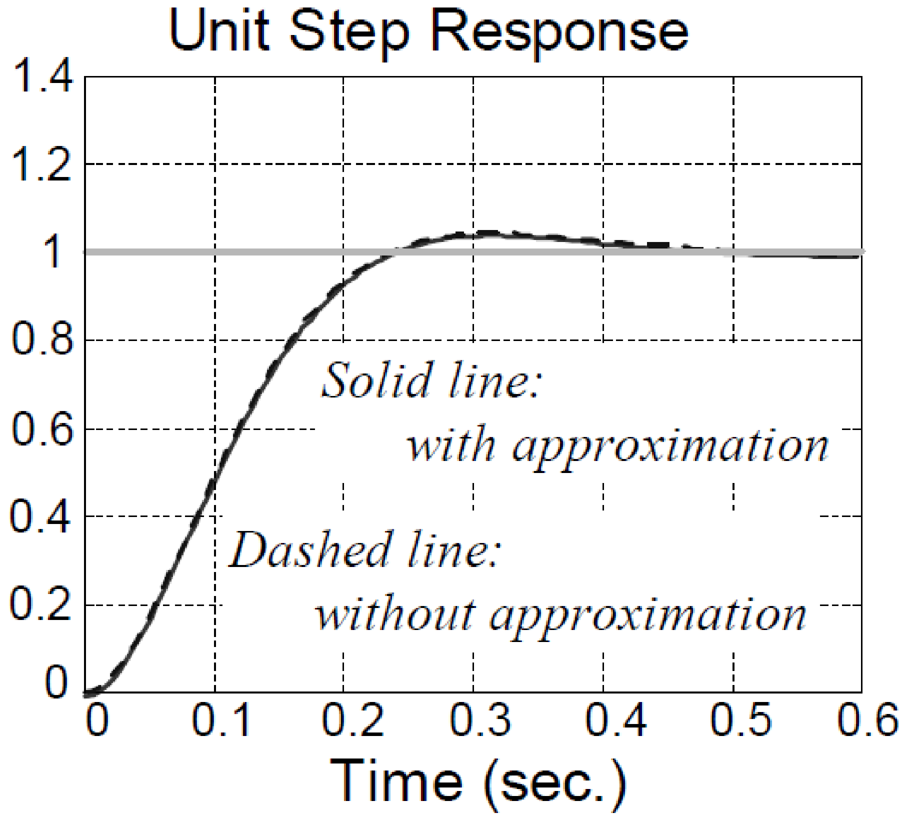
Modelling of disk drive control system, 5

$$T(s) = \begin{cases} \dfrac{200}{s^2 + 20s + 200} & \text{approximate} \\ \dfrac{200}{0.0001s^3 + 1.002s^2 + 20s + 200} & \end{cases}$$

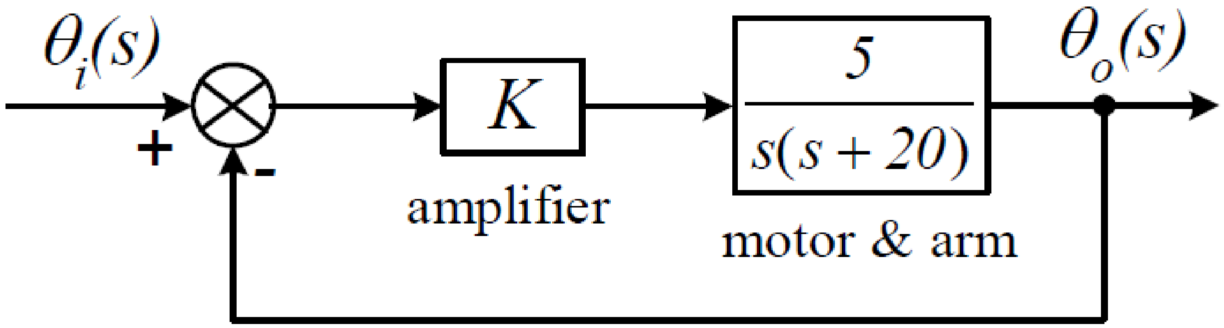
Assume: $K=40$

With or without approximation: the unit step response almost identical

Settling time: *0.4* seconds



Disk drive control system improvement, 1



K in the range of *10* to *1000* (Physical Restriction)

Settling time: *0.4* seconds when $K=40$

TF: $$T(s) = \frac{5K}{s^2 + 20s + 5K} \qquad \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\zeta\omega_n = 10$$

$$\left.\begin{aligned} 2\zeta\omega_n &= 20 \\ \omega_n^2 &= 5K \end{aligned}\right\} \Rightarrow \omega_n = \sqrt{5K}; \quad \zeta = \frac{10}{\sqrt{5K}}$$

Disk drive control system improvement, 2

$$\zeta\omega_n = 10 \quad \omega_n = \sqrt{5K} \quad \zeta = \frac{10}{\sqrt{5K}}$$

Underdamped design: $0 < \zeta < 1 \Rightarrow K > 20$

$$t_s = \frac{4}{\zeta\omega_n} = 0.4\,\text{sec}$$

Independent of K!! We cannot improve settling time by changing K alone.

$$\zeta = \frac{10}{\sqrt{5K}}; \quad PO = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

Depends on K.

K	30	40	100
ζ	0.82	0.71	0.45
PO	1.2%	4.3%	20.8%

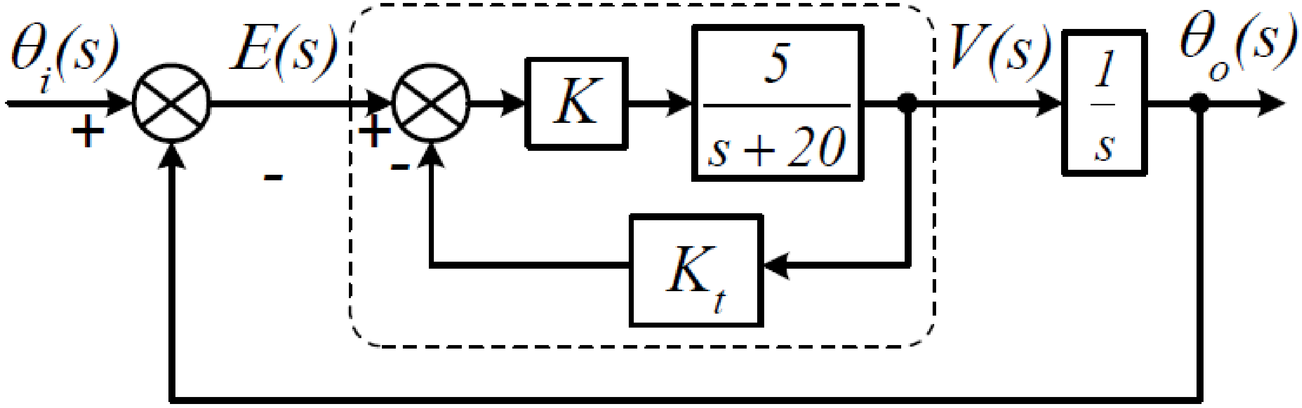
$t_s = 0.4\,\text{sec}$

Disk drive control system improvement, 3

- Performance can be improved by introducing angular velocity feedback.

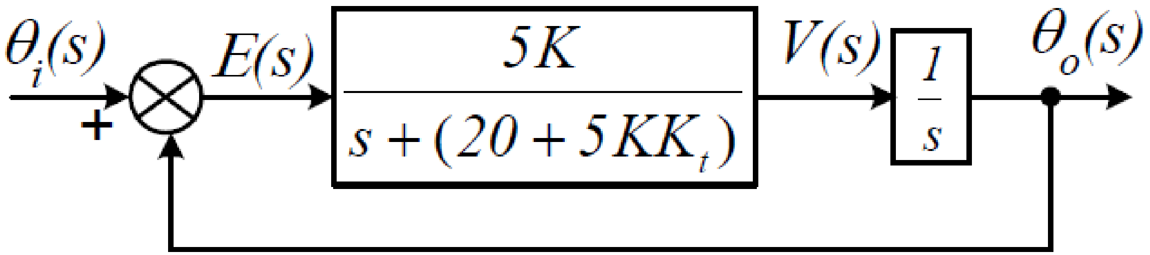
Design requirements: for a unit step input

$e_{ss}=0 \quad PO\leq 2\% \quad t_s \leq 0.08s$



$$\frac{V(s)}{E(s)}=\frac{\dfrac{5K}{s+20}}{1+\dfrac{5K}{s+20}K_t}=\frac{5K}{s+(20+5KK_t)}$$

Disk drive control system improvement, 4



Forward path TF: $$G(s) = \frac{5K}{s(s+(20+5KK_t))}$$

The overall closed-loop transfer function:

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{G(s)}{1+G(s)} = \frac{5K}{s^2+(20+5KK_t)s+5K}$$

Steady-state error for a unit step input:

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0 \qquad K_p = \lim_{s\to 0} G(s) = \infty$$

Disk drive control system improvement, 5

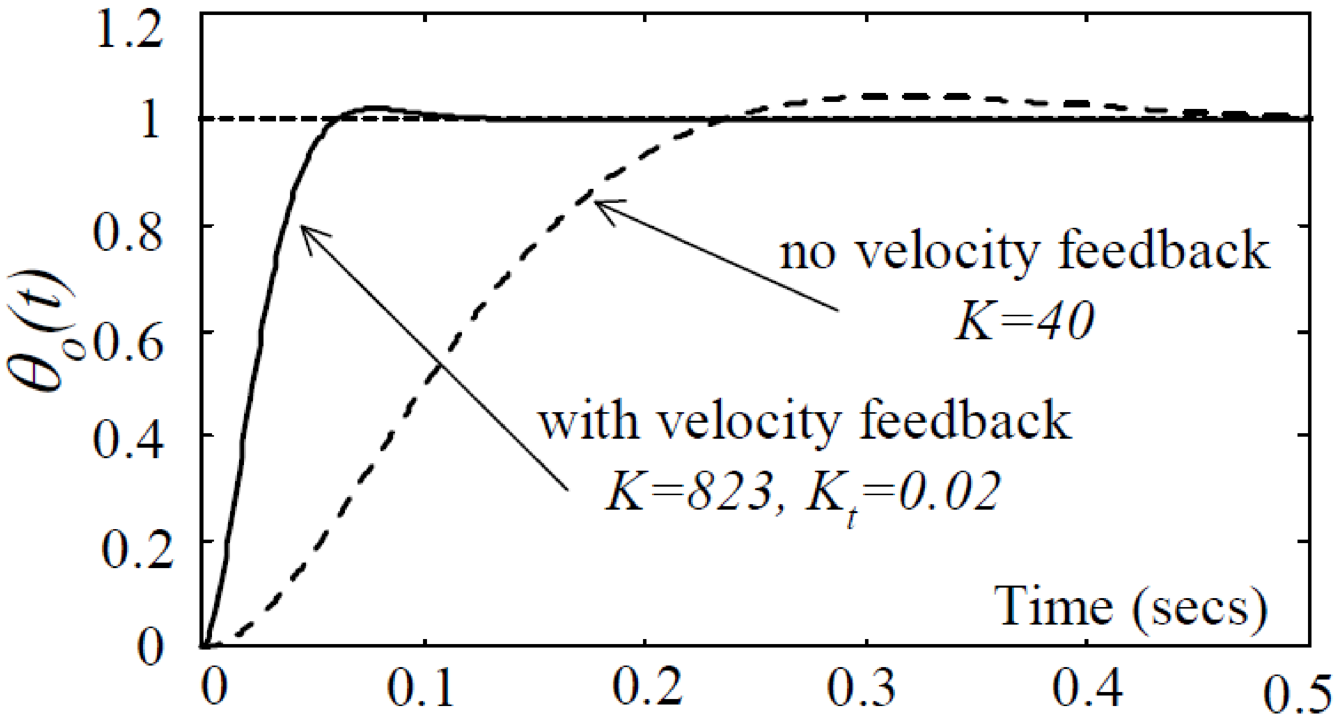
TF: $$T(s)=\frac{5K}{s^2+(20+5KK_t)s+5K} \qquad \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\left.\begin{aligned}2\zeta\omega_n &= 20+5KK_t\\ \omega_n^2 &= 5K\end{aligned}\right\} \Rightarrow \begin{cases}\omega_n = \sqrt{5K}\\ \zeta = \dfrac{20+5KK_t}{2\sqrt{5K}}\end{cases}$$

K	K_t	ζ	ω_n	PO	t_s
400	0.0216	0.707	44.721	4.3%	0.1265
823	0.02	0.78	64.1	2%	0.08
960	0.0204	0.85	69.282	0.63%	0.0679
46451	0.0034	0.84	481.93	0.77%	0.0099

Disk drive control system improvement, 6

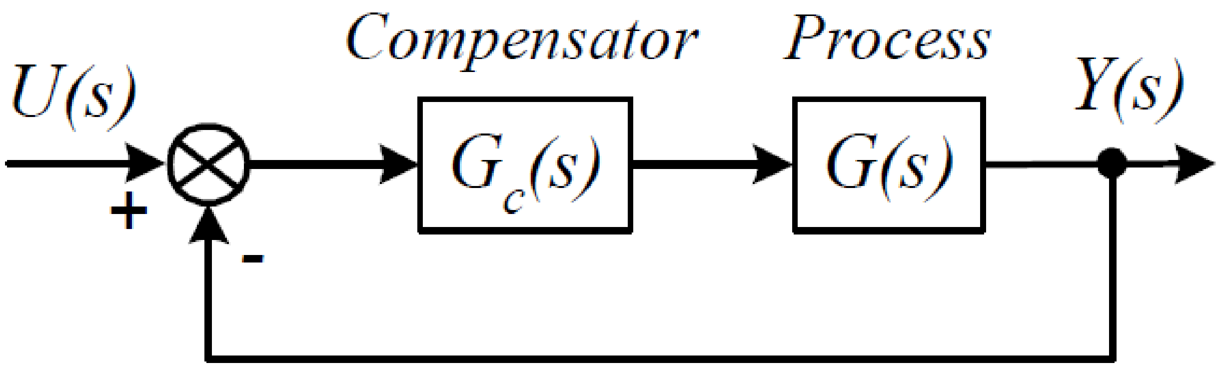
- Performance can be improved by introducing angular velocity feedback.

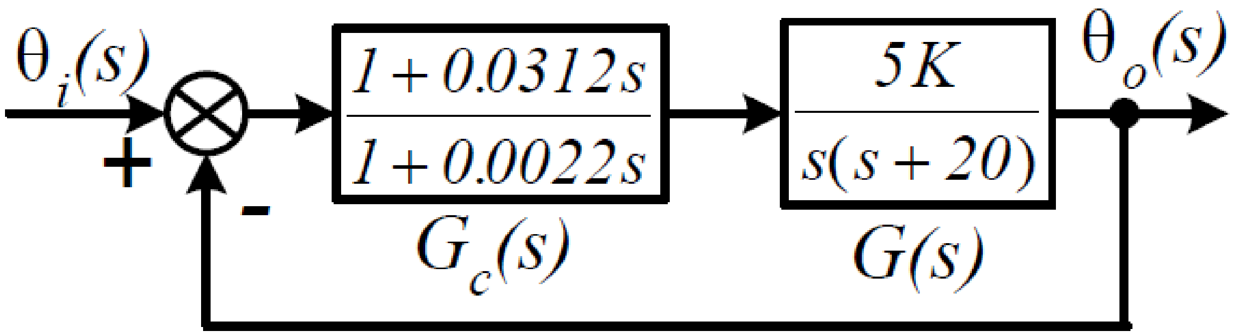


$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{4115}{s^2 + 100s + 4115}$$

Disk drive control system improvement, 6

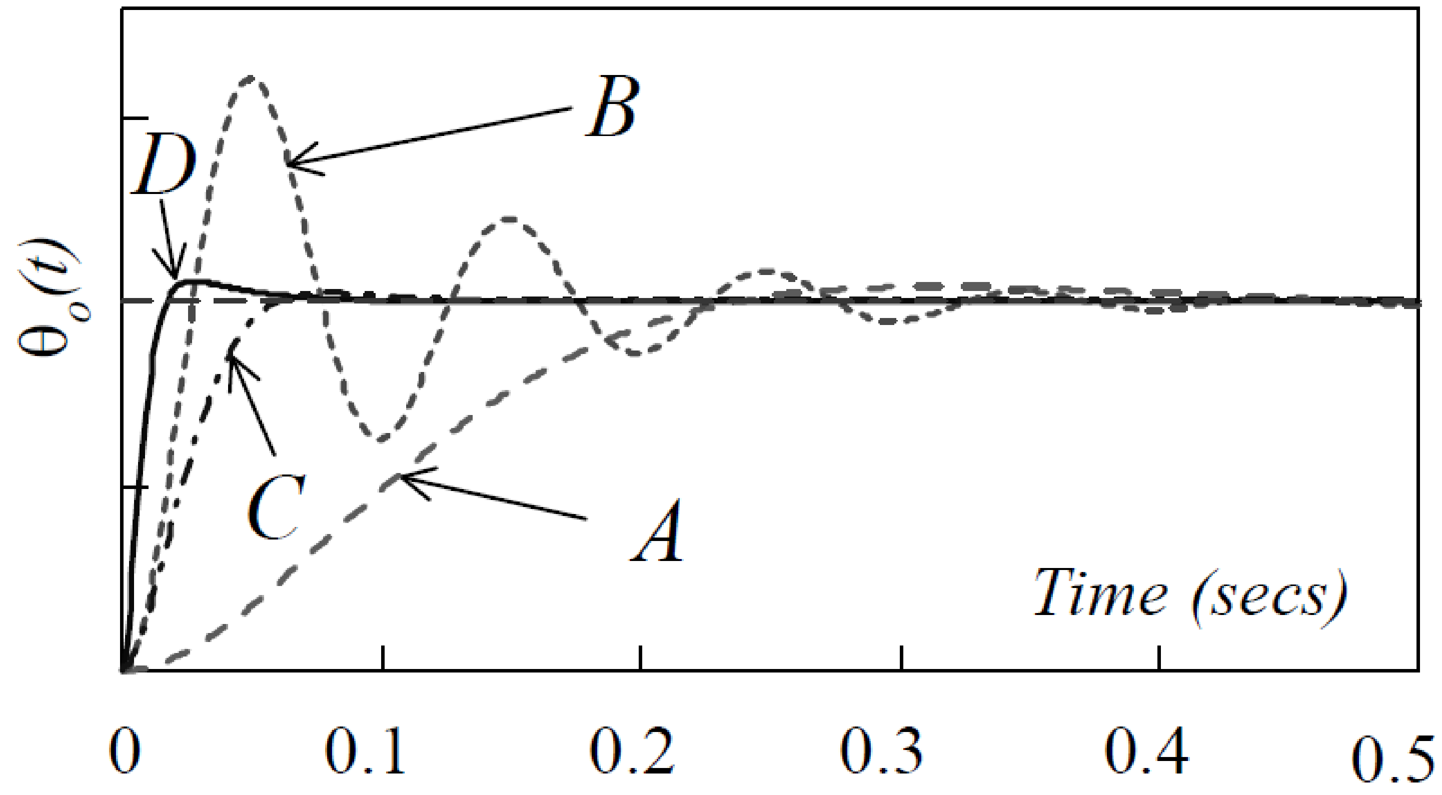
- An additional sensor (e.g. tachometer) which measures angular velocity is required to implement velocity feedback.
- A cheap alternative: use a compensator.





A compensator (controller) is inserted into the forward path of the system in order to improve control performance.

Disk drive control system improvement, 7



A: no compensator, no velocity feedback, $K=40$

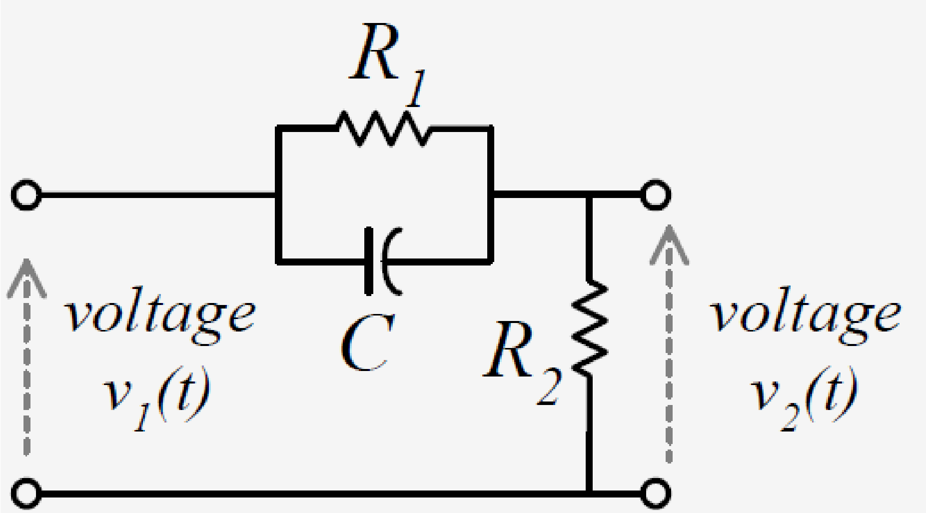
B: no compensator, no velocity feedback, $K=823$

C: no compensator, velocity feedback $K=823$, $K_t=0.02$

D with compensator, no velocity feedback, $K=823$

Disk drive control system improvement, 8

- Physical implementation of compensator.



$$G_N(s) = \frac{V_2}{V_1} = \frac{R_2}{R_2 + \frac{R_1 \frac{1}{CS}}{R_1 + \frac{1}{CS}}}$$

$$= \frac{R_2}{R_1 + R_2} \frac{(1 + R_1 Cs)}{1 + \frac{R_1 R_2 C}{R_1 + R_2} s}$$

Resistor:

$$v(t) = Ri(t)$$

$$\Rightarrow V(s) = \boxed{R}\ I(s)$$

Capacitor:

$$i(t) = C\frac{dv(t)}{dt}$$

$$\Rightarrow I(s) = CsV(s)$$

$$\Rightarrow V(s) = \boxed{\frac{1}{Cs}} I(s)$$

Disk drive control system improvement, 9

$$G_N(s) = \frac{R_2}{R_1 + R_2} \frac{(1 + R_1 C s)}{1 + \frac{R_1 R_2 C}{R_1 + R_2} s} \quad \alpha \equiv \frac{R_1 + R_2}{R_2} \quad T \equiv \frac{R_1 R_2 C}{R_1 + R_2}$$

$$G_N(s) = \left(\frac{1}{\alpha}\right)\left(\frac{1 + \alpha T s}{1 + T s}\right)$$

$$R_1 = 395\Omega,\ R_2 = 30\Omega,\ C = 78.9\mu F$$

$$G_N(s) = \left(\frac{1}{14.2}\right)\left(\frac{1 + 0.0312s}{1 + 0.0022s}\right)$$

The RC network connected to a amplifier of gain *14.2* will give us the required compensator transfer function.