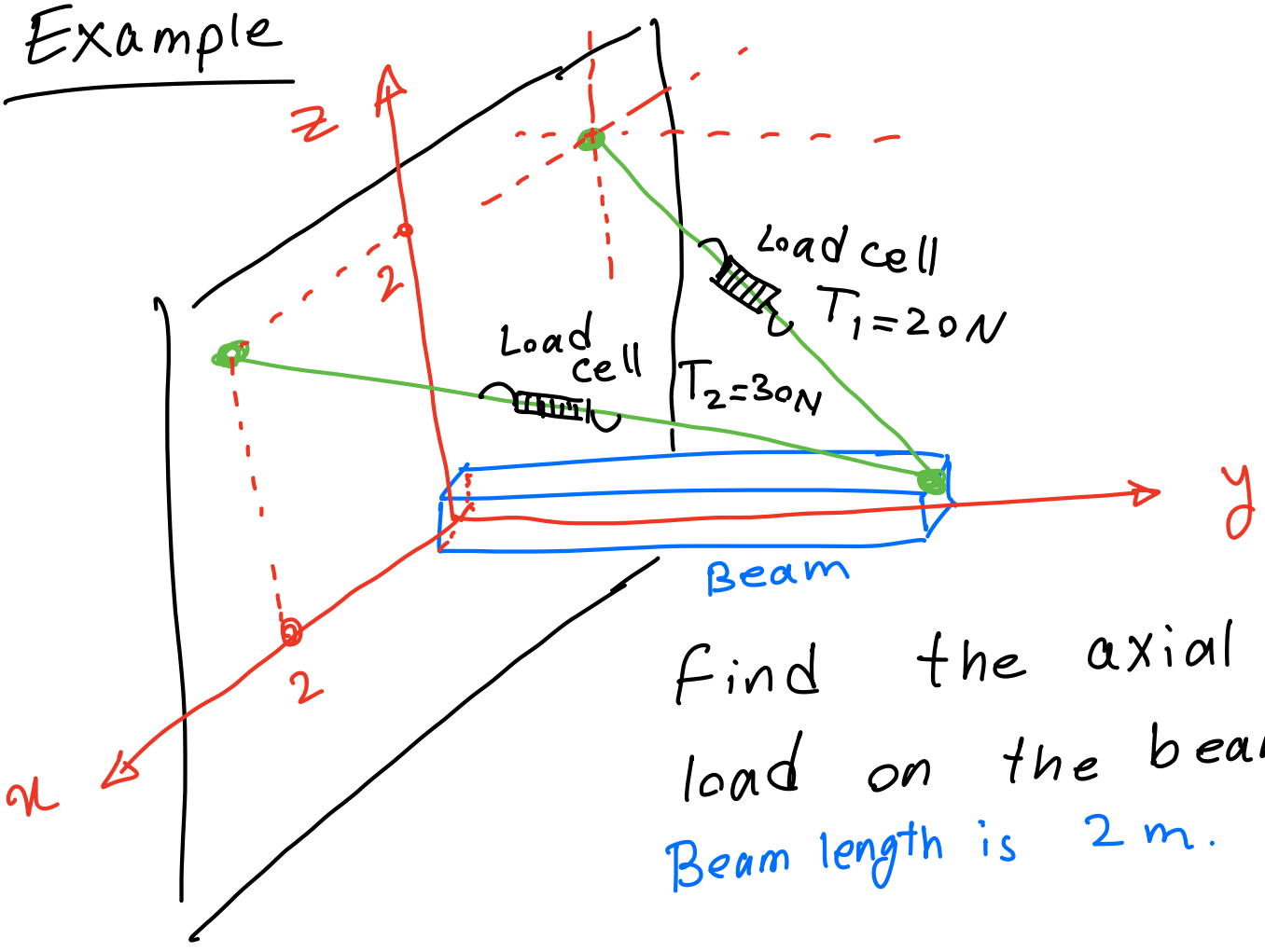
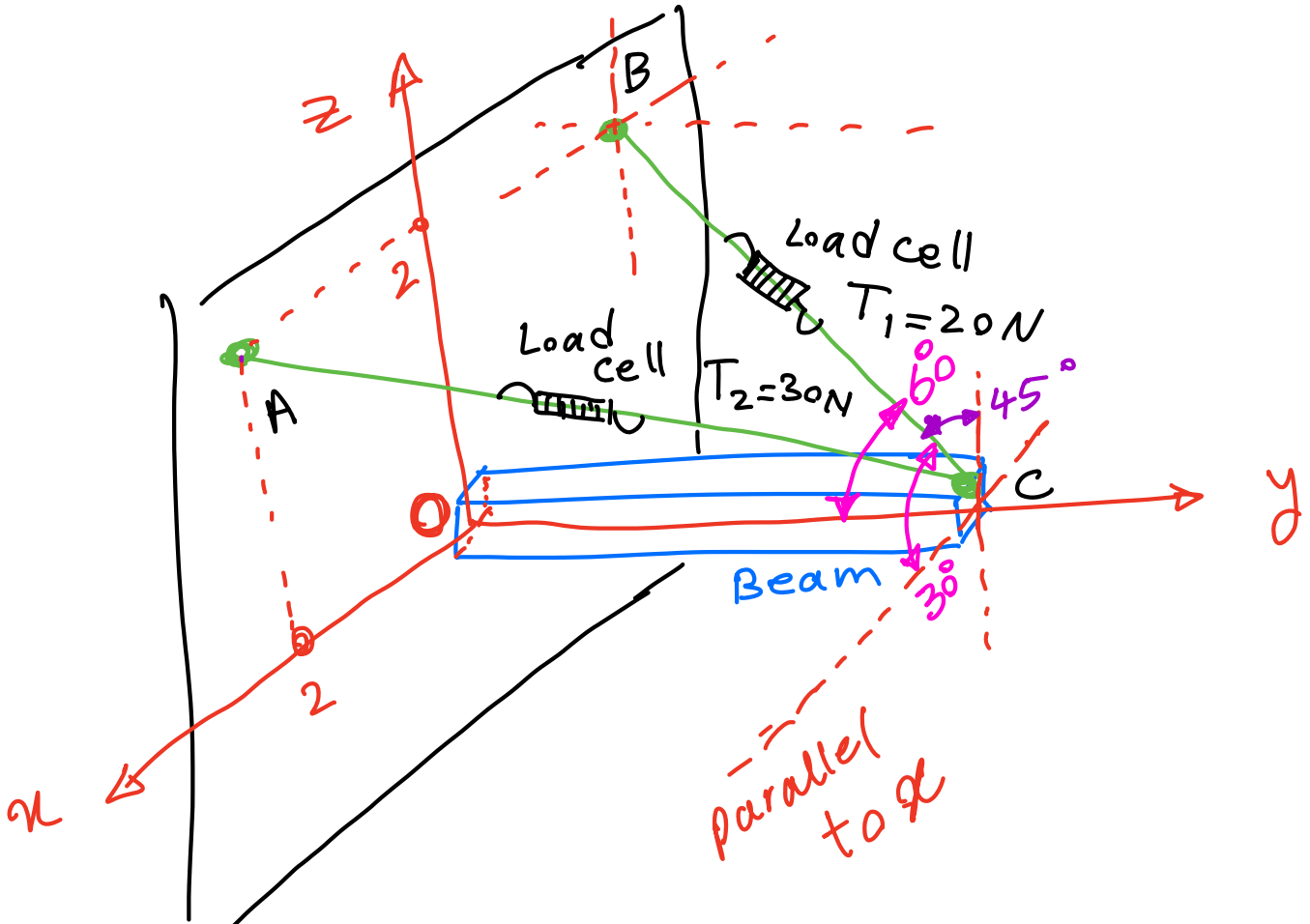
Example



Find the axial load on the beam.

Beam length is 2 m.



Vector $\vec{CA}$:

$$\vec{CA} = (x_A\hat{i} + y_A\hat{j} + z_A\hat{k}) - (x_C\hat{i} + y_C\hat{j} + z_C\hat{k})$$

$$= 2\hat{i} + 0\hat{j} + 2\hat{k} - (0\hat{i} + 4\hat{j} + 0\hat{k})$$

$$\vec{CA} = 2\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\vec{CA}_{unit} = \frac{2\hat{i} - 4\hat{j} + 2\hat{k}}{\sqrt{2^2 + 4^2 + 2^2}} = \frac{2\hat{i} - 4\hat{j} + 2\hat{k}}{\sqrt{24}}$$

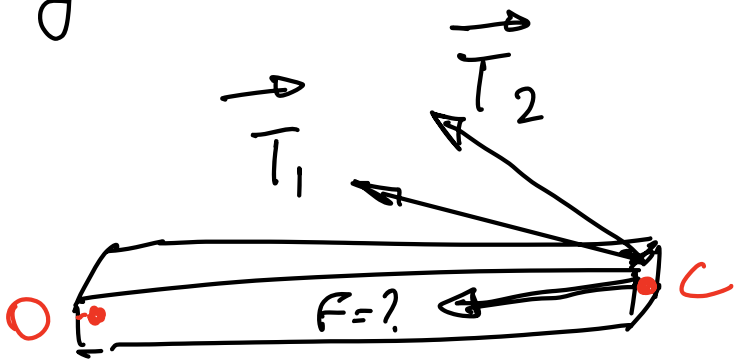
$$\vec{T}_2 = (30N)\,\frac{2\hat{i} - 4\hat{j} + 2\hat{k}}{\sqrt{24}}$$

$$\vec{T} = |\vec{T}_1|\,(\cos 30^\circ \hat{i} + \cos 60^\circ \hat{j} + \cos 45^\circ \hat{k})$$

---

we now have $\vec{T}_1$ and $\vec{T}_2$

we need to find the axial load along the beam.



find the unit vector for the beam

The projection of $\vec{T}_1$ and $\vec{T}_2$ can be found by the DOT product of the unit vector of the position of the beam along the Length.

$$\vec{OC}_{unit} = \frac{0\hat{i} + 2\hat{j} + 0\hat{k}}{\sqrt{4}} = \frac{2\hat{j}}{2} = \hat{j}$$

The force of $\vec{T}_1$ along the beam is:

$$\vec{T}_1 \cdot \vec{OC}_{unit} =$$

$$20\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}+\frac{\sqrt{3}}{2}\hat{k}\right)\cdot(\hat{j})$$

$$= 20\,(1/2)\,(1) = 10$$

$$\vec{T}_2 \cdot \vec{OC}_{unit} = (30)\left(\frac{2\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{12}}\right)\cdot\hat{j}$$

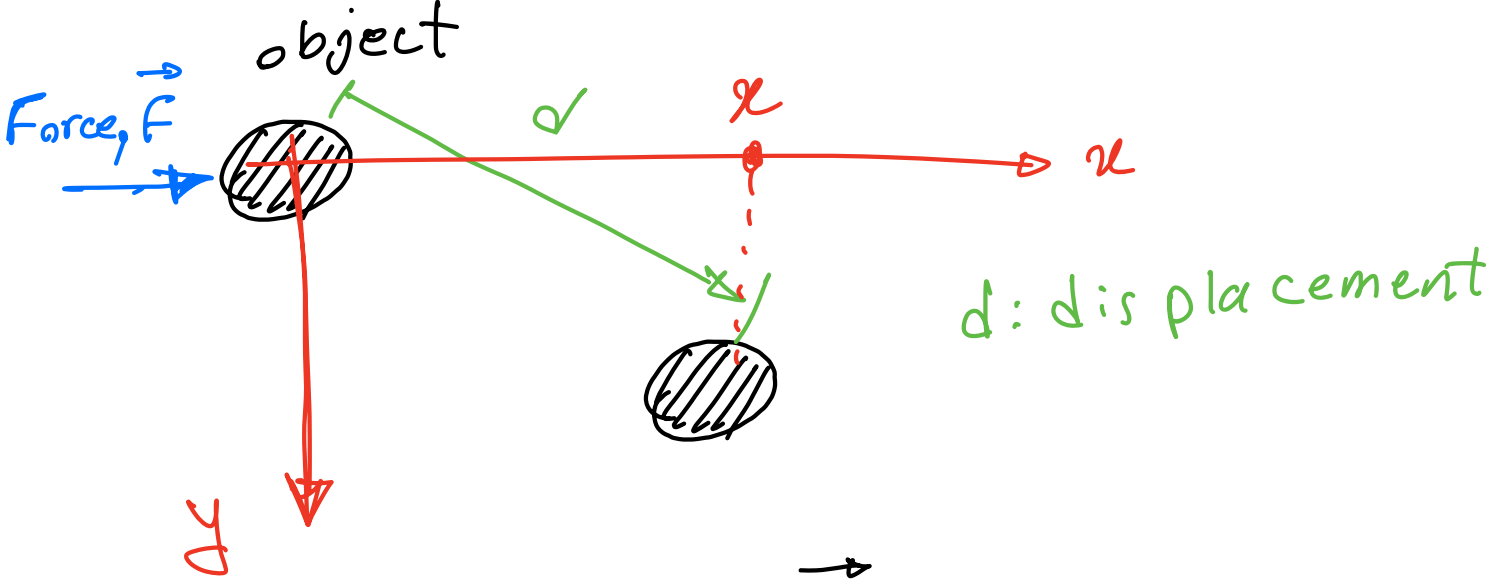
$$= \frac{30}{\sqrt{12}}(-2)$$

The total axial load on the beam is $\vec{T}_1 \cdot \vec{OC}_{unit} + \vec{T}_2 \cdot \vec{OC}_{unit}$

Anothe application of DOT product

Work

work done by a force

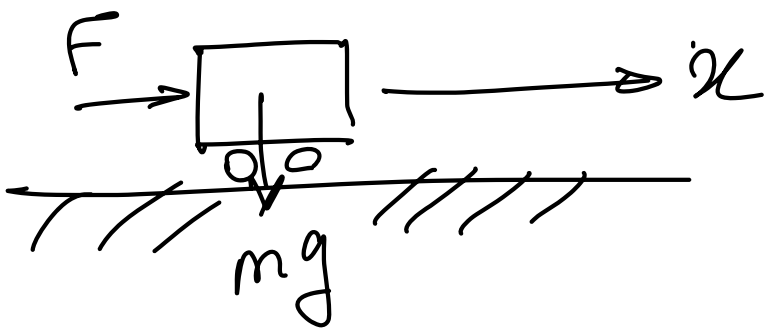


$$\text{work} = \vec{F} \cdot \vec{d}$$

For example: $\vec{F} = |\vec{F}|\,\vec{i}$

$$\vec{d} = x\vec{i} + y\vec{j}$$

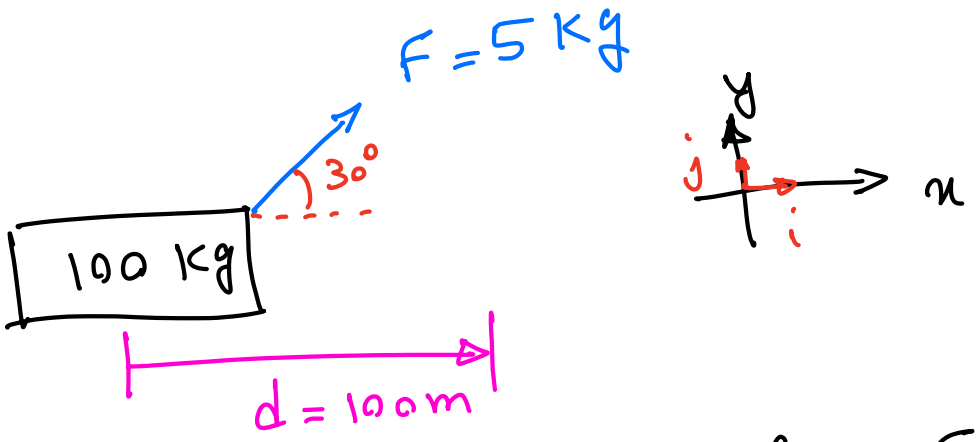
$$\text{work} = |\vec{F}|\,\vec{i} \cdot (x\vec{i} + y\vec{j}) = |\vec{F}|\,x$$



mg does not do work in

x direction

Example



Find the work done by force F.

$$\vec{f} = F\cos 30^\circ \vec{i} + F\sin 30^\circ \vec{j}$$

$$\vec{d} = 100\,\vec{i}$$

$$\text{work} = \vec{F}\cdot\vec{d} = \left(F\cos 30^\circ \vec{i} + F\sin 30^\circ \vec{j}\right)\cdot 100\vec{i}$$

$$= (5\cos 30^\circ)100$$

$$= 500\cos 30^\circ$$

Cross product

next week