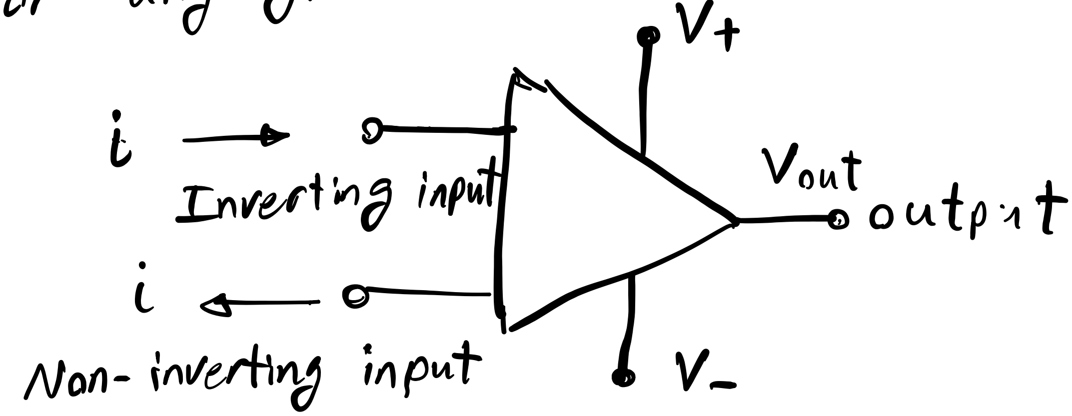
Operational Amplifiers

Introduction

An operational Amplifier, usually referred to as op-Amp, is a DC-coupled high-gain electronic voltage amplifier with two differential inputs and a single output. In its normal operation, the output of the op-Amp is controlled be negative feedback which, due to the amplifier's high gain, will completely control the output voltage for any given input.



Ideal op-Amp

The ideal op-Amp has the following characteristics:

- Infinite voltage gain
- Infinite inputinpedance
- Zero output inpedance
- Infinite bandwidth
- Zero input offset voltage

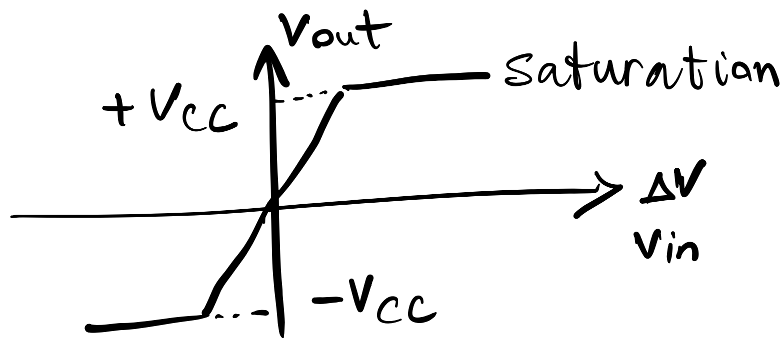
In a typical op-Amp, the circuit has characteristics close to the ideal one.

A camparison between an ideal and a typical op Amp

Parameter	Ideal	Typical
Differential voltage gain	∞	$10^5 - 10^9$
Voltage gain	0	10^{-5}
Gain bandwidth	∞	1-20 MHz
Input impedance	∞	$10^6\Omega - 10^{12}\Omega$
output impendance	0	$100 - 1000\Omega$

The output voltage is a function of the differential input voltage times the gain of the amplifier.

The output voltage saturates to the supply source voltage:



op-Amp rules:

- The voltage rule:

  The output attempts to do whatever is necessary to make the voltage difference between the inputs zero.

- The current rule:

  The inputs draw no current.

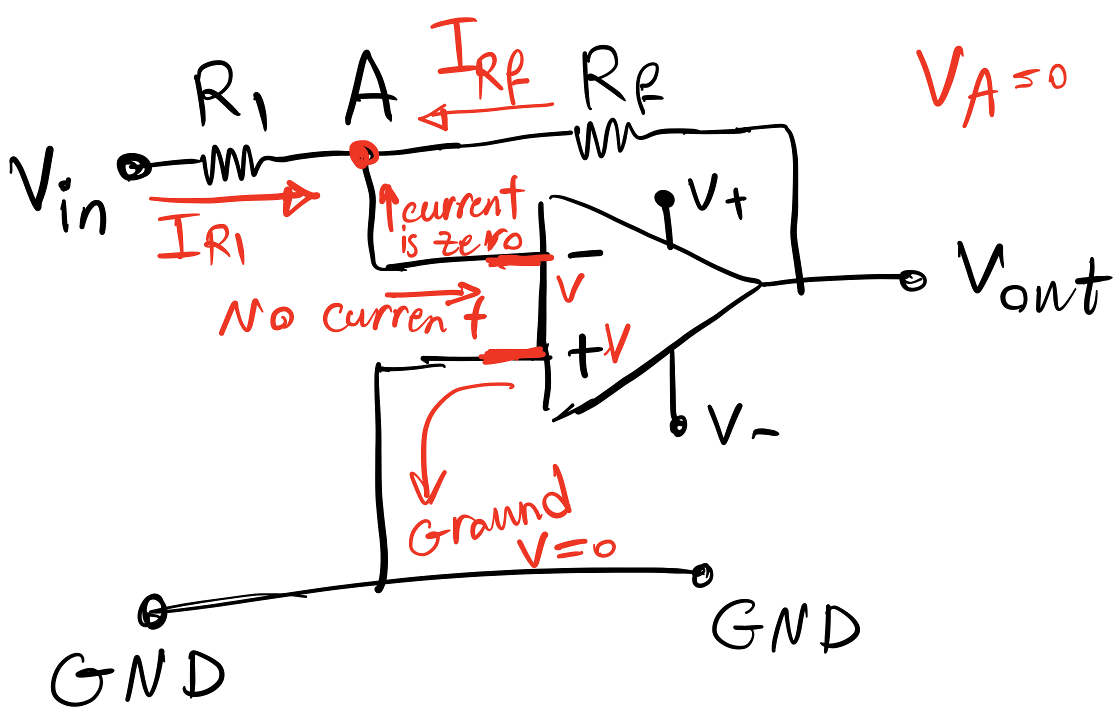
Comments on rule 1: The voltage gain of a real op-Amp is so high that a fraction of a mV input will swing the output over its full range.

Comment on rule 2: The input current is so low (0.08 μA).

Inverting Amplifier

The behaviour of most configurations of Op-Amps can be determined by applying the "op-Amp" rules.

For an inverting amplifier, the current rule tries to drive the current to zero at point A.



$$I_{R_1} = \frac{V_{in}}{R_1} \qquad I_{R_f} = \frac{V_{out}}{R_f}$$

$$I_{Rf} + I_{R1} = 0$$

$$I_{Rf} = -I_{R1}$$

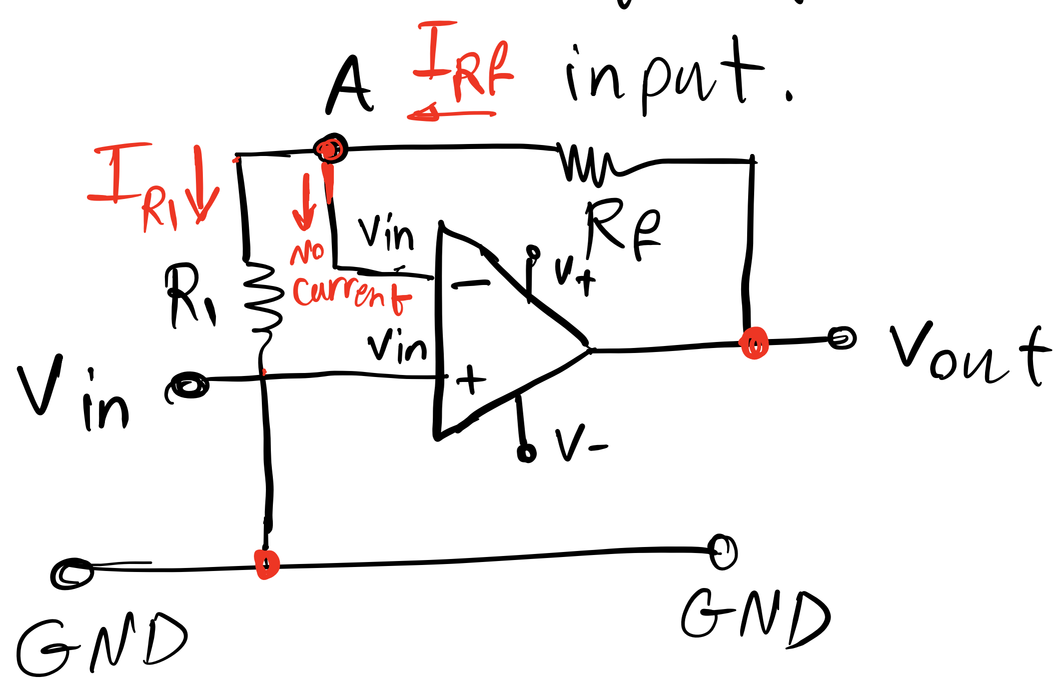
$$\frac{V_{out}}{R_f} = -\frac{V_{in}}{R_1}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_1}$$

Non-Inverting amplifier.

current rule → drive the current to zero at point A

Voltage rule → Make the Voltage At A equal to the input.



$$I_{R_1} = \frac{V_{in}}{R_1}$$

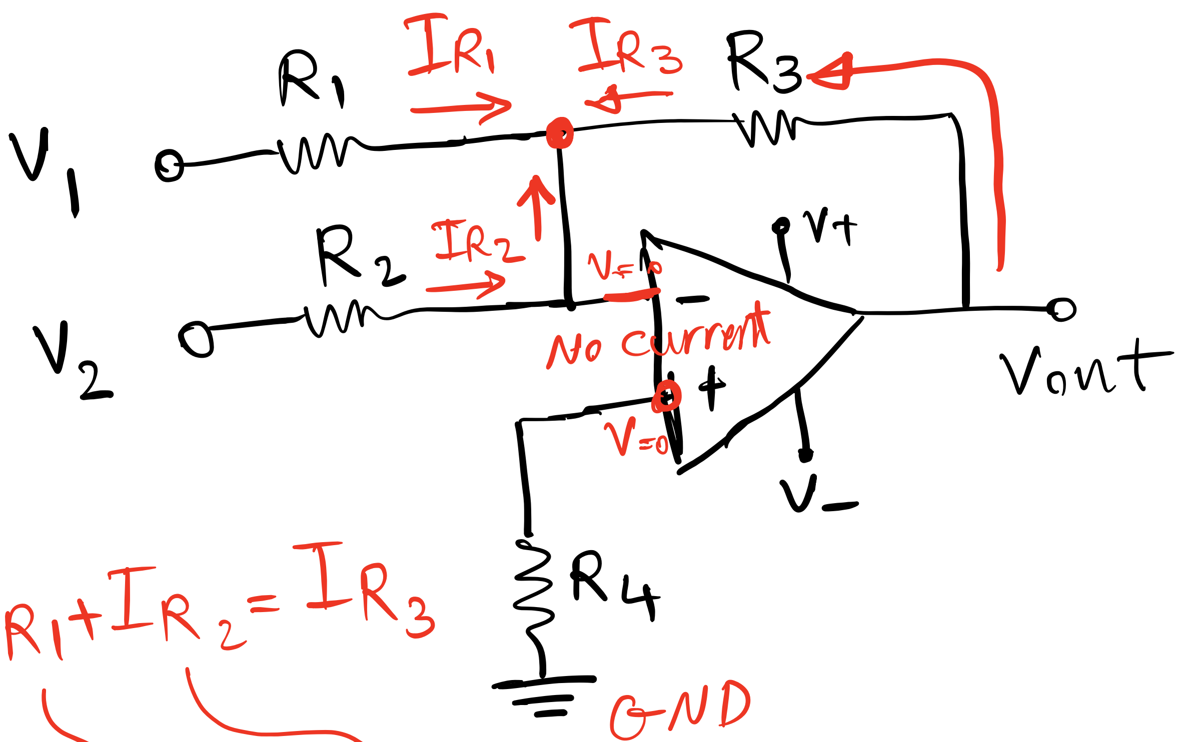
$$I_{R_f} = \frac{V_{out} - V_{in}}{R_f}$$

$$I_{R_1} = I_{R_f}$$

$$\frac{V_{in}}{R_1} = \frac{V_{out} - V_{in}}{R_f}$$

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_1}$$

Summing amplifier

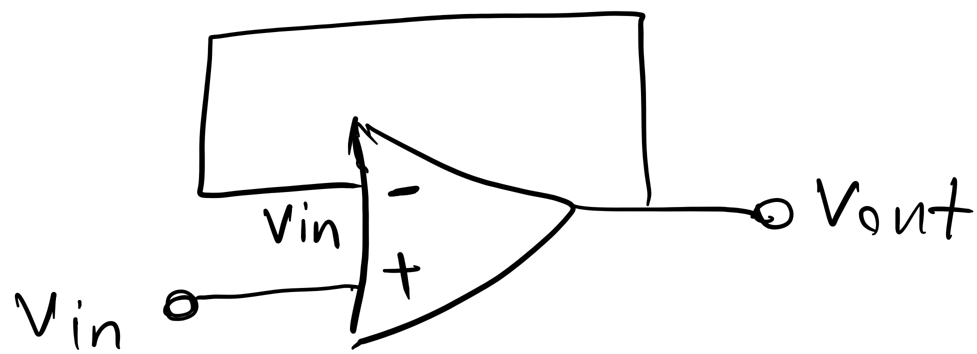


$$I_{R_1} + I_{R_2} = I_{R_3}$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_{out}}{R_3}$$

$$R_1 = R_2 = R_3 \implies V_1 + V_2 = -V_{out}$$

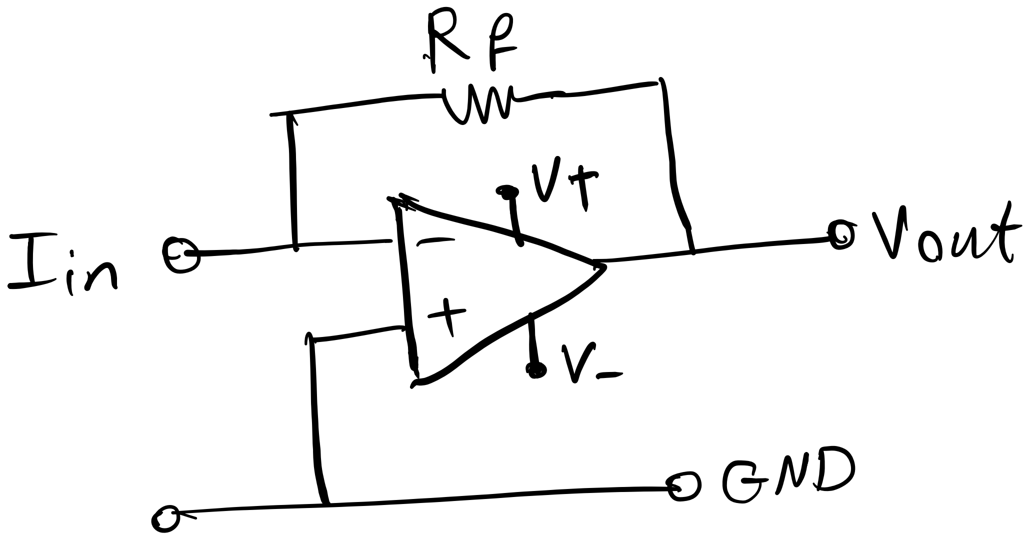
Voltage Follower



$$V_{out} = V_{in}$$

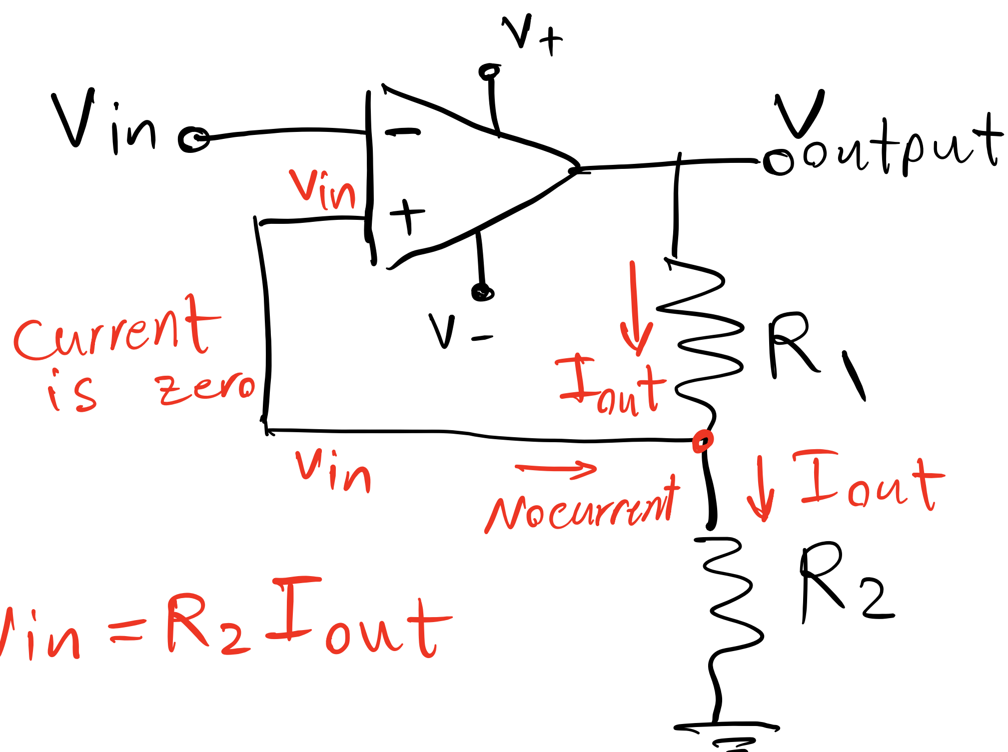
The voltage follower give effective isolation output from the input source to avoide overloading effect. (used in construction of buffers in logic circuits).

Current to voltage Amplifier



$$V_{out} = -I_{in} R_f$$

Voltage to current amplifier



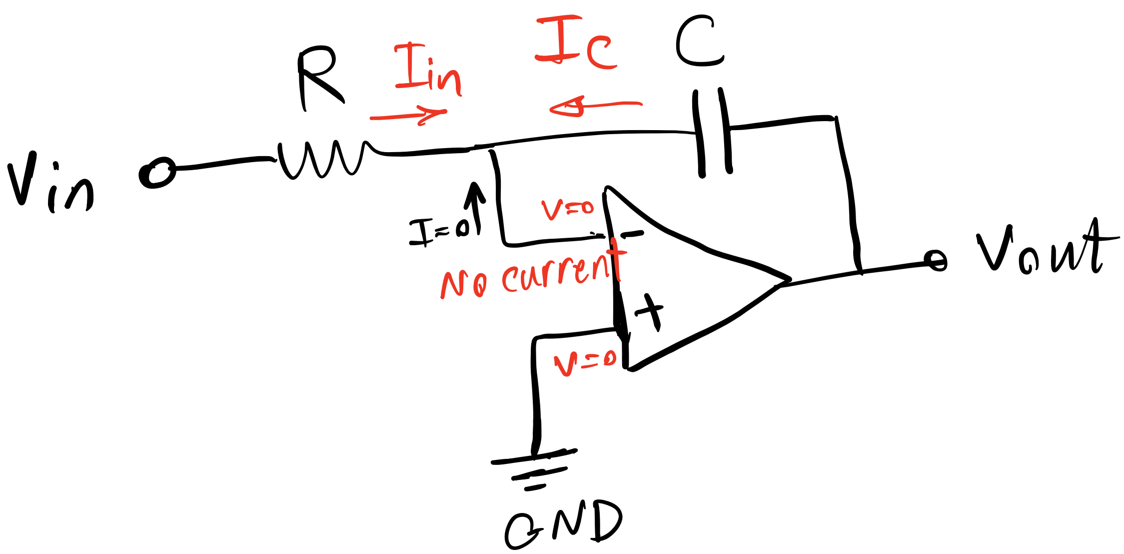
$V_{in} = R_2 I_{out}$

$V_{out} - V_{in} = I_{out}(R_1)$

$V_{out} = I_{out}(R_1 + R_2)$

Integrator circuits

For this circuit, all we have to do is to swap the capacitor and resistor in the previous circuit.



$$I_c + I_{in} = 0 \implies I_c = -I_{in}$$

$$I_c = C\frac{dV_{out}}{dt}$$

$$V_{in} = R I_{in}$$

$$I_{in} = \frac{V_{in}}{R}$$

$$C\frac{dV_{out}}{dt} = -\frac{V_{in}}{R}$$

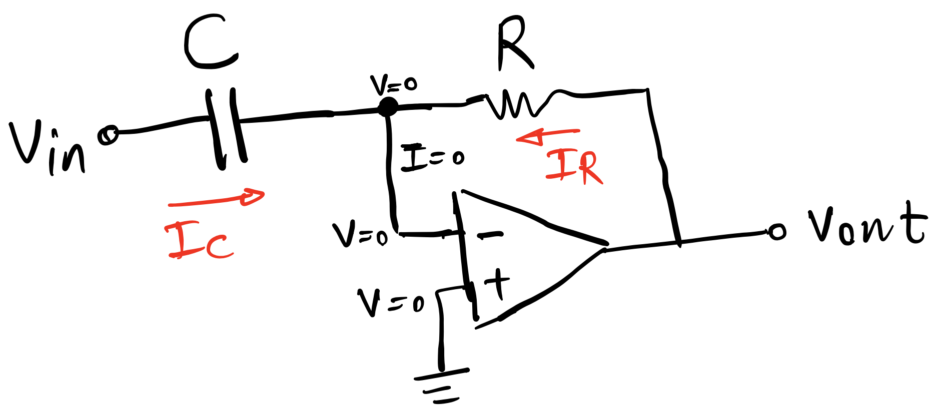
$$\frac{dV_{out}}{dt} = -\frac{V_{in}}{RC}$$

$$dV_{out} = -\frac{V_{in}}{RC}\,dt$$

$$\int dV_{out} = -\int \frac{V_{in}}{RC}\,dt + \text{constant}$$

$$V_{out} = \int -\frac{V_{in}}{RC}\,dt + \text{constant}$$

Differentiator circuit:

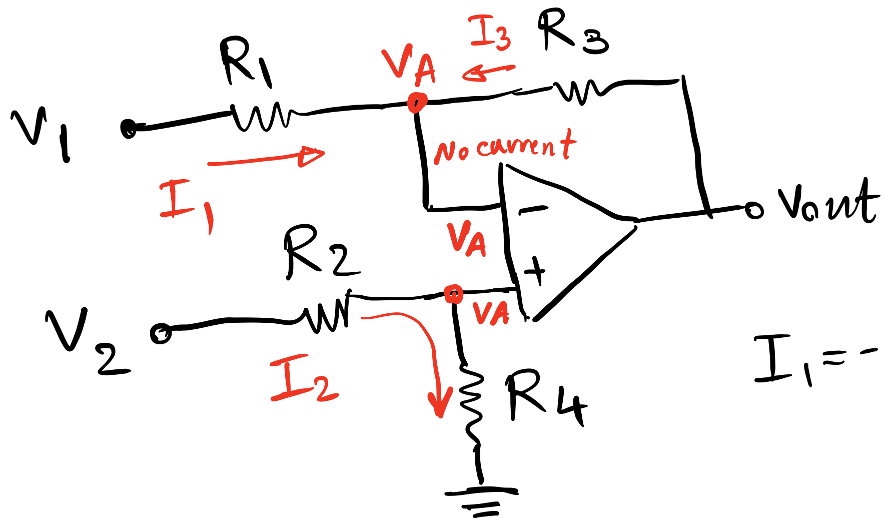


$$I_C = C\frac{dV_{in}}{dt} \qquad I_R = \frac{V_{out}}{R}$$

$$I_R + I_C = 0 \qquad I_R = -I_C$$

$$\frac{V_{out}}{R} = -C\frac{dV_{in}}{dt} \Rightarrow V_{out} = -RC\frac{dV_{in}}{dt}$$

Differential Amplifier



$$I_1 = -I_3$$

$$I_1 = \frac{V_A - V_1}{R_1} \qquad I_3 = \frac{V_A - V_{out}}{R_3}$$

$$I_2 = \frac{V_2 - V_A}{R_2} \qquad I_2 = \frac{V_A}{R_4}$$

$$\begin{cases} \dfrac{V_A - V_1}{R_1} = -\dfrac{V_A - V_{out}}{R_3} \\ \dfrac{V_2 - V_A}{R_2} = \dfrac{V_A}{R_4} \Rightarrow \dfrac{V_2}{R_2} - \dfrac{V_A}{R_2} = \dfrac{V_A}{R_4} \end{cases}$$

$$\frac{V_2}{R_2} = V_A\left(\frac{1}{R_2} + \frac{1}{R_4}\right) \Rightarrow \frac{V_2}{R_2} = V_A \frac{R_2+R_4}{R_2R_4}$$

$$V_A = \frac{V_2}{R_2}\left(\frac{R_2R_4}{R_2+R_4}\right)$$

$$\frac{V_A - V_1}{R_1} = -\frac{V_A - V_{out}}{R_3}$$

$$\frac{V_A}{R_1} + \frac{V_A}{R_3} = \frac{V_1}{R_1} + \frac{V_{out}}{R_3}$$

$$V_A\left(\frac{R_1+R_3}{R_1R_3}\right) = \frac{V_1}{R_1} + \frac{V_{out}}{R_3}$$

$$V_A = \frac{V_2}{R_2}\left(\frac{R_2R_4}{R_2+R_4}\right)$$

$$\frac{V_2}{\cancel{R_2}}\left(\frac{\cancel{R_2} R_4}{R_2+R_4}\right)\left(\frac{R_1+R_3}{R_1 R_3}\right) = \frac{V_1}{R_1} + \frac{V_{out}}{R_3}$$

$$V_{out} = V_2\,\frac{(R_3+R_1)R_4}{(R_4+R_2)\,R_1} - V_1\,\frac{R_3}{R_1}$$

Differential amplifier

Example: $R_1 = R_2 = R_3 = R_4$

$$\boxed{V_{out} = V_2 - V_1}$$

Difference between input voltages V_1 and V_2