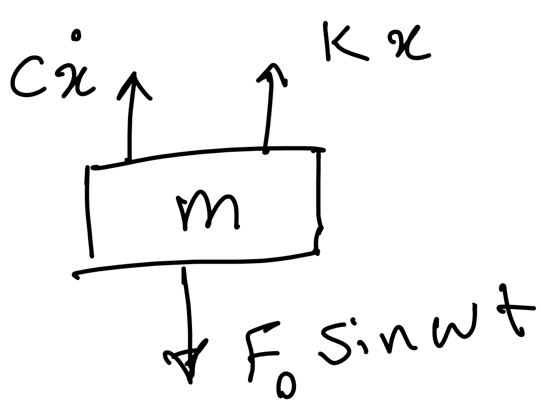
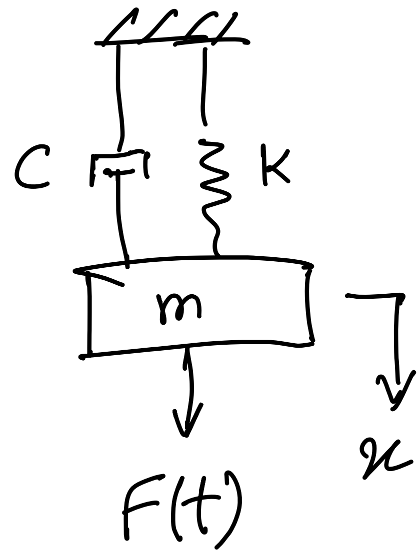
chapter 3

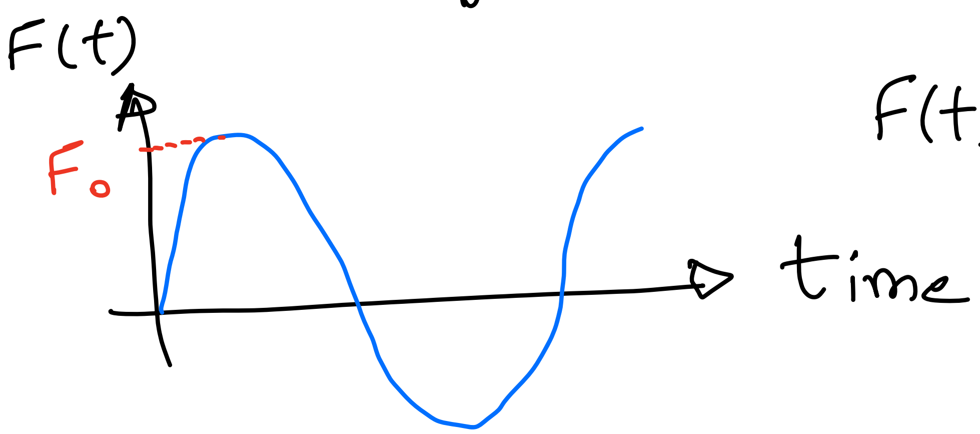
Harmonically Excited vibration

First we consider single-DOF system with viscous damping.

F.B.D







$F(t) = F_0 \sin\omega t$

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$$

The solution to this equation consists of two parts, the complementary function, which is the solution of the homogeneous equation, and the particular part.

The complementary is a damped FREE vibration that was discussed in Chapter 2.

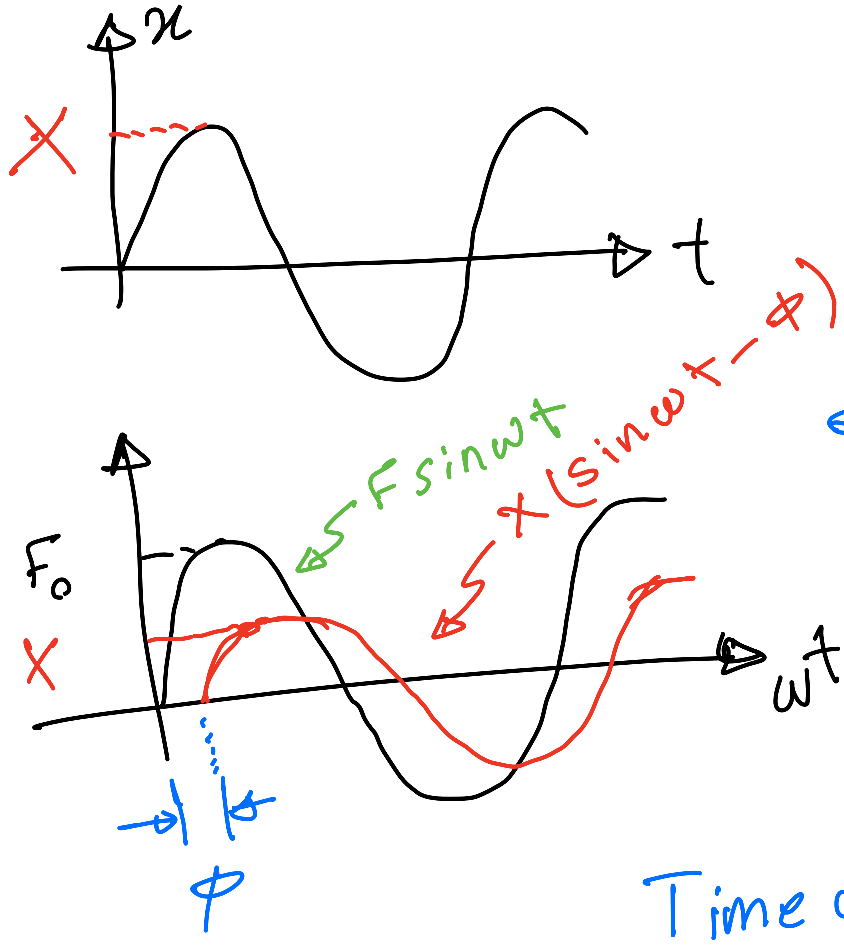
The particular solution is a steady-state oscilation of the same frequency ω as the excitation ($F_0 \sin\omega t$)

The particular solution:

$$x = X \sin(\omega t + \phi)$$

The amplitude of oscillation

phase of the displacement with respect to the exciting force.



$$\phi = \frac{\text{Time delay}}{T} 2\pi$$

T : period of oscilation

$$\text{Time delay} = \frac{\phi T}{2\pi}$$

Excitation force:

$$F_0 (\cos\omega t + i \sin\omega t) = F_0 e^{i\omega t}$$

$i = \sqrt{-1}$

phase shift

$$x = X e^{i(\omega t - \phi)} = \underbrace{\left(X e^{-i\phi}\right)}_{\bar{X}} e^{i\omega t}$$

$$= \bar{X} e^{i\omega t}$$

$$\bar{X} = X e^{-i\phi}$$

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$x = \bar{X} e^{i\omega t}$$

$$\boxed{i \cdot i = -1}$$

$$\dot{x} = \bar{X} i\omega e^{i\omega t}$$

$$\ddot{x} = -\bar{X}\omega^2 e^{i\omega t}$$

$$F(t) = F_0 e^{i\omega t}$$

$$-m\bar{X}\omega^2 e^{i\omega t} + c\bar{X} i\omega e^{i\omega t} + k\bar{X} e^{i\omega t} = F_0 e^{i\omega t}$$

$$\bar{X}\cancel{e^{i\omega t}}\left[-m\omega^2 + c\omega i + k\right] = F_0 \cancel{e^{i\omega t}}$$

$$\left(-\omega^2 m + ic\omega + k\right)\bar{X} = F_0$$

$$\bar{X} = \frac{F_0}{(k - \omega^2 m) + i(c\omega)}$$

$z = a + ib$ Real Imag

$|z| = \sqrt{a^2 + b^2}$

$F_0 = F_0 + 0 \times i$

$$|\bar{X}| = \frac{|F_0|}{|(K-\omega^2 m) + i(c\omega)|}$$

magnatute

$$|F_0| = \sqrt{F_0^2 + 0^2} = F_0$$

$$|\bar{X}| = \frac{F_0}{\sqrt{(K-\omega^2 m)^2 + (c\omega)^2}}$$

natural frequency of undamed oscilation

$$\omega_n = \sqrt{\frac{K}{m}}$$

critical damping $\quad c_c = 2m\omega_n$

Damping coefficient

Damping factor (ratio) $\quad \zeta = \frac{c}{c_c}$

$$\frac{c\omega}{K} = \frac{c}{c_c}\frac{c_c\omega}{K} = 2\zeta\frac{\omega}{\omega_n}$$

$$z = a + ib \qquad \angle z = \tan^{-1}\frac{b}{a}$$

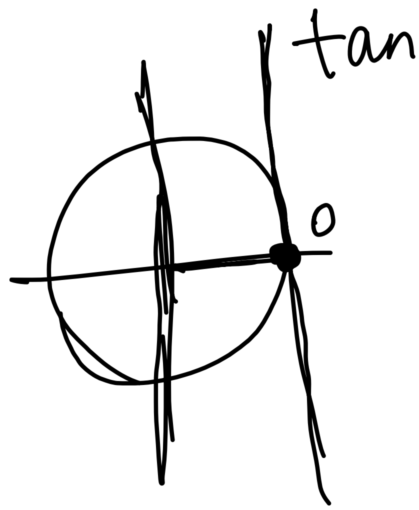
$$\angle \frac{z_1}{z_2} = \angle z_1 - \angle z_2$$

z_2

$$\angle \bar{X} = \frac{F_0 + i\,0}{\underbrace{(K-\omega^2 m)}_{\text{Real}} + i\,\underbrace{(c\omega)}_{\text{Imag.}}}$$

$$\Phi = \tan^{-1}\frac{0}{F_0} - \tan^{-1}\frac{c\omega}{K-\omega^2 m}$$

$$\Phi = 0 - \tan^{-1}\frac{c\omega}{K-\omega^2 m}$$



$$\Phi = -\tan^{-1}\frac{c\omega}{K-\omega^2 m}$$

Instead of $c, K, m \xrightarrow[\text{in terms of}]{\text{write}} \omega_n, \zeta$

$$|\bar{X}| = \frac{F_0}{\sqrt{(K-\omega^2 m)^2 + (c\omega)^2}}$$

Divide the numerator and the denominator by K

$$\bar{X} = \frac{F_o/K}{\sqrt{\left(1-\frac{m\omega^2}{K}\right)^2 + \left(\frac{c\omega}{K}\right)^2}}$$

ξ, ω_n

$\omega_n = \sqrt{\frac{K}{m}}$ or $\omega_n^2 = \frac{K}{m}$

$\frac{c}{K}$

$\xi = \frac{c}{c_c}$ $\cancel{C_c = 2\sqrt{Km}}$

$\cancel{\xi = \frac{c}{2\sqrt{Km}}} \Rightarrow \cancel{\xi^2 = \frac{c^2}{(\sqrt{Km})^2}}$

$\cancel{\xi^2 = \frac{c^2/m}{4K}}$

$$\boxed{C_c = 2m\omega_n}$$

$\xi = \frac{c}{c_c}$

$\omega_n = \sqrt{\frac{K}{m}}$ $\omega_n = \sqrt{\frac{K}{c_c/2\omega_n}}$

$\omega_n^2 = \frac{K(2\omega_n)}{c_c} \Rightarrow \cancel{\omega_n^2} = \frac{K(2\cancel{\omega_n})}{c/\xi}$

$$\boxed{\frac{c}{K} = \frac{2\xi}{\omega_n}}$$

$$\bar{X} = \frac{F_0/K}{\sqrt{\left(1-\frac{m\omega^2}{K}\right)^2 + \left(\frac{c\omega}{K}\right)^2}}$$

$$\frac{XK}{F_0} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\phi = -\tan^{-1}\frac{c\omega}{K-\omega^2 m}$$

$$\tan\phi = \frac{-c\omega}{K-\omega^2 m}$$

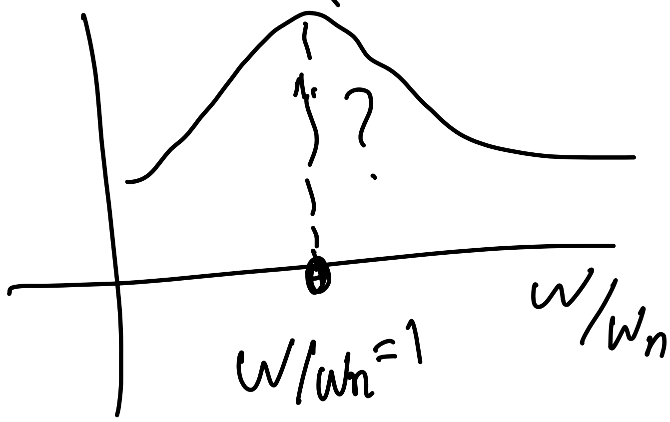
$$\tan\phi = \frac{-c/K\ \omega}{1-\omega^2 m/K}$$

$$\frac{c}{K} = 2\xi/\omega_n$$

$$\tan\Phi = \frac{-2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{m}{K} = \frac{1}{\omega_n^2}$$

$$\frac{\omega}{\omega_n} = ?$$



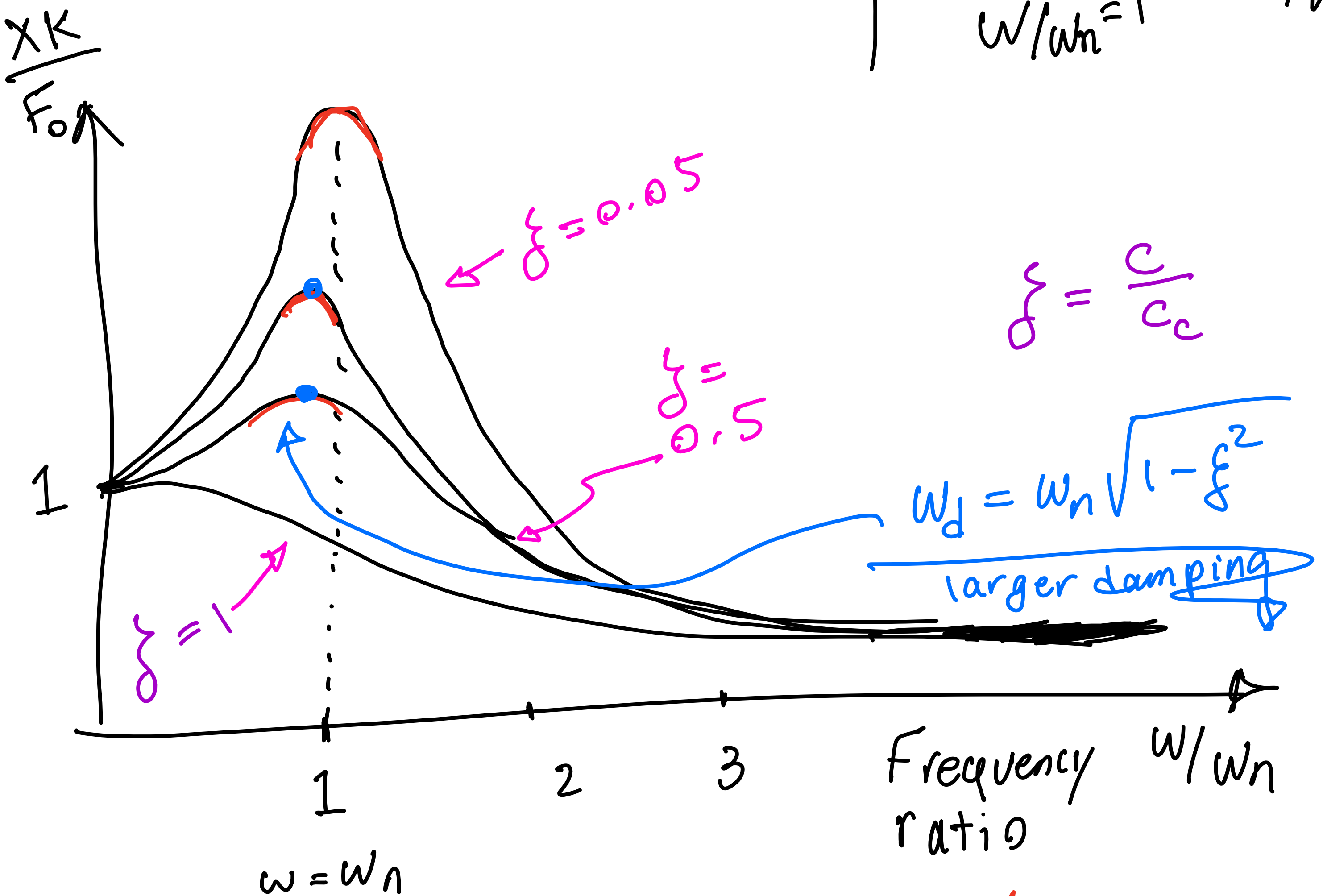


Fig. 3.1-3 Book

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{m}\sin\omega t$$

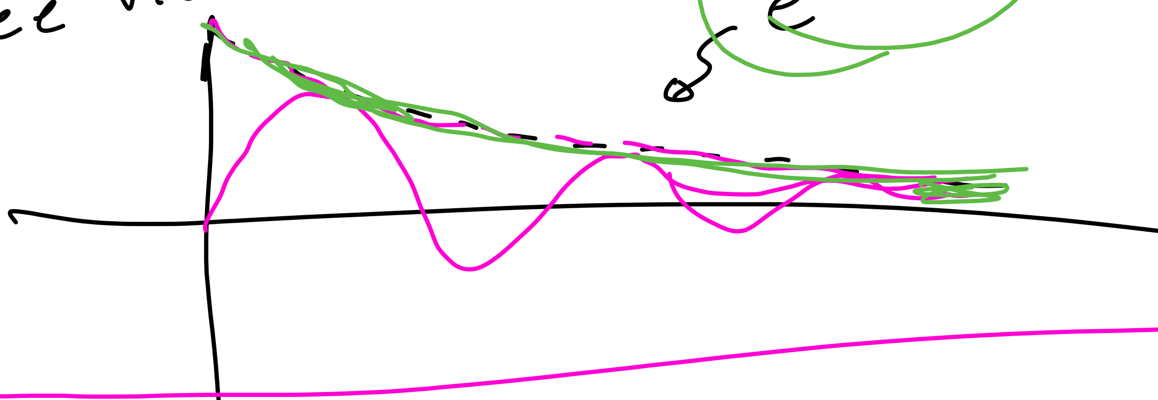
$$x(t) = \frac{F_0}{K}\frac{\sin(\omega t - \phi)}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$+ X_1 e^{-\xi\omega_n t}\sin\left(\sqrt{1-\xi^2}\,\omega_n t + \phi_1\right)$$

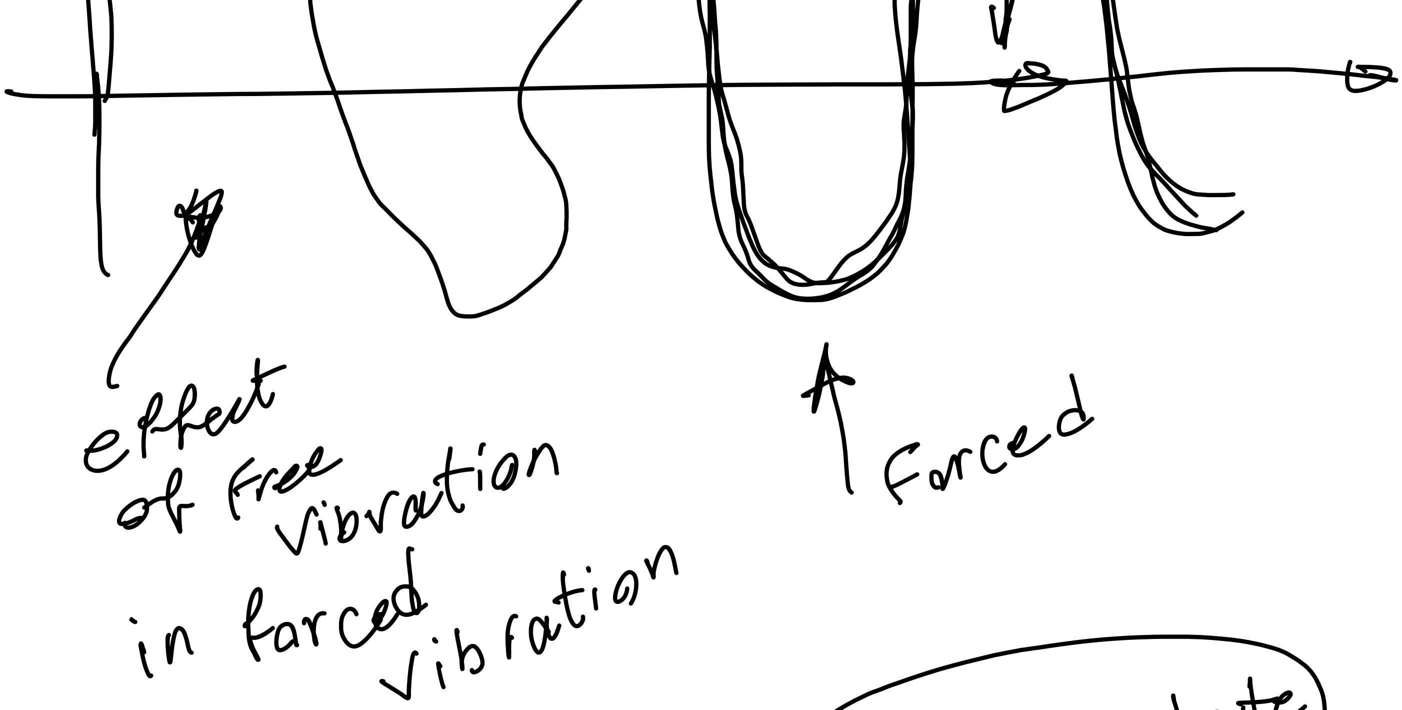
$$e^{-\xi\omega_n t} = \frac{1}{e^{\xi\omega_n t}}$$

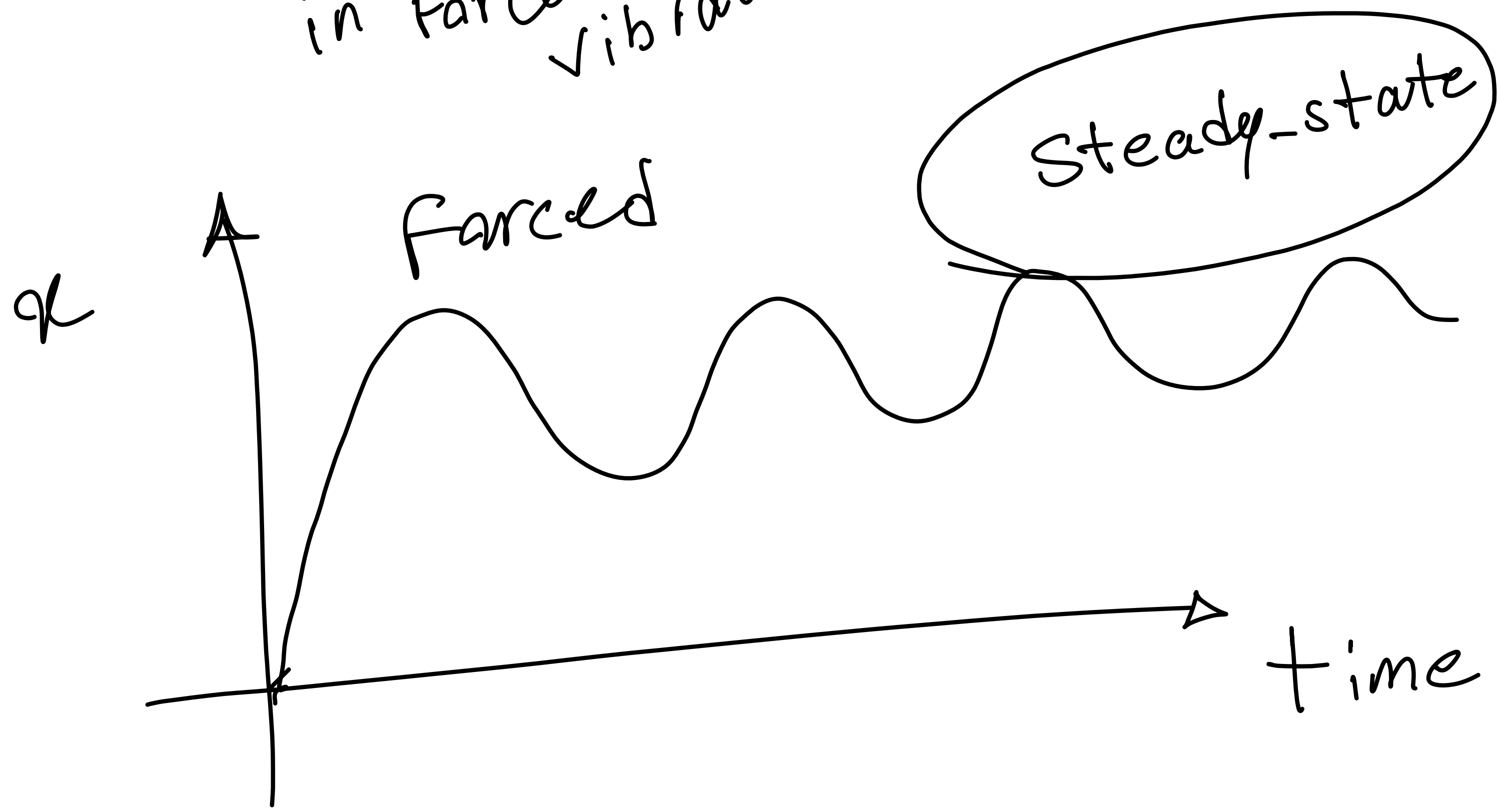
Free vibratian

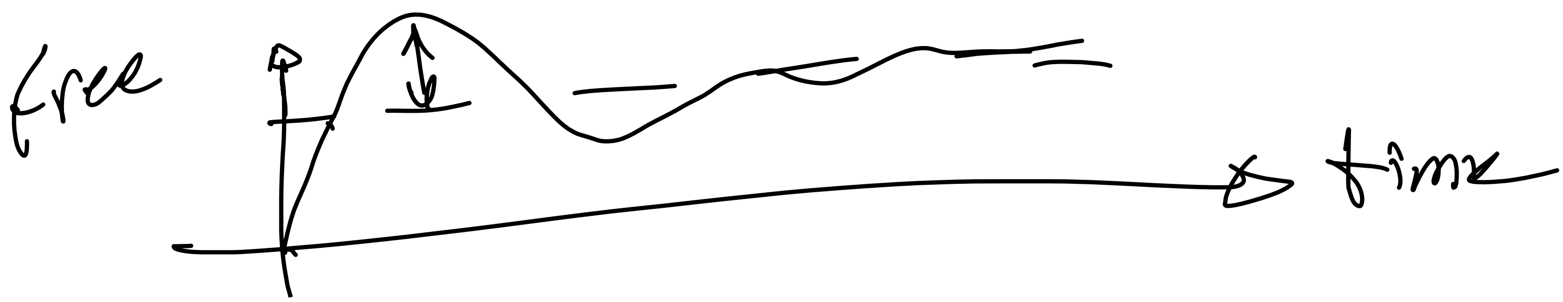
$e^{-\xi\omega_n t}$



transient


effect of free vibration in forced vibration
Forced


Steady-state
forced
x
time


free
time

Next time

Rotating unbalance