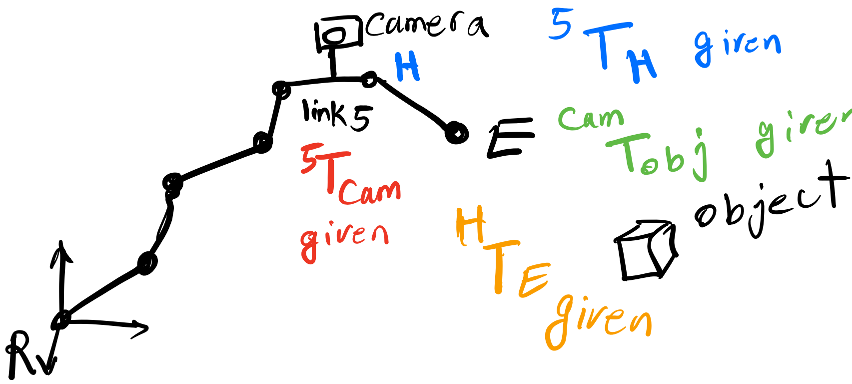
Example 2.15

In a robotic set-up, a camera is attached to the fifth link of a 6-DOF robot. It observes an object and determines its frame relative to the camera's frame. Using the following information, determine the necessary motion the end effector must make to get to the object.



$^{5}T_{H}$ given

$^{cam}T_{obj}$ given

$^{5}T_{cam}$ given

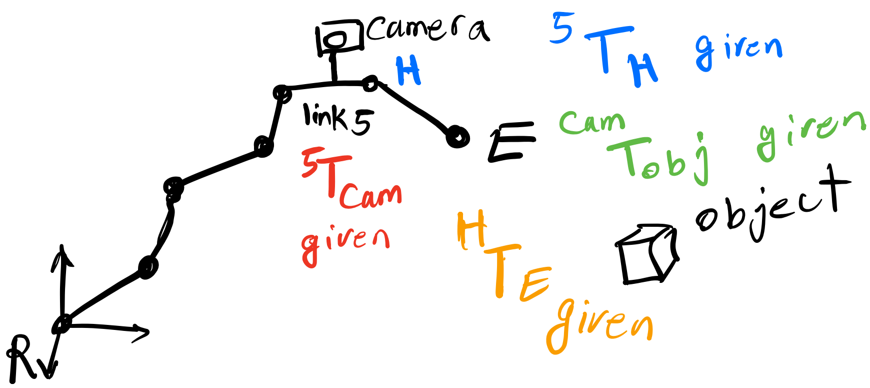
$^{H}T_{E}$ given

$$
{}^{5}T_{cam} = \begin{bmatrix} 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}T_{H} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{cam}T_{obj} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{H}T_{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



$$\underset{?}{{}^{R}T_5} \times \underset{\text{given}}{{}^{5}T_H} \times \underset{\text{given}}{{}^{H}T_E} \times \underset{?}{{}^{E}T_{obj}} = \underset{?}{{}^{R}T_5} \times \underset{\text{given}}{{}^{5}T_{cam}} \times \underset{\text{given}}{{}^{cam}T_{obj}}$$

$$\underbrace{({}^{R}T_5)^{-1}\,{}^{R}T_5}_{I} \times {}^{5}T_H \times {}^{H}T_E \times \overset{?}{{}^{E}T_{obj}} = \underbrace{({}^{R}T_5)^{-1}\,{}^{R}T_5}_{I} \times {}^{5}T_{cam} \times {}^{cam}T_{obj}$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I \times {}^{5}T_{H} \times {}^{H}T_{E} \times \overset{?}{{}^{E}}T_{obj} = I \times {}^{5}T_{cam} \times {}^{cam}T_{obj}$$

$${}^{5}T_{H} \times {}^{H}T_{E} \times \overset{?}{{}^{E}}T_{obj} = {}^{5}T_{cam} \times {}^{cam}T_{obj}$$

$$\underbrace{({}^{5}T_{H})^{-1}\,{}^{5}T_{H}}_{I} \times {}^{H}T_{E} \times {}^{E}T_{obj} = ({}^{5}T_{H})^{-1}\,{}^{5}T_{cam} \times {}^{cam}T_{obj}$$

$${}^{H}T_{E} \times {}^{E}T_{obj} = ({}^{5}T_{H})^{-1} \times {}^{5}T_{cam} \times {}^{cam}T_{obj}$$

$$\underbrace{({}^{H}T_{E})^{-1}\,{}^{H}T_{E}}_{I} \times {}^{E}T_{obj} = ({}^{H}T_{E})^{-1}({}^{5}T_{H})^{-1} \times {}^{5}T_{cam} \times {}^{cam}T_{obj}$$

$$ {}^{E}T_{obj} = ({}^{H}T_{E})^{-1} \times {}^{5}(T_{H})^{-1} \times {}^{5}T_{cam} \times {}^{cam}T_{obj} $$

$$ {}^{H}T_{E}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

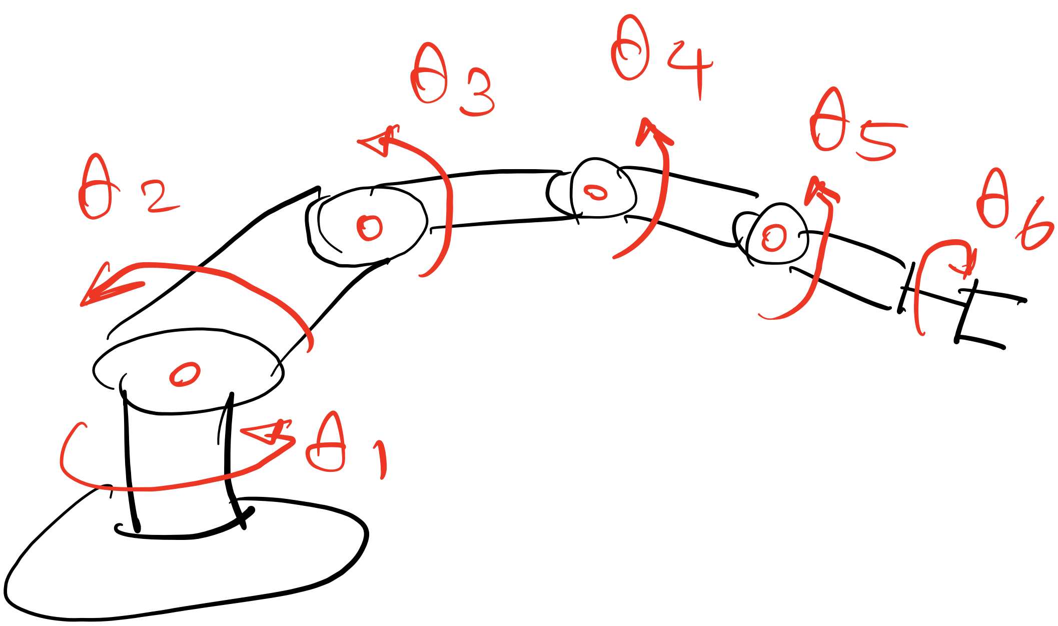
$$ {}^{5}T_{H}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{E}T_{obj} = \begin{bmatrix} -1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

See example 2.15 in the book

Forward and inverse Kinematics of robots (sec. 2.8 Book)

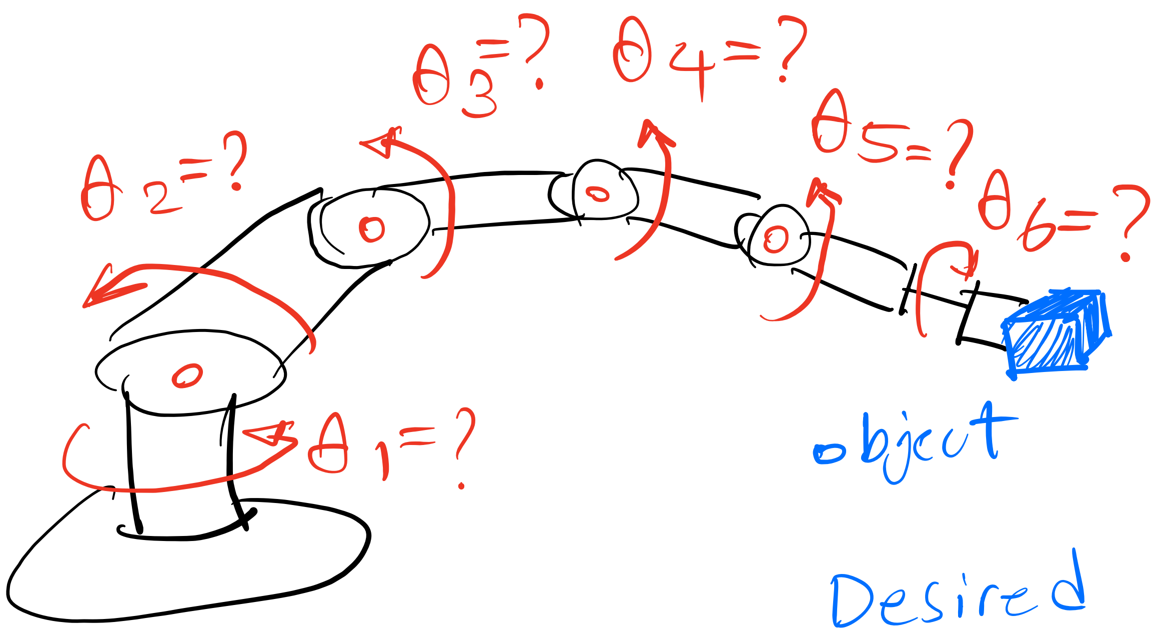
Forward Kinematics



$\theta_1, \theta_2, \cdots \theta_6$ will be given to the motors.

The hand will move to a position based on the given θ values.

Inverse Kinematics:



Find $\theta_1, \theta_2, \theta_3, \dots, \theta_6$ for the robot hand to move to a desired position and orientation.

Forward and Inverse Kinematic Equations: Position (sec. 2.9)

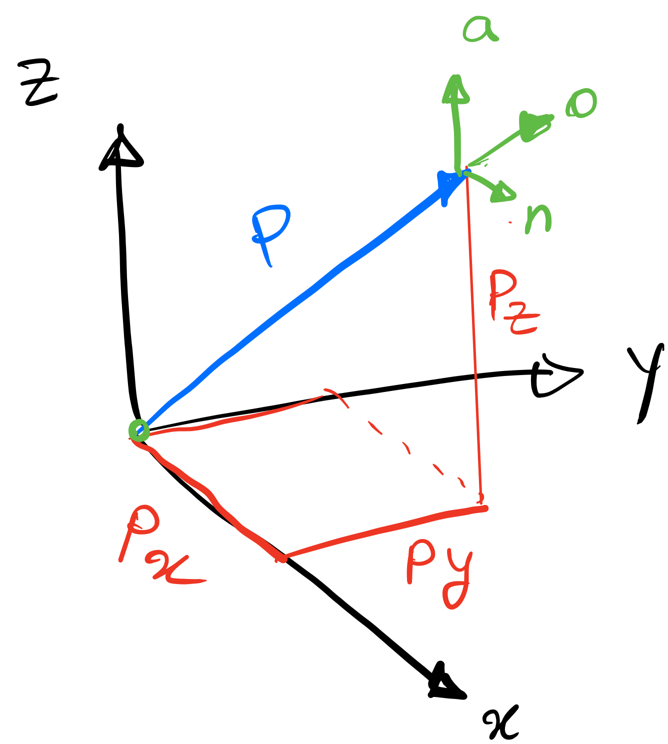
(a) Cartesian (gantry, rectangular) coordinates

(b) cylindrical coordinates

(c) spherical coordinates

(d) Articulated (All-revolute) coordinates

Cartesian (gantry, Rectangular) coordinates



The transformation matrix representing the forward Kinematic equation of the position of the hand of the robot in a Cartisian Coordinate system will be:

$$ {}^{R}T_{P} = T_{cart}(P_x, P_y, P_z) = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

For the inverse Kinematic solution, simply set the desired position equal to P.

Example (2.16 book)

It is desired to position the origin of the hand frame of a cartesian robot at point $P = [3, 4, 7]^T$. calculate the necassary Cartesian coordinate motions that need to be made.

Solution:

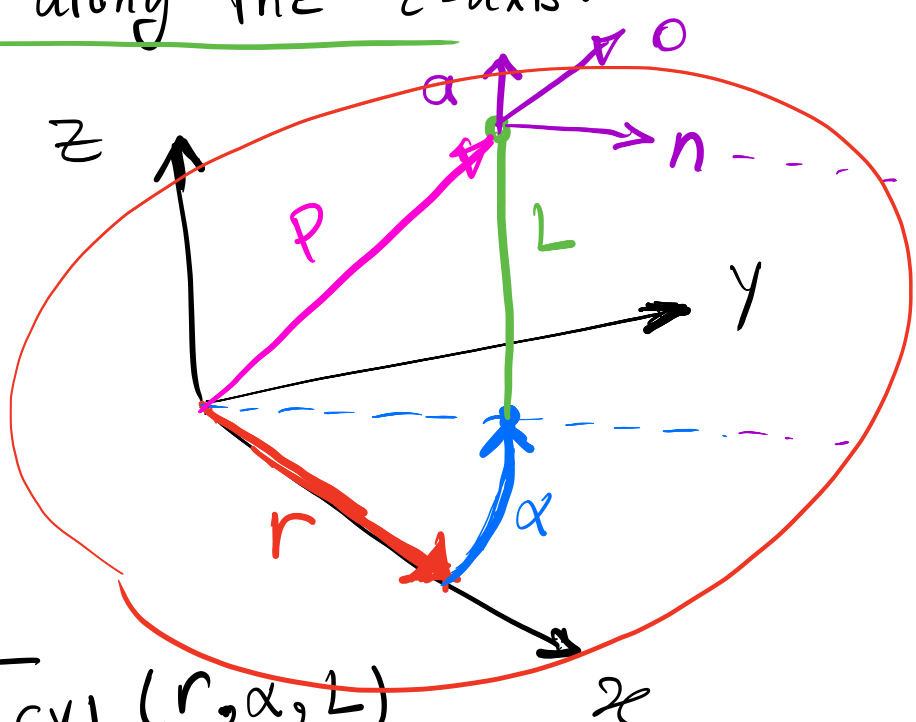
$$
{}^{R}T_{P} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$P_x = 3$, $P_y = 4$, $P_z = 7$

Cylindrical coordinates

A cylindrical coordinate system includes two linear translations (2 prismatic joints, or 2 linear actuators) and one rotation (one revolute joint or one motor).

The sequence is tranlation of r along the x-axis, a rotation of α about the z-axis, and a translation of L along the z-axis.



$$^{R}T_P = T_{cyl}(r, \alpha, L)$$

$$= \text{Trans}(0,0,L)\,\text{Rot}(z,\alpha)\,\text{Trans}(r,0,0)$$

$$
{}^{R}T_{p} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c\alpha & -s\alpha & 0 & 0 \\ s\alpha & c\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{R}T_{p} = T_{cyl}(r, \alpha, L) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

You may restore the original orientation of the frame by rotating the n, o, a frame about the a-axis an angle of $-\alpha$, which is equivalent of post-multiplying the cylindrical coordinate matrix by a ratatoin matrix of

$Rot(a, -\alpha)$:

$T_{cyl} \times Rot(a, -\alpha)$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c(-\alpha) & -s(-\alpha) & 0 & 0 \\ s(-\alpha) & c(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & rc\alpha \\ 0 & 1 & 0 & rs\alpha \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example (2.17 book)