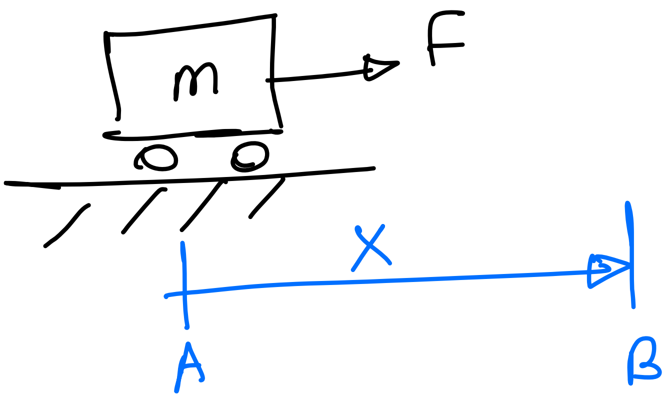
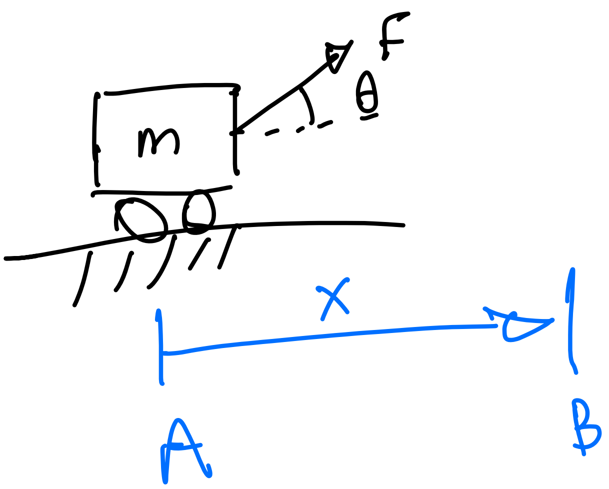
Virtual work

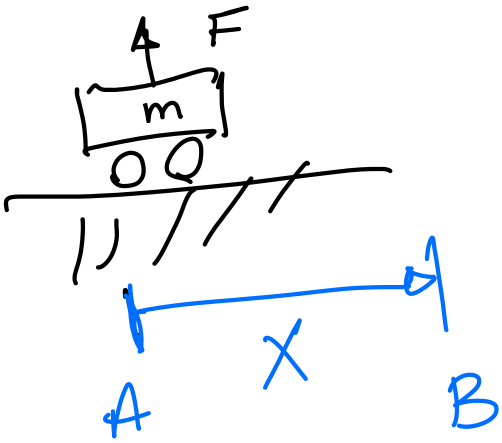
work = Force × displacement
(Force in the direction of displacement)



work = F x



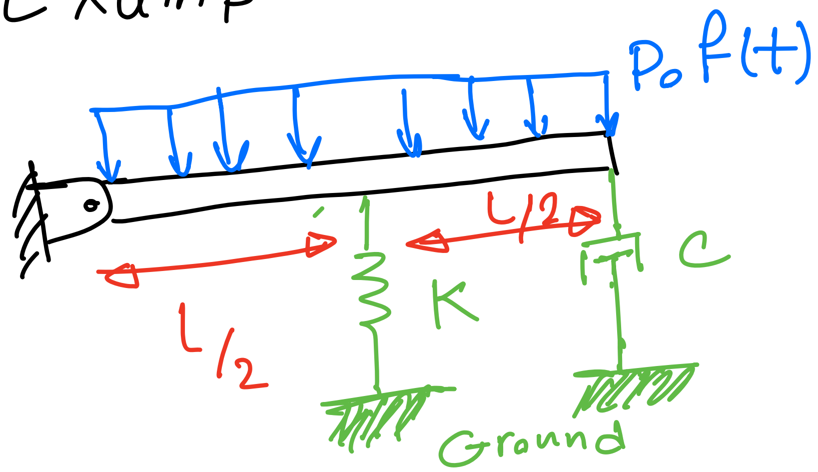
work = (F cos θ) x



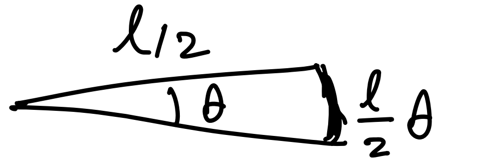
work = 0

$$\text{Work} = \underbrace{\vec{F} \cdot \vec{x}}_{\text{Dot product}} = |F|\,|x| \cos\theta$$

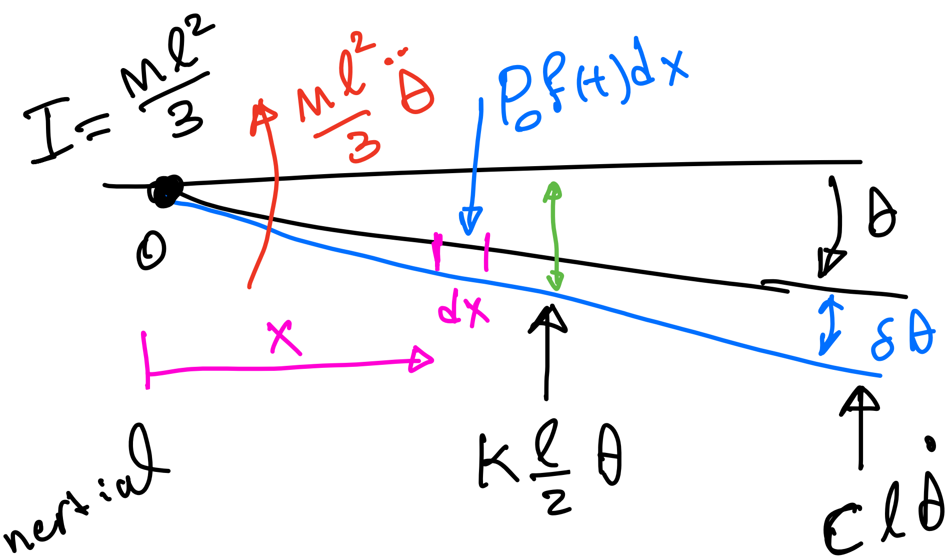
Example (2.5-1 Book)

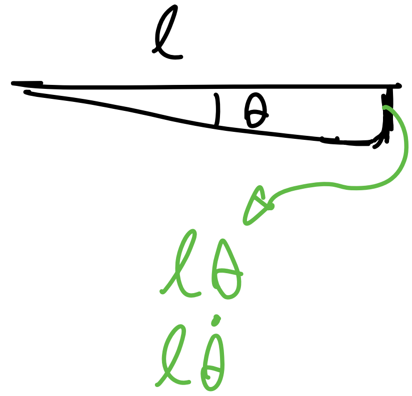


Using the virtual work, determine the equation of Motion for the rigid beam of mass M.



$\sin\theta \to \theta$
small displacement





Inertial Force
tation

ular
ration

Rot

$I\alpha \rightarrow$ angu accele

$I \rightarrow$ moment of inertia

$$\boxed{\sum M_0 = I\alpha} \qquad \boxed{\sum F = ma}$$

Ratation $\qquad$ Translation

$\alpha \rightarrow \ddot{\theta}$

Inertia force $\quad \delta W = -\left(\frac{ml^2}{3}\ddot{\theta}\right)\delta\theta$

spring force $\quad \delta W = -\left(k\frac{l}{2}\theta\right)\frac{l}{2}\delta\theta$

Damper fore $\quad \delta w = -(c l\dot{\theta})\, l\, \delta\theta$

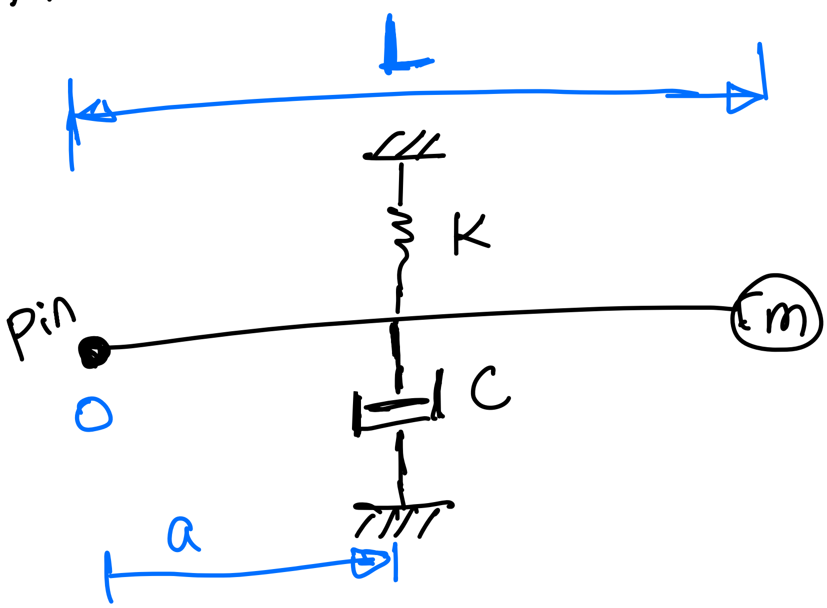
uniform load $\quad \delta W = \int (P_0 f(t)\, dx)\, x\, \delta\theta$

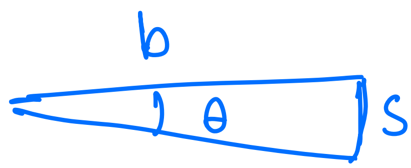
Summing the virtual work an equating to zero gives the differential equation of motian.

$$\left(\frac{Ml^2}{3}\right)\ddot{\theta} + (c l^2)\dot{\theta} + k\frac{l^2}{4}\theta = P_0\frac{l^2}{2}f(t)$$

we covered section 2.6 in the previans lectures

Problem 2-41 (Book)



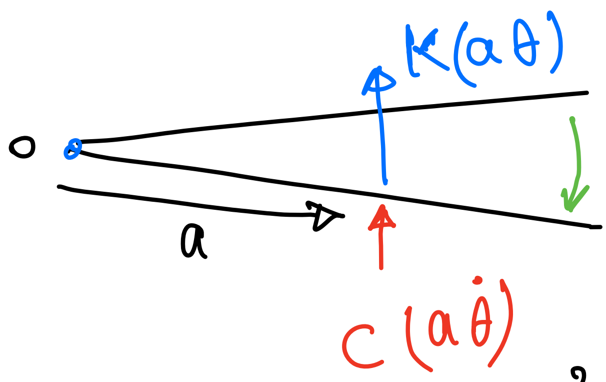


$s = b\theta$ for small displacemnt

Set up the differential equation of Motion :

$\Sigma M_o = I\ddot{\theta}$

F.B.D.



$$\Sigma M_o = -K(a\theta)\,a - a\,c\,(a\dot{\theta}) = m\ell^2\ddot{\theta}$$

$$\ddot{\theta} + \underbrace{\frac{c}{m}\left(\frac{a}{\ell}\right)^2}_{2\xi\omega_n}\dot{\theta} + \underbrace{\frac{K}{m}\left(\frac{a}{\ell}\right)^2}_{\omega_n^2}\theta = 0$$

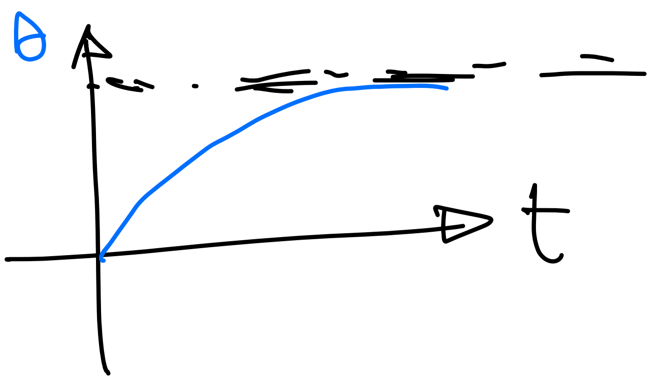
Solution : $\theta = e^{st}$

$$s^2 + \frac{c}{m}\left(\frac{a}{l}\right)^2 s + \frac{k}{m}\left(\frac{a}{l}\right)^2 = 0$$

$$s_{1,2} = -\frac{c}{2m}\left(\frac{a}{l}\right)^2 \pm \sqrt{\left(\frac{c\,a^2}{2ml^2}\right)^2 - \frac{k}{m}\left(\frac{a}{l}\right)^2}$$

what is the critical damping ?

Critical Damping



$$\sqrt{\left(\frac{c\,a^2}{2ml^2}\right)^2 - \frac{k}{m}\left(\frac{a}{l}\right)^2} = 0$$

$$\frac{c_c\,a^2}{2ml^2} = \frac{a}{l}\sqrt{\frac{k}{m}} \Rightarrow c_c = 2\,\frac{l}{a}\sqrt{km}$$

Damped natural frequency.

$$\omega_d = \frac{a}{l}\sqrt{\frac{k}{m} - \left(\frac{ca}{2ml}\right)^2}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\zeta = \frac{c}{C_c}$$

$\omega_n =$

standard form of a second order system:

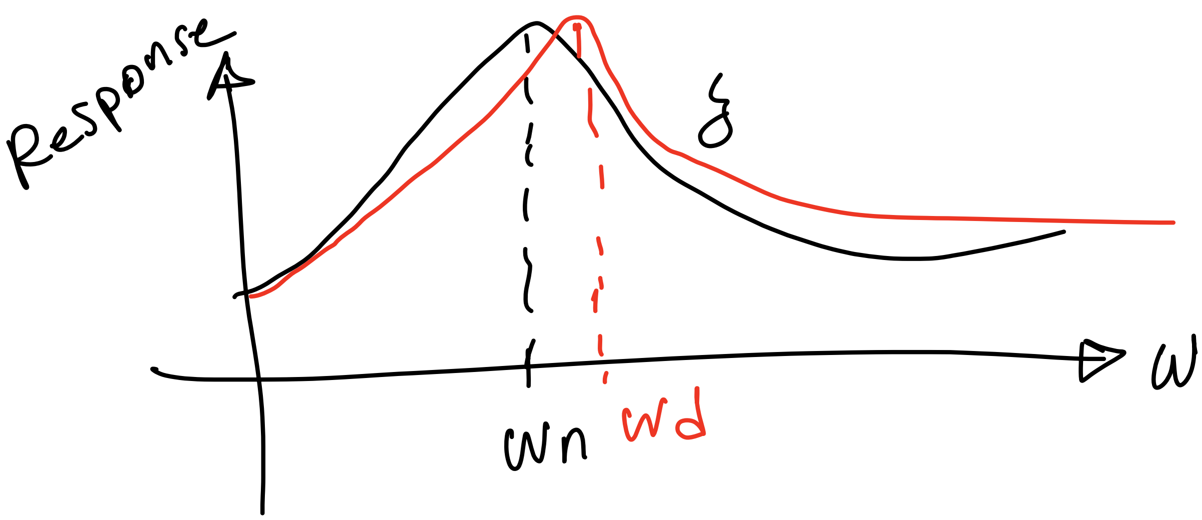
$$\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

$$\omega_n^2 = \frac{k}{m}\left(\frac{a}{l}\right)^2 \Longrightarrow \omega_n = \frac{a}{l}\sqrt{\frac{K}{m}}$$

$$2\xi\omega_n = \frac{c}{m}\left(\frac{a}{l}\right)^2$$

$$2\xi\frac{a}{l}\sqrt{\frac{K}{m}} = \frac{c}{m}\left(\frac{a}{l}\right)^2$$

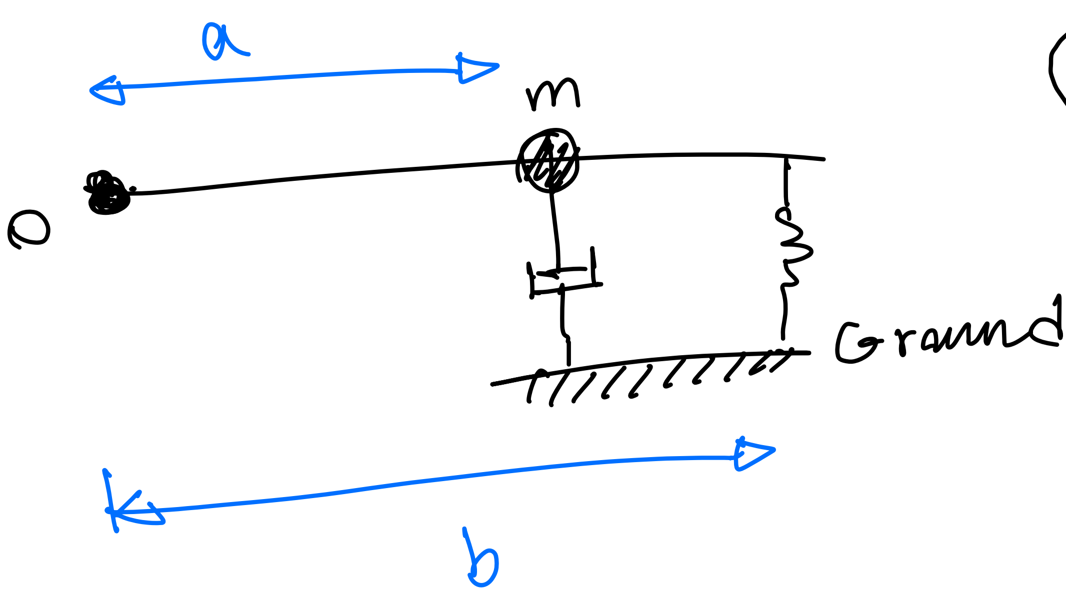
$$\xi = \frac{ca}{2l\sqrt{Km}}$$ Damping ratio



$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

small $\xi \rightarrow \omega_d = \omega_n$

Problem 2.42



$\ell = b$

Equation of Motion ?

$\omega_d = ?$ ω_d is natural frequency of damped oscilation

$C_c = ?$ critical Damping coefficient

$$\sum M_o = m a^2 \ddot{\theta} = -K b^2 \theta - c a^2 \dot{\theta}$$

$$\ddot{\theta} + \frac{c}{m} \dot{\theta} + \frac{K}{m} \left(\frac{b}{a}\right)^2 \theta = 0$$

$$\omega_n = \frac{b}{a} \sqrt{\frac{K}{m}} \qquad \omega_d = \sqrt{\frac{K}{m} \left(\frac{b}{a}\right)^2 - \left(\frac{c}{2m}\right)^2}$$

$$C_c = \frac{2b}{a} \sqrt{Km}$$