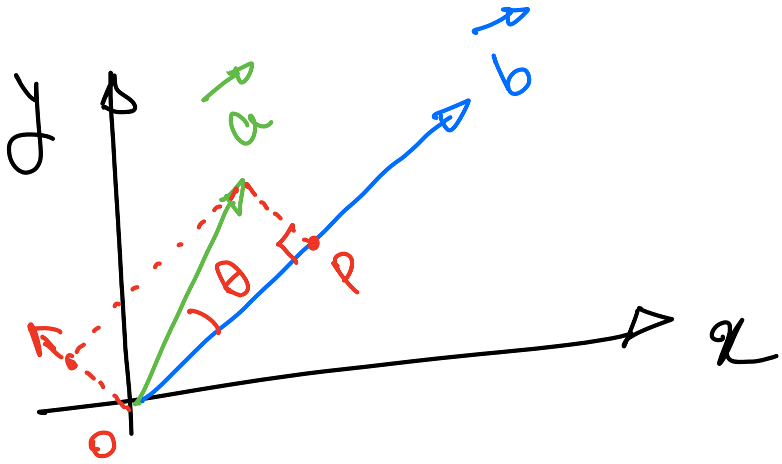
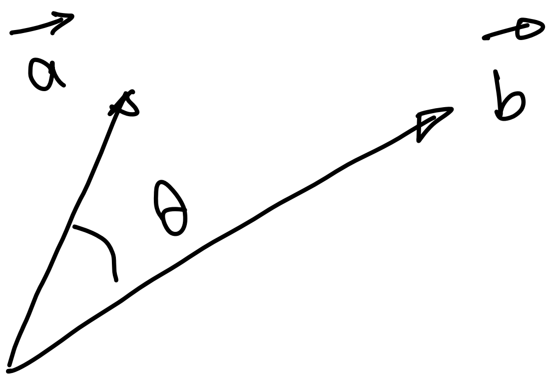
Projection of a vector along another vector:



Find the projection of $\vec{a}$ along vector $\vec{b}$

$$\overline{OP} = \underbrace{|\vec{a}|}_{\text{The magnitude of } \vec{a}} \cos\theta$$

Dot products:



$$\vec{a} \cdot \vec{b} = |\vec{a}|\, |\vec{b}| \cos\theta$$

Dot product

magnitude of $\vec{a}$

magnitude of $\vec{b}$

The answer is a Scalar (Not a vector)

Example

$$\vec{a} = 2\hat{i} + 3\hat{j}$$

$$\vec{b} = 4\hat{i} + 2\hat{j}$$

$$\vec{a} \cdot \vec{b} = ?$$

$$\vec{a} \cdot \vec{b} = (2\hat{i} + 3\hat{j})(4\hat{i} + 2\hat{j})$$

$$= 8(\hat{i} \cdot \hat{i}) + 4(\hat{i} \cdot \hat{j}) + 12(\hat{j} \cdot \hat{i}) + 6(\hat{j} \cdot \hat{j})$$

$$\hat{i}\cdot\hat{i} = |\hat{i}|\,|\hat{i}|\,\cos(\widehat{i,i})$$

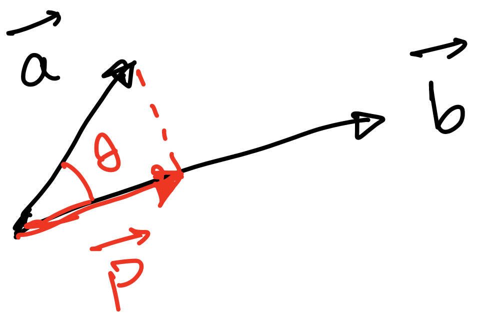
$$= 1 \times 1\,\cos(0) = 1$$

$$\hat{i}\cdot\hat{j} = |\hat{i}|\,|\hat{j}|\,\cos(\widehat{i,j})$$

$$= 1 \times 1\,\cos(90^\circ) = 0$$

$$\vec{a}\cdot\vec{b} = 8 + 6 = 14$$

Now, find the projection of $\vec{a}$ along $\vec{b}$:



$$\vec{P} = ?$$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\,\cos(\widehat{\vec{a},\vec{b}})$$

$$\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|} = \cos(\widehat{\vec{a},\vec{b}})$$

$$\vec{b}_{unit} = \frac{\vec{b}}{|\vec{b}|}$$

$$\vec{a} \cdot \vec{b}_{unit} = |\vec{a}| \cos\theta$$

Therefor, $|\vec{P}| = |\vec{a}| \cos\theta = \vec{a} \cdot \vec{b}_{unit}$

To find vector $\vec{P}$ we need the direction for $|\vec{P}|$.

The direction of $\vec{P}$ is the same as $\vec{b}$.

$$\vec{P} = \underbrace{(\vec{a} \cdot \vec{b}_{unit})}_{\text{magnitude only}} \underbrace{\vec{b}_{unit}}_{\text{direction}}$$

Example:

what is the projection of $\vec{b}$ along $\vec{a}$, as a vector.

$$\vec{a} = 3\hat{i} - 4\hat{j}$$

$$\vec{b} = 4\hat{k} + 8\hat{j}$$

$$\vec{P} = (\vec{b} \cdot \vec{a}_{unit})\, \vec{a}_{unit}$$

$$\vec{P} = \left[\underbrace{(4\hat{k} + 8\hat{j})}_{(0\hat{i} + 8\hat{j} + 4\hat{k})} \cdot \left(\frac{3\hat{i} - 4\hat{j}}{\sqrt{3^2 + 4^2}} \right) \right] \left(\frac{3\hat{i} - 4\hat{j}}{\sqrt{3^2 + 4^2}} \right)$$

$$\vec{P} = \frac{-32}{5} \left(\frac{3\hat{i} - 4\hat{j}}{5} \right)$$

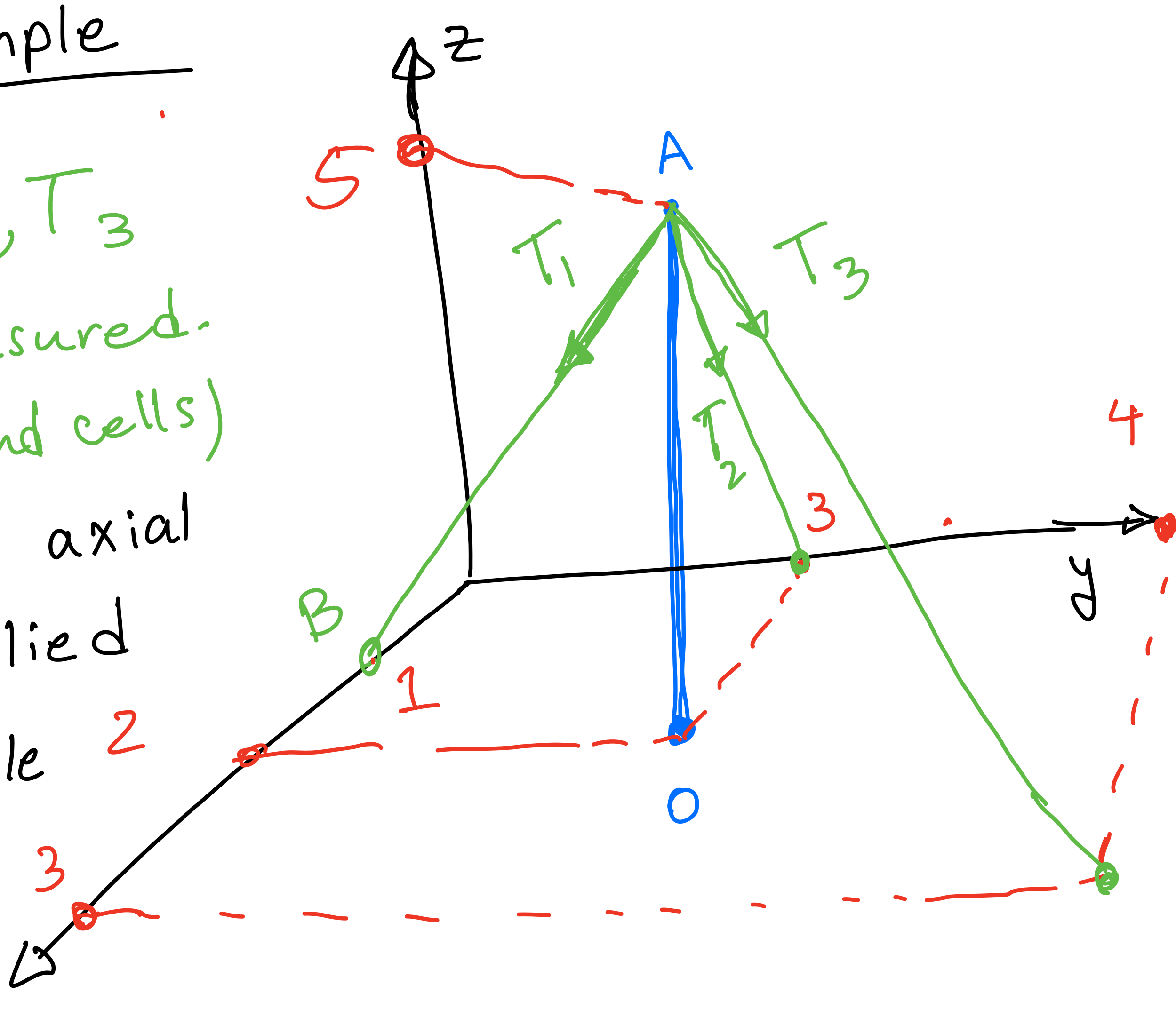
Example

T_1, T_2, T_3

were measured.

(using load cells)

Find the axial load applied to the pole OA



due to the cables tension loads.

$$\text{Vector } \overrightarrow{AB} = x_B\hat{i} + y_B\hat{j} + z_B\hat{k} - (x_A\hat{i} + y_A\hat{j} + z_A\hat{k})$$

$$B \begin{cases} x_B = 1 \\ y_B = 0 \\ z_B = 0 \end{cases} \qquad A \begin{cases} x_A = 2 \\ y_A = 3 \\ z_A = 5 \end{cases}$$

$$\vec{AB} = (1-2)\hat{i} + (0-3)\hat{j} + (0-5)\hat{k}$$

$$= -\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\vec{T_1} = \underbrace{|\vec{T_1}|}_{\text{given}} \vec{AB}_{unit} = |\vec{T_1}| \left(\frac{-\hat{i} - 3\hat{j} - 5\hat{k}}{\sqrt{1^2+3^2+5^2}} \right)$$

$$\vec{OA} = x_A\hat{i} + y_A\hat{j} + z_A\hat{k} - (x_o\hat{i} + y_o\hat{j} + z_o\hat{k})$$

$$\vec{P_1} = (\vec{T_1} \cdot \vec{OA}_{unit})\, \vec{OA}_{unit}$$

projection of $\vec{T_1}$ along the pole

$$\vec{P_2} = (\vec{T_2} \cdot \vec{OA}_{unit})\, \vec{OA}_{unit}$$

$$\vec{P}_3 = (\vec{T}_3 \cdot \overrightarrow{OA}_{unit})\overrightarrow{OA}_{unit}$$

The total axial load due to $\vec{T}_1$, T_2 and T_3 along the pole is the sum of $\vec{P}_1$, $\vec{P}_2$ and $\vec{P}_3$.