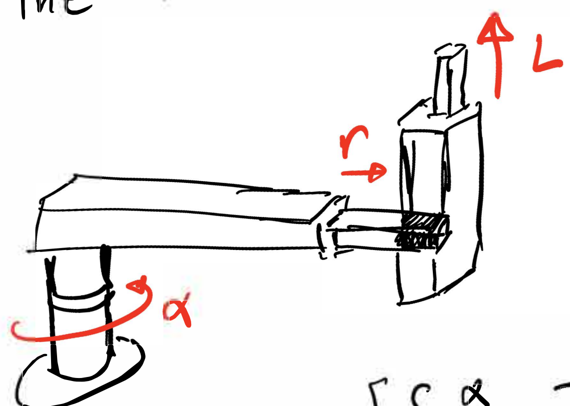
Example

suppose we desire to place the origin of the hand frame of a cylindrical robot at $[3,4,7]^T$. Calculate the joint variables of the robot.



$$T_{cyl}(r,\alpha,L) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

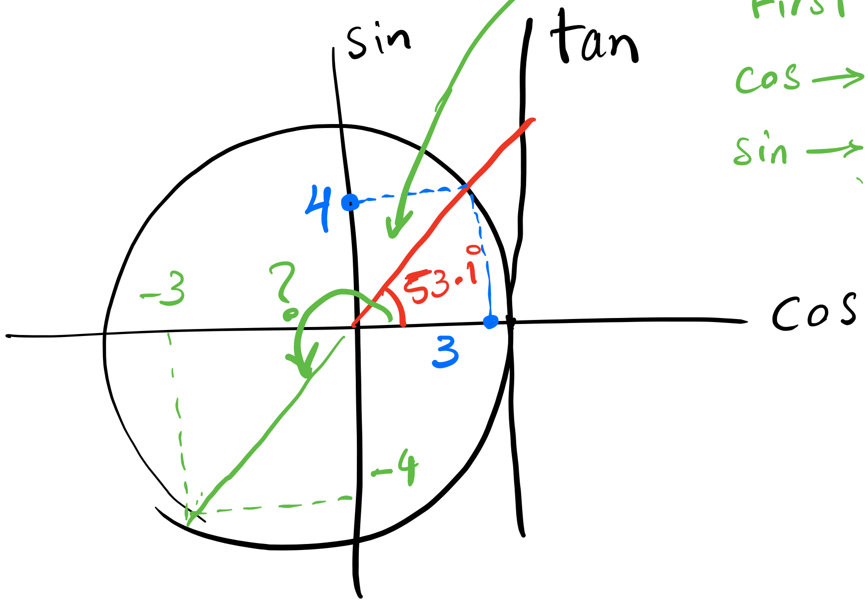
$$\left.\begin{aligned} L &= 7 \\ rc\alpha &= 3 \\ rs\alpha &= 4 \end{aligned}\right\} \Rightarrow \frac{rs\alpha}{rc\alpha} = \frac{4}{3}$$

$$\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.1^\circ$$

First quadrant

cos → positive

sin → positive



$$r\cos\alpha = 3$$

$$r\cos 53.1^\circ = 3 \Rightarrow r = 5$$

Example

The position and restored orientation of a cylindrical robot are given. Find the matrix representing the original position and orientation of the robot befor it was restored.

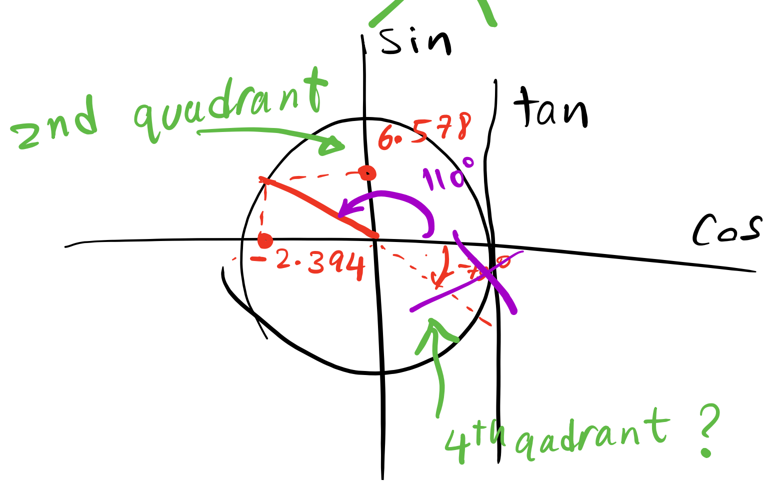
$$T = \begin{bmatrix} 1 & 0 & 0 & -2.394 \\ 0 & 1 & 0 & 6.578 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & rc\alpha \\ 0 & 1 & 0 & 0 & rs\alpha \\ 0 & 0 & 1 & L & \\ 0 & 0 & 0 & 1 & \end{bmatrix}$$

$$L = 9$$

$$\tan\alpha = \frac{r\sin\alpha}{r\cos\alpha} = \frac{6.578}{-2.394} = -2.748$$

$$\alpha = ? \qquad \cancel{-70}^{\circ}\ ?$$



$\alpha = 180^\circ - 70^\circ = 110^\circ$

$r \sin(\alpha) = 6.578 \Rightarrow r$

substititing these values in the originay T_{cyl} transformation matrix will give the original orientation of the robot.

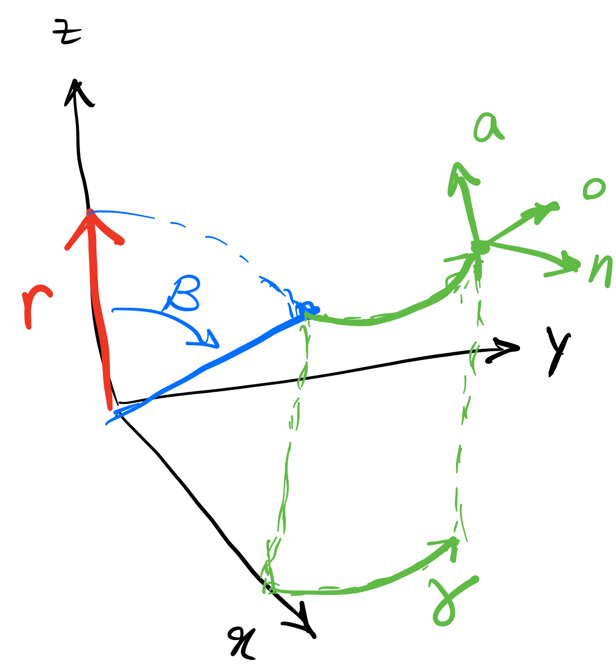
$$T_{cyl}(r, \alpha, L) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.342 & -0.9397 & 0 & -2.394 \\ 0.9397 & -0.342 & 0 & 6.578 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical coordinates (section 2.9.3) Book

A spherical coordinate system consists of one linear motion and two rotations.

The sequence is a translation of r along the z-axis, a rotation of β about the y-axis, and a rotation of γ about the z-axis as shown below:



$$^{R}T_{P} = T_{sph}(r, \beta, \gamma)$$

$$= Rot(z, \gamma)\, Rot(y, \beta)\, Trans(0,0,r)$$

$$^{R}T_{P} = \begin{bmatrix} c\gamma & -s\gamma & 0 & 0 \\ s\gamma & c\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta & 0 \\ 0 & 1 & 0 & 0 \\ -s\beta & 0 & c\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{sph}(r, \beta, \gamma) = \begin{bmatrix} c\beta c\gamma & -s\gamma & s\beta c\gamma & rs\beta c\gamma \\ c\beta s\gamma & c\gamma & s\beta s\gamma & rs\beta s\gamma \\ -s\beta & 0 & c\beta & rc\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>Example</u> (2.19 Book)

suppose we now want to place the origin of the hand of a spherical robot at $[3, 4, 7]^T$. calculate the joint variables of the robot.

$$T_{sph}(r,\beta,\gamma) = \begin{bmatrix} c\beta c\gamma & -s\gamma & s\beta c\gamma & r s\beta c\gamma \\ c\beta s\gamma & c\gamma & s\beta s\gamma & r s\beta s\gamma \\ -s\beta & 0 & c\beta & r c\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} r s\beta c\gamma &= 3 \\ r s\beta s\gamma &= 4 \end{aligned}\right\} \quad \frac{\cancel{r}\,\cancel{s\beta}\, s\gamma}{\cancel{r}\,\cancel{s\beta}\, c\gamma} = \frac{4}{3}$$

$$r c\beta = 7$$

$$\tan\gamma = 4/3 \Rightarrow \gamma = 53.1^\circ \quad \text{or} \quad 233.1^\circ$$

$$s\gamma = 0.8 \quad \text{or} \quad -0.8$$

$$c\gamma = 0.6 \quad \text{or} \quad -0.6$$

$$r s\beta = \frac{3}{0.6} = 5 \quad \text{or} \quad -5$$

$$r c\beta = 7 \Rightarrow \tan\beta = \frac{5}{7}$$

$$\Rightarrow \beta = 35.5^\circ \quad \text{or} \quad -35.5^\circ$$

Articulate coordinates:

we will develop the matrix representation for this later, when we discuss the Denavit-Hartenberg representation

Section 2.10

Farward and Inverse Kinematic Equations: oriention