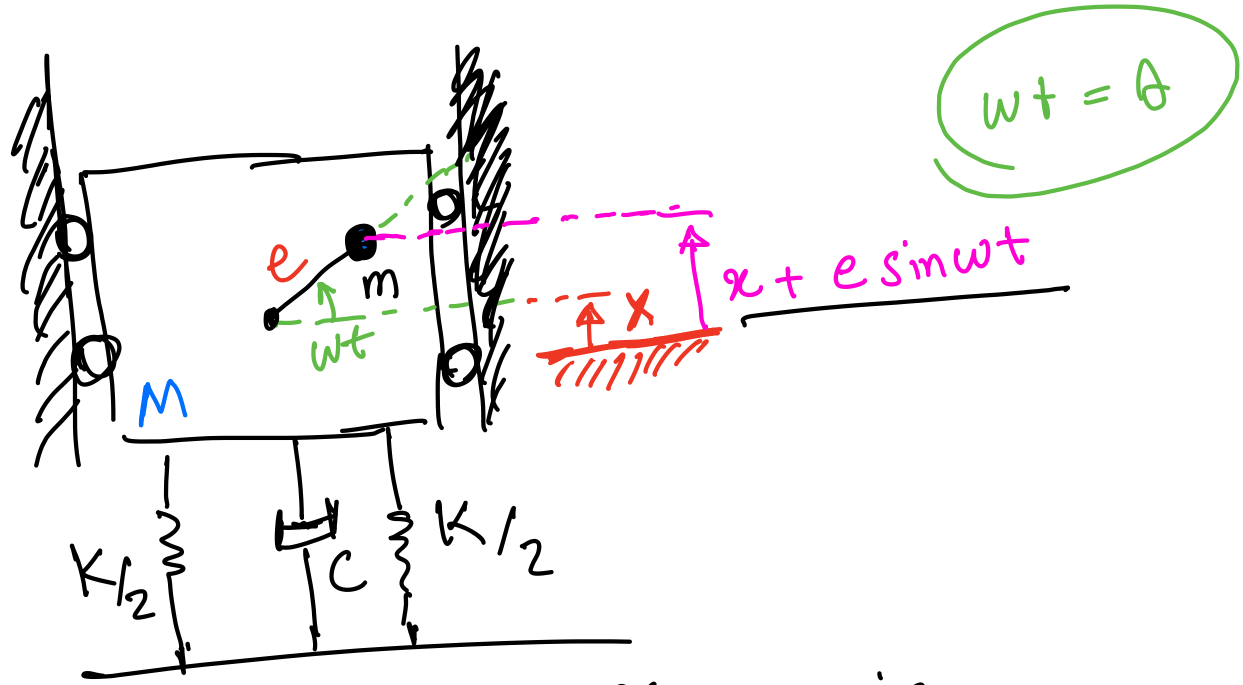
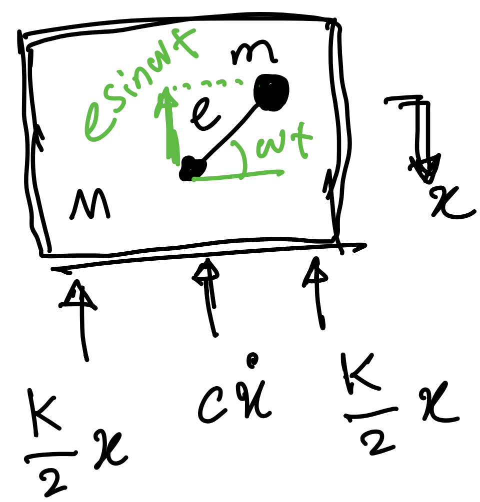
Rotating unbalance (section 3.2) Book



Displacement of mass m is

$$x + e\sin\omega t$$

Equation of motion



$$(M-m)\ddot{x} + m\underbrace{\frac{d^2}{dt^2}(x + e\sin\omega t)}_{\ddot{x} - e\omega^2\sin\omega t} = -Kx - c\dot{x}$$

Rearrange:

$$M\ddot{x} + c\dot{x} + Kx = (me\omega^2)\sin\omega t$$

This equation is similar to:

$$m\ddot{x} + c\dot{x} + Kx = F_o \sin\omega t$$

From the previous lecture (Eq. 3.1-1) Book

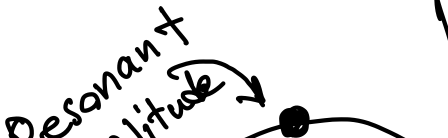
Steady state solution will be similar to the steady state solution of Eq. 3.1-1:

$$X = \frac{me\omega^2}{\sqrt{(K - M\omega^2)^2 + (c\omega)^2}}$$

$$\tan\phi = \frac{c\omega}{K - M\omega^2}$$

for non dimensional form:

$$\frac{M}{m}\frac{X}{e} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}}$$



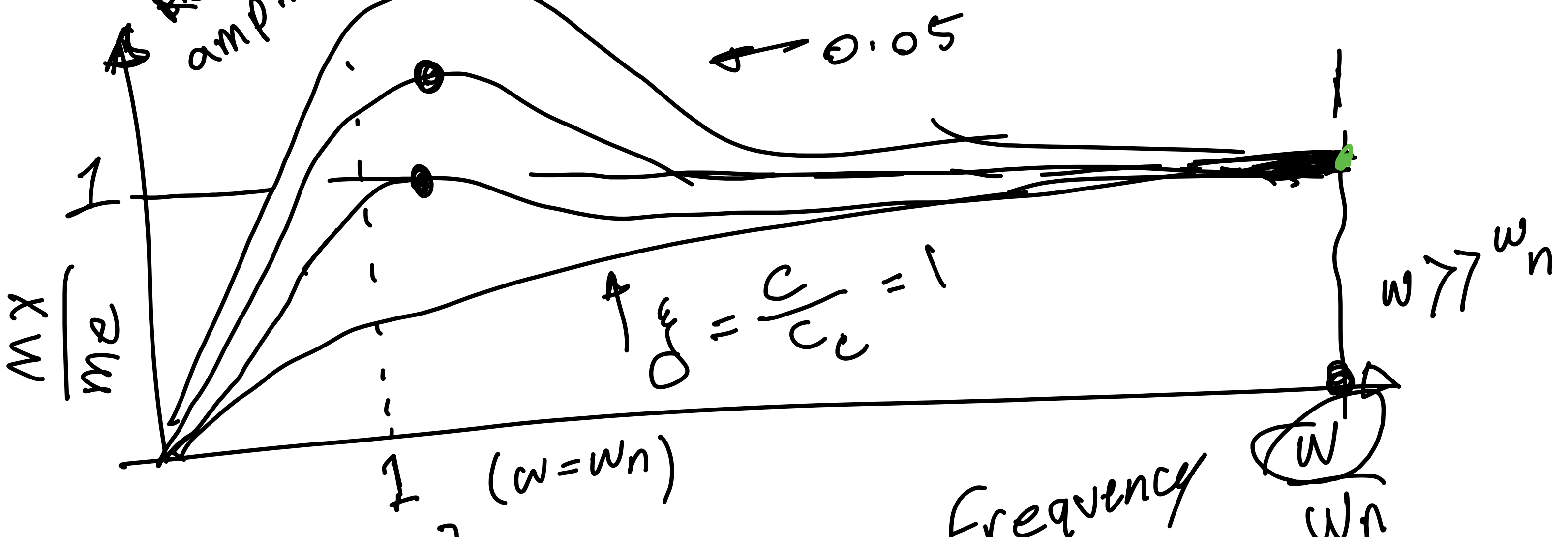


Fig. 3.2-2

$$\tan\phi = \frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

The complete solution is:

$$x(t) = X_1 e^{-\xi\omega_n t}\sin\left(\sqrt{1-\xi^2}\,\omega_n t + \phi_1\right) + \frac{me\omega^2}{\sqrt{(K-M\omega^2)^2+(c\omega)^2}}\sin(\omega t + \phi)$$

(Eq. 3.2-6)

Example:

A counterrotating eccentric weight is used to produce the forced oscillation of a spring supported mass. The resonant amplitude is 0.6cm (measured experimentally) when the speed of rotating was increased considerably beyond the resonant frequency, the amplitude appeared to approach a fixed value of 0.08 cm.

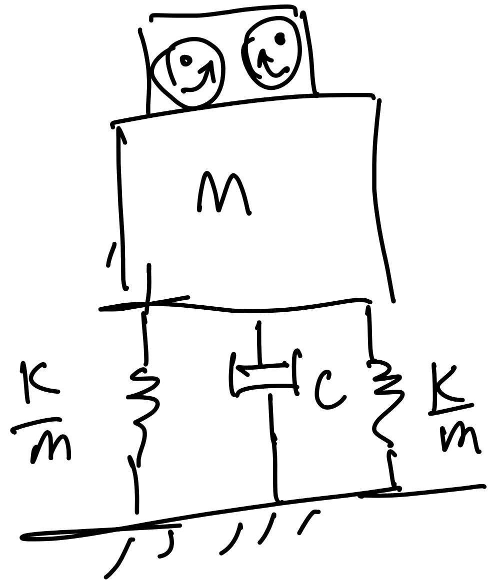
Determine the damping factor (ratio) ξ of the system.

$$\frac{M}{m}\frac{X}{e} = \frac{\left(\frac{w}{w_n}\right)^2}{\sqrt{\left[1-\left(\frac{w}{w_n}\right)^2\right]^2+\left[2\xi\frac{w}{w_n}\right]^2}}$$

The resonant amplitude:

X when $\omega = \omega_n$:

$$\frac{M}{m}\frac{X}{e} = \frac{1}{2\zeta}$$

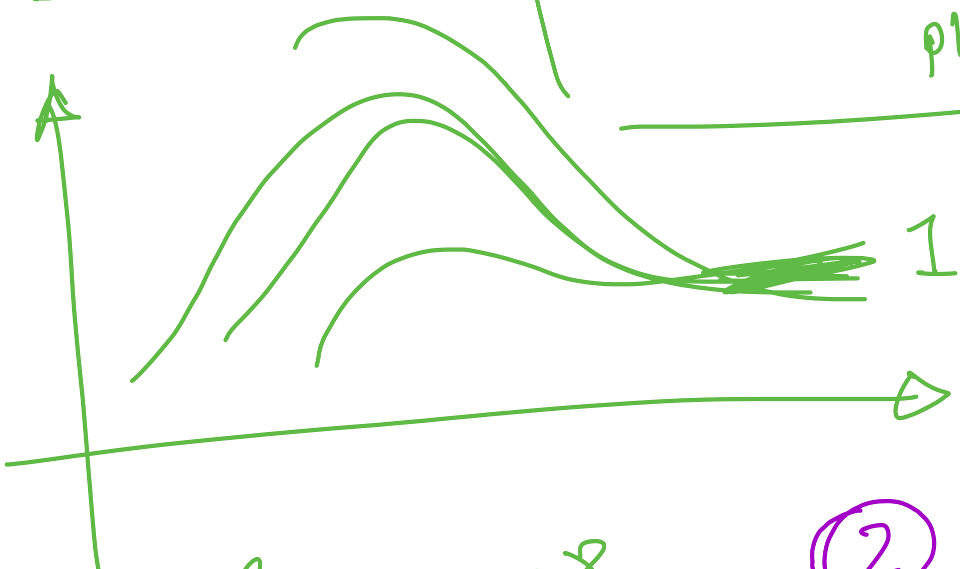


$$X = \frac{\frac{me}{M}}{2\zeta} = 0.6 \text{ cm}$$ (1)

when ω is very much greater than ω_n the equation becomes:

$XM = me$ from the plot

$X = 0.08$



$$X = \frac{me}{M} = 0.08$$ (2)

From Eq. 1 and Eq. 2:

$$\xi = ?$$

$$\boxed{X = \frac{\frac{me}{M}}{2\xi} = 0.6 \text{ cm} \qquad X = \frac{me}{M} = 0.08}$$

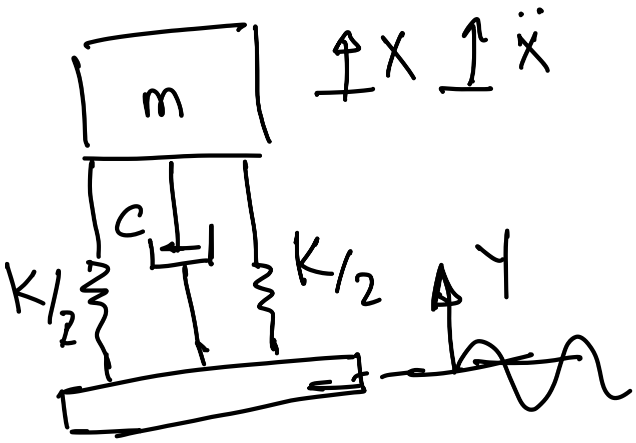
$$\xi = \frac{0.08}{2 \times 0.6}$$

$$\xi = 0.0666$$

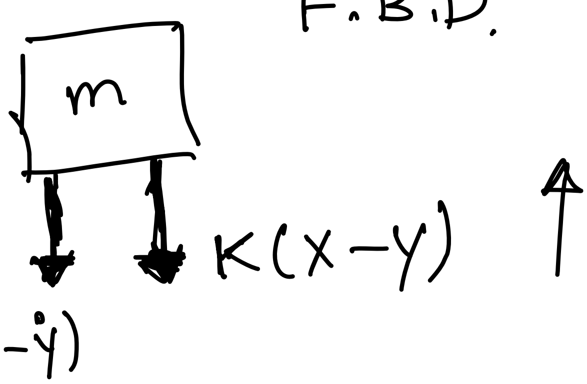
Support Motion (section 3.5 Book)

(support Excitation)

(or Base excitation)



F.B.D.



Assume $X > Y \rightarrow$ spring in tension

$\oplus\uparrow \; \sum F = m\ddot{x}$

$$-K(x-y) - c(\dot{x}-\dot{y}) = m\ddot{x}$$

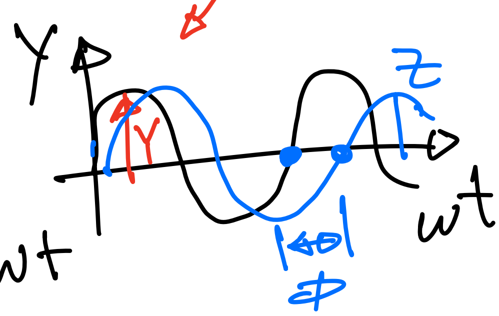
choose $\quad z = x-y \longrightarrow \dot{z} = \dot{x}-\dot{y}$

$$\ddot{z} = \ddot{x}-\ddot{y}$$

$$m\ddot{z} + c\dot{z} + Kz = -m\ddot{y}$$

Assume $\quad y(t) = Y\sin\omega t \longrightarrow \ddot{y} = -Y\omega^2\sin\omega t$

the excitation of the base



$$m\ddot{z} + c\dot{z} + Kz = m\omega^2 Y\sin\omega t$$

Assume $\quad z = Z\sin(\omega t - \Phi)$

same as $z = Ze^{i(\omega t - \Phi)}$

$$Z = \frac{m\omega^2 Y}{\sqrt{(K-m\omega^2)^2 + (c\omega)^2}}$$

$$\tan\Phi = \frac{c\omega}{K-m\omega^2}$$

$$Ze^{-i\phi} = \frac{m\omega^2 Y}{k - m\omega^2 + i\omega c}$$

$$y = Ye^{i\omega t}$$

$$z = Ze^{i(\omega t - \phi)} = Ze^{-i\phi}\, e^{i\omega t}$$

$$x = Xe^{i(\omega t - \psi)} = \left(Xe^{-i\psi}\right) e^{i\omega t}$$

$$x = (Ze^{i\phi} + Y)\, e^{i\omega t}$$

$$\boxed{Z = x - y}$$

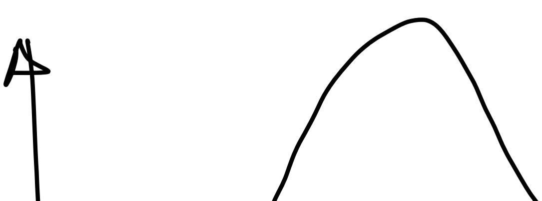
$$x = \left(\frac{k - i\omega c}{k - m\omega^2 + i\omega c}\right) Ye^{i\omega t}$$

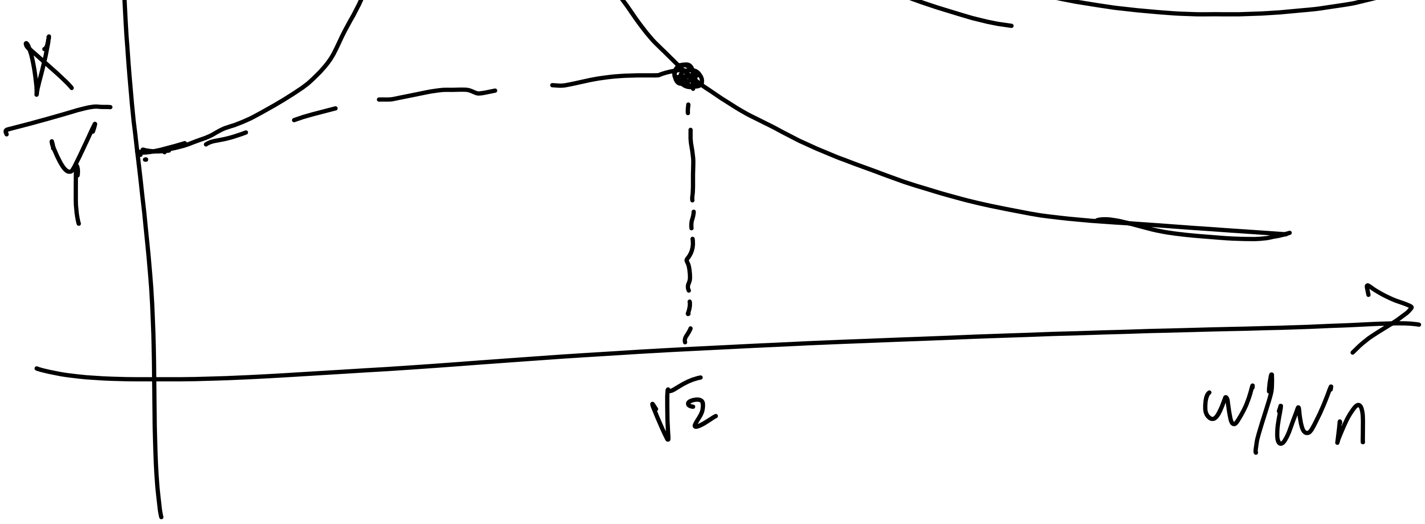
Transfer function form $= \dfrac{\text{output}}{\text{Input}}$

$$\left|\frac{X}{Y}\right| = \sqrt{\frac{k^2 + (\omega c)^2}{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\tan\psi = \frac{mc\omega^3}{k(k - m\omega^2) + (c\omega)^2}$$

$$\left|\frac{X}{Y}\right| = 1 \quad ?$$





X/Y
√2
ω/ωn