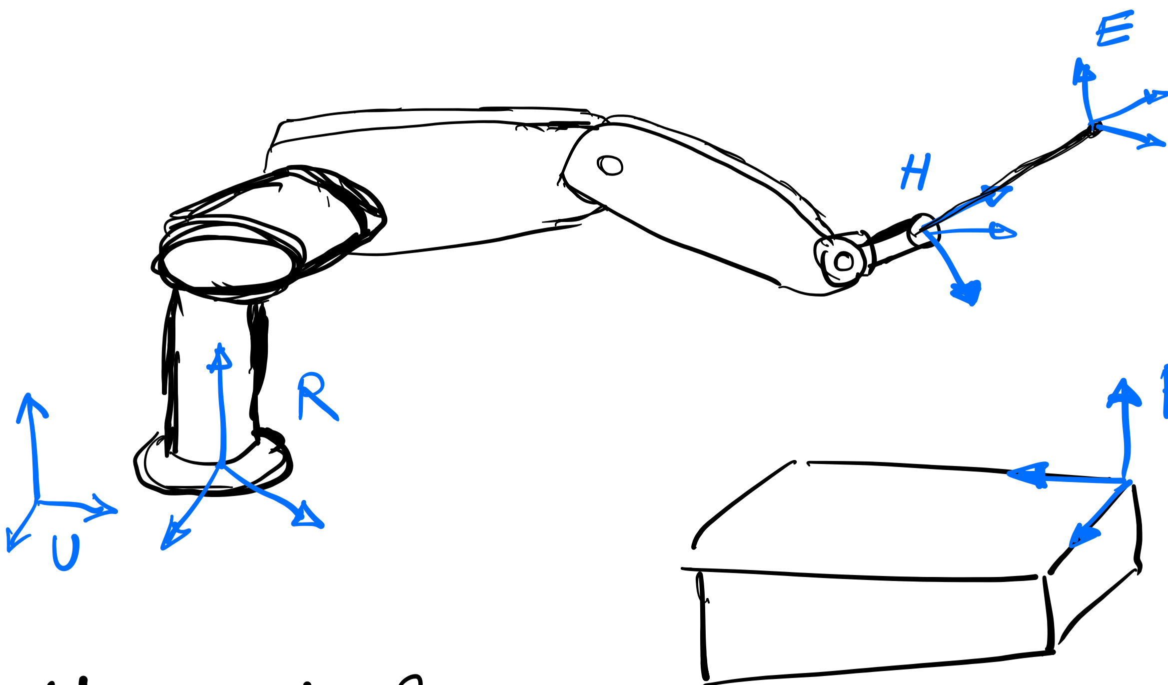
2.7. Inverse of Transformation Matrices



$$ {}^{U}T_{E} = {}^{U}T_{R}\,{}^{R}T_{H}\,{}^{H}T_{E} = {}^{U}T_{P}\,{}^{P}T_{E} $$

Known ? Known Known Known

We must know where to drill (using a camera)

$$ {}^{U}T_{R}\,{}^{R}T_{H}\,{}^{H}T_{E} = {}^{U}T_{P}\,{}^{P}T_{E} $$

$$ \overbrace{({}^{U}T_{R})^{-1}\,{}^{U}T_{R}}^{I}\,{}^{R}T_{H}\,\overbrace{{}^{H}T_{E}\,({}^{H}T_{E})^{-1}}^{I} = ({}^{U}T_{R})^{-1}\,{}^{U}T_{P}\,{}^{P}T_{E}\,({}^{H}T_{E})^{-1} $$

$$ ({}^{U}T_{R})^{-1}\,{}^{U}T_{R} = I $$

(identity matrix)

$$ I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{H}T_{E}\,({}^{H}T_{E})^{-1} = I $$

$$ {}^{R}T_{H} = ({}^{U}T_{R})^{-1}\,{}^{U}T_{P}\,{}^{P}T_{E}\,({}^{H}T_{E})^{-1} $$

$$ ({}^{U}T_{R})^{-1} \longrightarrow {}^{R}T_{U} $$

$$ ({}^{H}T_{E})^{-1} \longrightarrow {}^{E}T_{H} $$

$${}^{R}T_H = {}^{R}T_U \, {}^{U}T_P \, {}^{P}T_E \, {}^{E}T_H$$

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only for transformation matrices, there is an easy way of calculating the inverse of a transformation matrix:

$$T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\vec{P} \cdot \vec{n} \\ o_x & o_y & o_z & -\vec{P} \cdot \vec{o} \\ a_x & a_y & a_z & -\vec{P} \cdot \vec{a} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vec{P} = P_x \vec{i} + P_y \vec{j} + P_z \vec{k}$$

$$\vec{n} = n_x \vec{i} + n_y \vec{j} + n_z \vec{k}$$

$$\vec{P} \cdot \vec{n} = P_x n_x + P_y n_y + P_z n_z$$

$$\vec{P} \cdot \vec{o} = P_x o_x + P_y o_y + P_z o_z$$

Example

Calculate the matrix representing $Rot(x, 40^\circ)^{-1}$.

$$Rot(x, 40^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.766 & -0.643 & 0 \\ 0 & 0.643 & 0.766 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The inverse of this matrix is:

$$Rot(x, 40^\circ)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.766 & 0.643 & 0 \\ 0 & -0.643 & 0.766 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example

Calculate the inverse of the given transformation matrix

$$T = \begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0.866 & 0 & -0.5 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-(3 \times 0.5 + 2(0.866) + 5(0)) = -3.23$$

$$-(3 \times 0 + 2(0) + 5(1)) = -5$$

$$-(3 \times 0.866 + 2(-0.5) + 5(0) = -1.6$$

$$T^{-1} = \begin{bmatrix} 0.5 & 0.866 & 0 & -3.23 \\ 0 & 0 & 1 & -5 \\ 0.866 & -0.5 & 0 & -1.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 2.15