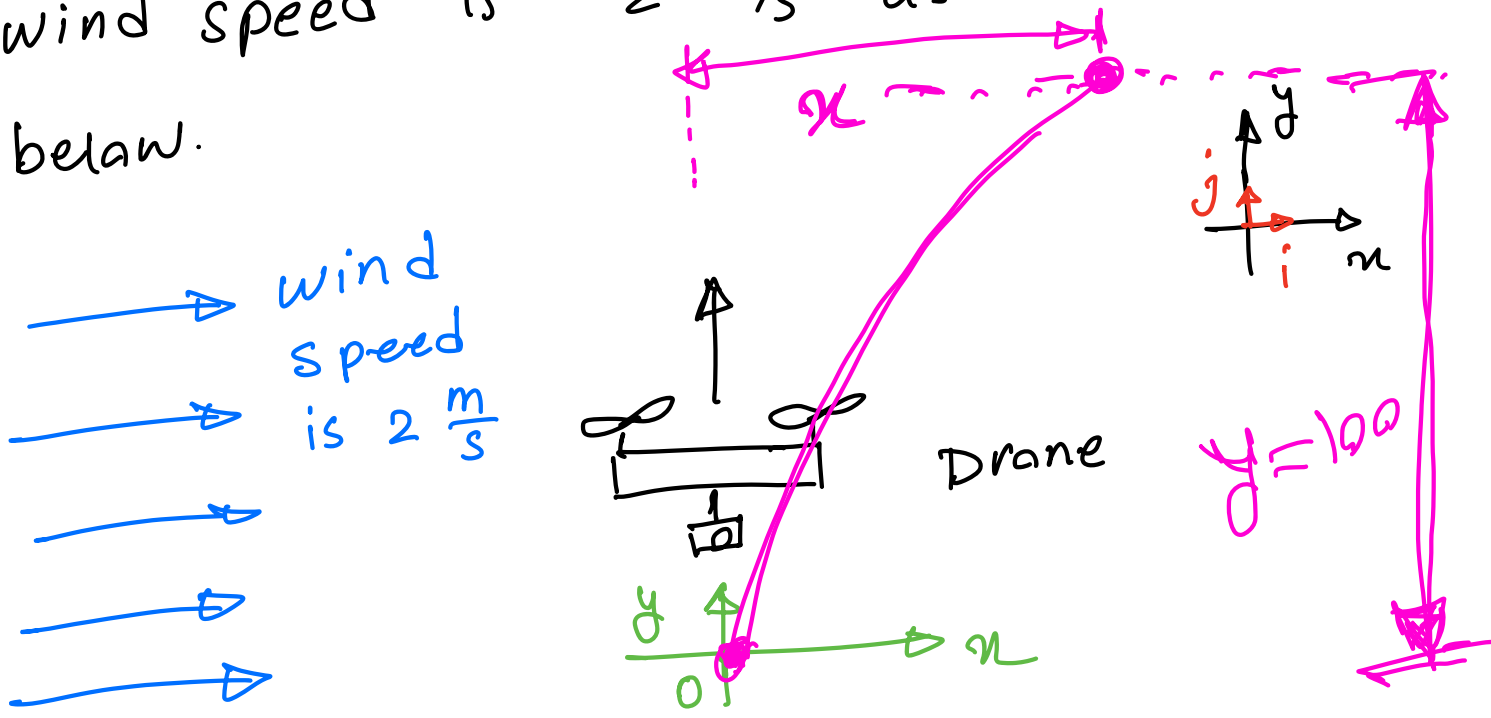
Relative Motion

Example

A drone is flying with the speed of $1 \frac{m}{s}$, and the wind speed is 2 m/s as shown below.



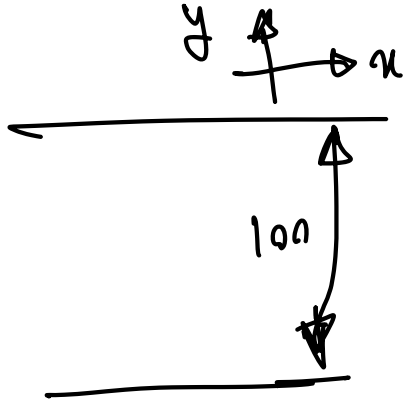
IF the drone starts at $x=0$, $y=0$, Find the position for x when $y=100$ m

$$V_{Drone/wind} = V_{Drone} - V_{wind}$$

Given ? Given

$$1\hat{j} = \vec{V_D} - 2\hat{i}$$

$$\vec{V_D} = 2\hat{i} + 1\hat{j}$$



$y = 100$ m

$$\text{velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$\text{time} = \text{Displacement} / \text{velocity}$$

$$\text{time} = y/1 = 100 \text{ second}$$

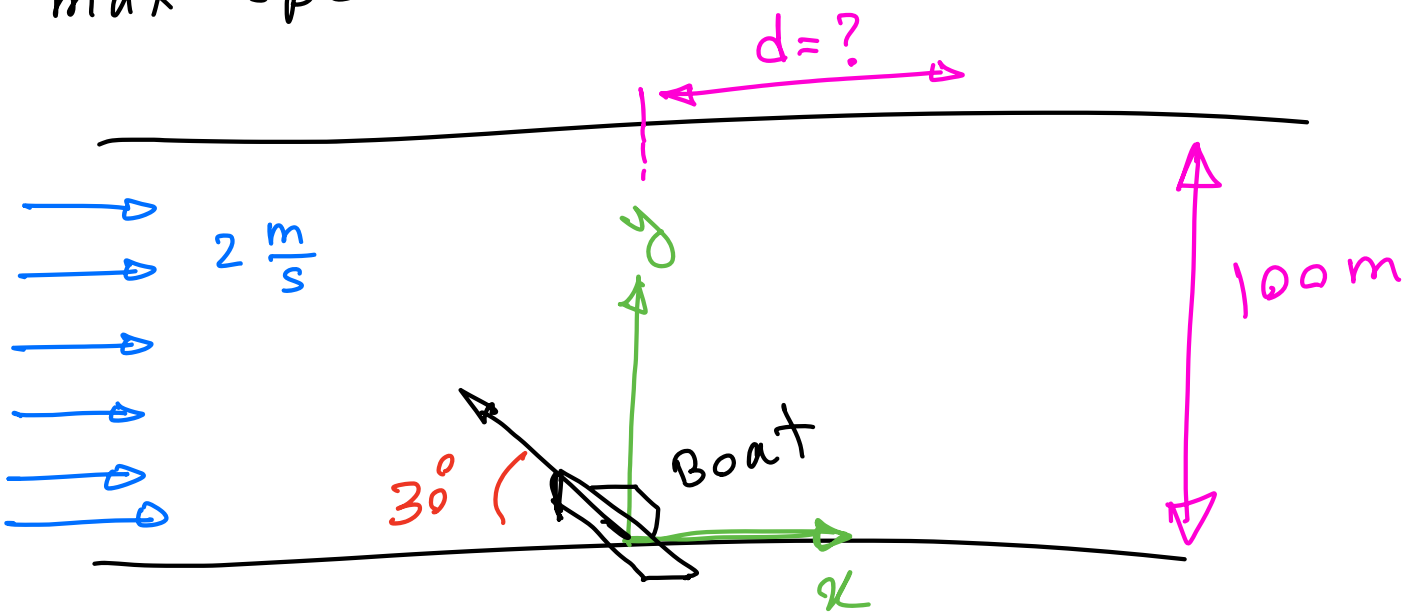
$$x = \text{velocity} \times \text{time}$$

$$x = 2 \frac{m}{s} \times 100\ s = 200\ m$$

Example

A boat is traveling in a river as shown. The speed of the water is 2 m/s. Find the distance "x"

d along the x-axis, when the boat travels 100 m in y direction. The max speed of the boat is $1 \frac{m}{s}$ and the boat is traveling with it's max speed.



$$\vec{V}_R = 2\,\hat{i}$$

$$\vec{V}_{B/R} = 1\,(-\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j})$$

$$\vec{V}_{B/R} = \vec{V}_B - \vec{V}_R$$

$$\vec{V}_B = \vec{V}_{B/R} + \vec{V}_R$$

$$\vec{V}_B = -\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j} + 2\hat{i}$$

$$= (-\cos 30^\circ + 2)\,\hat{i} + \sin 30^\circ\,\hat{j}$$

$$= \left(2 - \frac{\sqrt{3}}{2}\right)\hat{i} + \frac{1}{2}\,\hat{j}$$

Displacement = velocity × time

$$\left(d\,\hat{i} + 100\,\hat{j}\right) = \left[\left(2 - \frac{\sqrt{3}}{2}\right)\hat{i} + \frac{1}{2}\,\hat{j}\right] t$$

$$\begin{cases} 100 = \frac{1}{2}\,t \implies t = 200 \text{ seconds} \\ d = \left(2 - \frac{\sqrt{3}}{2}\,t\right) \end{cases}$$

$$\Rightarrow d \simeq 225 \text{ m}$$

Example

Find the vector of a magnitute of 5 along vector $\vec{a}$.

$$\vec{a} = 1\,\hat{i} + 2\,\hat{j} - \hat{k}$$

~~$5\vec{a} = 5(1\hat{i} + 2\hat{j} - \hat{k})$~~

step 1: Find the unit vector for $\vec{a}$

(It gives the direction of $\vec{a}$ with magnitute of 1)

$$\vec{a}_{unit} = \frac{\vec{a}}{\text{magnitude of } \vec{a}}$$

$$\text{magnitute of } \vec{a} = \sqrt{1^2 + 2^2 + (-1)^2}$$

$$\vec{a}_{unit} = \frac{1\hat{i} + 2\hat{j} - 1\hat{k}}{\sqrt{1+4+1}}$$

step 2: Multiply by 5:

Therefore, the vector with the magnitude of 5 in the direction of vector $\vec{a}$ is:

$$5\,\vec{a}_{unit} = 5\left(\frac{\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}}\right)$$

Dot products and cross products next week