Instrumentation and Controls

ETM 3301

Lecture 6

Instructor

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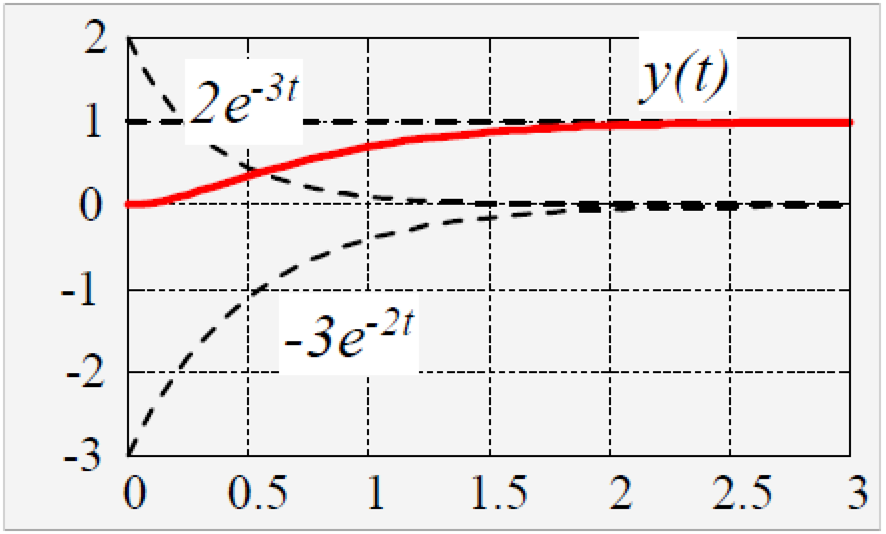
Responses and Transfer function

System 1 $G(s) = \frac{Y(s)}{U(s)} = \frac{6}{s^2 + 5s + 6}$ $u(t) = 1\ ; \quad U(s) = \frac{1}{s}$

$$Y(s) = \frac{6}{s(s^2 + 5s + 6)} = \frac{6}{s(s+2)(s+3)} = \frac{1}{s} - \frac{3}{s+2} + \frac{2}{s+3}$$

$$y(t) = 1 - 3e^{-2t} + 2e^{-3t}$$

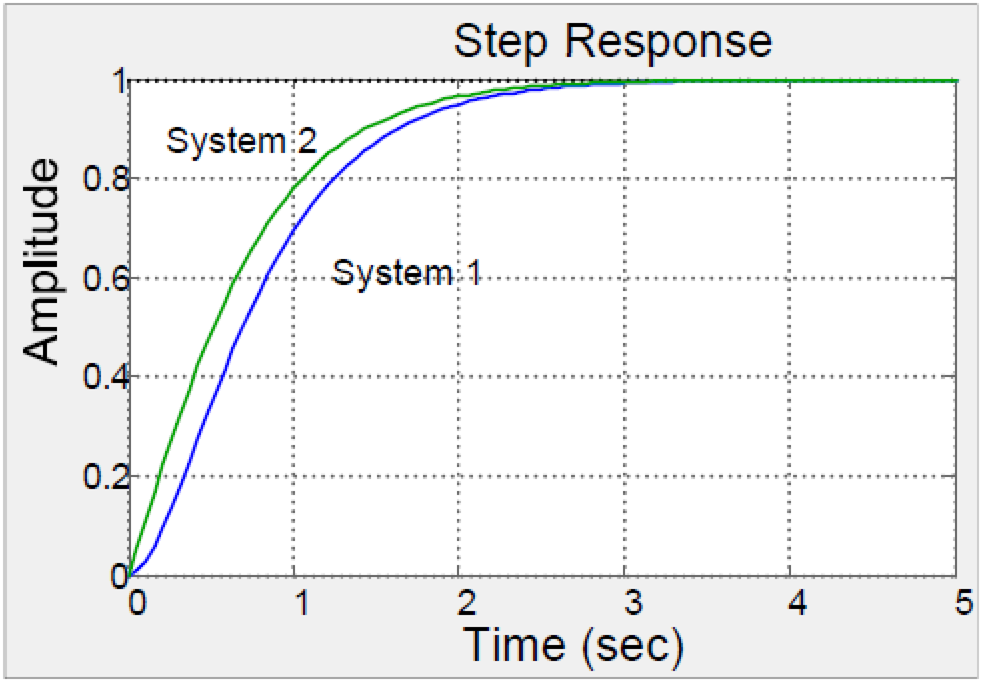


Responses and Transfer function

System 2 $$G(s)=\frac{Y(s)}{U(s)}=\frac{s+6}{s^2+5s+6}$$ $$u(t)=1;\quad U(s)=\frac{1}{s}$$

$$Y(s)=\frac{s+6}{s(s^2+5s+6)}=\frac{s+6}{s(s+2)(s+3)}=\frac{1}{s}-\frac{2}{s+2}+\frac{1}{s+3}$$

$$y(t)=1-2e^{-2t}+e^{-3t}$$



The system response characteristics mainly determined by the transfer function denominator.

Responses and Transfer function

System 3

$$G(s)=\frac{Y(s)}{U(s)}=\frac{5}{s^2+2s+5}$$

$$u(t)=1\,; \quad U(s)=\frac{1}{s}$$

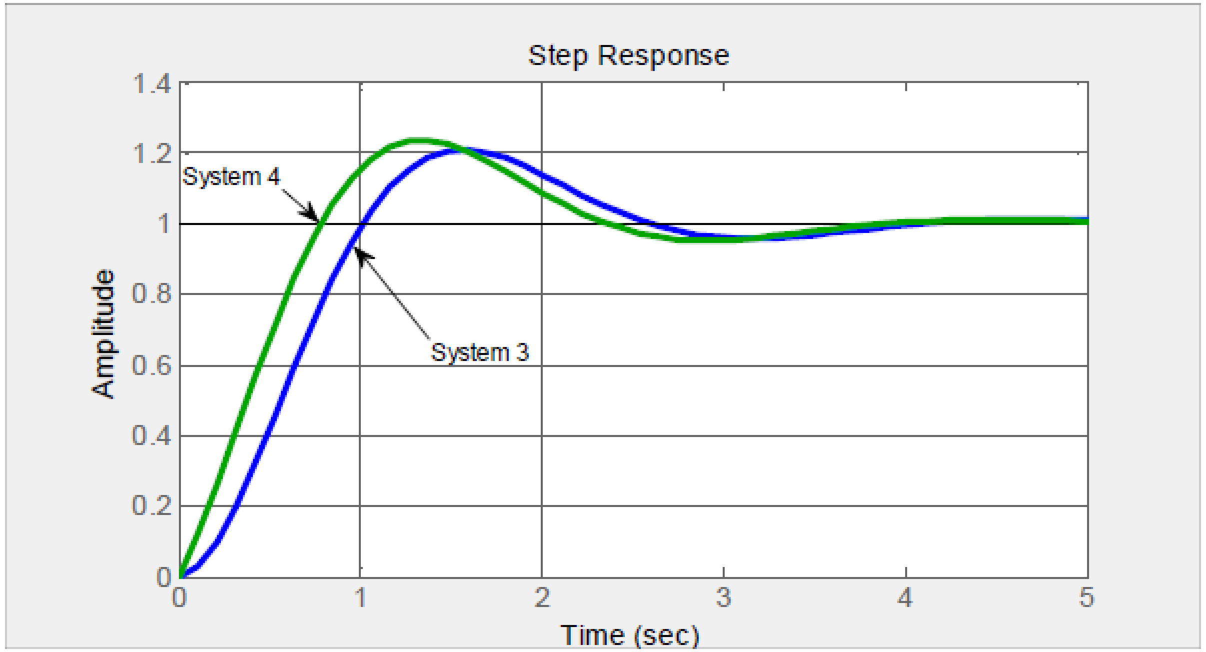
$$Y(s)=\frac{5}{s(s^2+2s+5)}=\frac{1}{s}-\frac{s+1}{(s+1)^2+2^2}-\frac{1}{(s+1)^2+2^2}$$

$$y(t)=1-1.18e^{-t}\sin(2t+63.4°)$$

System 4

$$G(s)=\frac{s+5}{s^2+2s+5}$$

The system response characteristics mainly determined by the transfer function denominator.



Responses and Transfer function

System 5

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3}{s^2 + 4s + 3}$$

$$u(t) = 1; \quad U(s) = \frac{1}{s}$$

$$Y(s) = \frac{3}{s(s+1)(s+3)} = \frac{1}{s} - \frac{3}{2}\frac{1}{s+1} + \frac{1}{2}\frac{2}{s+3}$$

$$y(t) = 1 - \frac{3}{2}e^{-t} + \frac{1}{2}e^{-3t}$$

System 6

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s+3}{s^2 + 4s + 3}$$

$$u(t) = 1; \quad U(s) = \frac{1}{s}$$

$$Y(s) = \frac{2s+3}{s(s+1)(s+3)} = \frac{1}{s} - \frac{1}{2}\frac{1}{s+1} - \frac{1}{2}\frac{1}{s+3}$$

$$y(t) = 1 - \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}$$

The system response characteristics mainly determined by the transfer function denominator.

Properties of Transfer Functions

- Consider a general transfer function ($n \geq m$)

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

- The **order** of the system: n
  - The order of denominator polynomial.

$D(s)$	the **characteristic polynomial** (CP) of $G(s)$
$D(s)=0$	the **characteristic equation** (CE) of $G(s)$
poles	the roots of the equation $D(s)=0$
zeros	the roots of the equation $N(s)=0$

Poles, Zeros and Transfer Functions

- Rewrite a transfer function

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

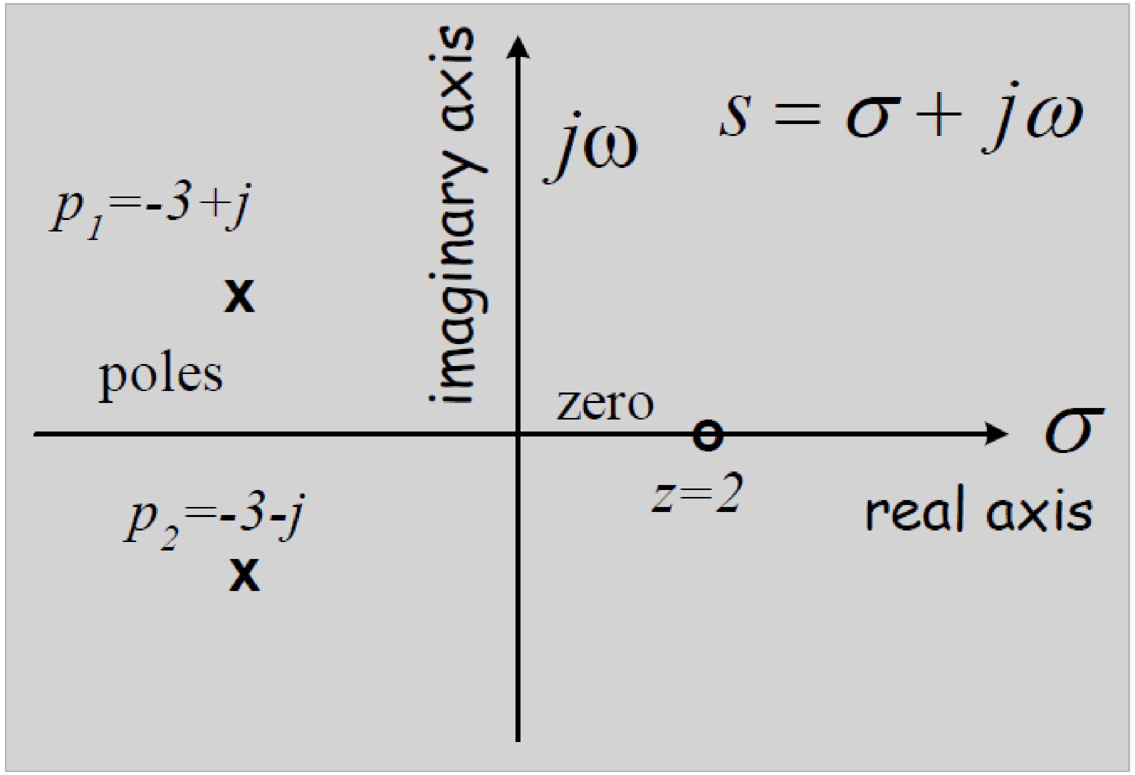
$$= K \frac{(s + z_1)(s + z_2) \cdots (s + z_m)}{(s + p_1)(s + p_2) \cdots (s + p_n)}$$

poles	$-p_1, -p_2, \ldots, -p_n$
zeros	$-z_1, -z_2, \ldots, -z_m$

- *zeros* ($-z_i$) and poles ($-p_i$) are either real or occur in complex conjugate pairs.

s-plane (complex plane) and pole/zero plot

- s: s is a complex variable
- s-plane: a complex plane



- Pole/zero plot: a plot of poles and zeros in the s-plane
- Poles shown by "x"
- Zeros shown by "o"

Transfer Function Example (1)

$$G(s) = \frac{3}{s^2 + 4s + 3} = \frac{3}{(s+1)(s+3)}$$

System order = order of denominator = 2

Characteristic polynomial (CP): $s^2 + 4s + 3$

Characteristic equation (CE): $s^2 + 4s + 3 = 0$

Poles: solution to $s^2 + 4s + 3 = 0$

$$s^2 + 4s + 3 = (s+1)(s+3) = 0 \Rightarrow s_1 = -1, \;\; s_2 = -3$$

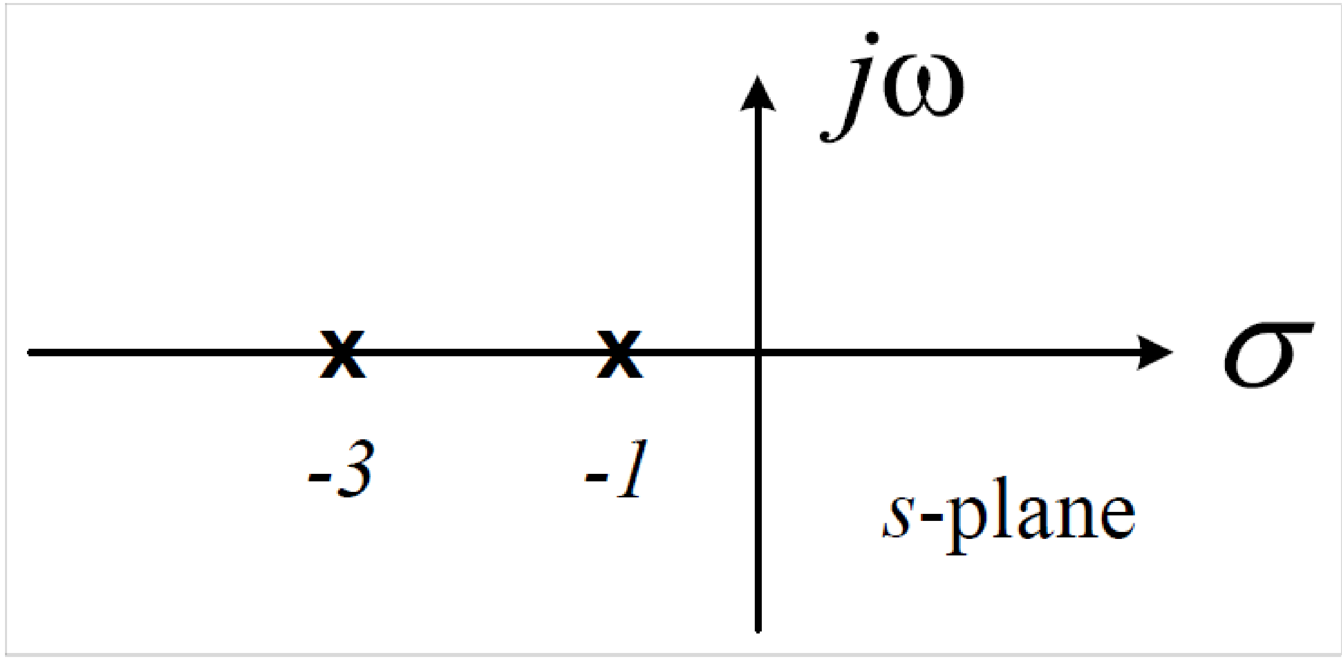
Zeros: no zeros.

Transfer Function Example (2)

Pole/Zero Plot

Poles: $s_1 = -1, \; s_2 = -3$

Zeros: no zeros.



Transfer Function Example (3)

$$G(s) = \frac{2s+4}{s^3+5s^2+11s+15} = \frac{2(s+2)}{(s+3)(s^2+2s+5)}$$

System order = order of denominator = 3

Characteristic polynomial (CP): $s^3+5s^2+11s+15$

Characteristic equation (CE): $s^3+5s^2+11s+15 = 0$

Poles: solution to $s^3+5s^2+11s+15 = 0$

$$s^3+5s^2+11s+1 = (s+3)(s^2+2s+5) = 0$$

$$s+3=0; \quad s^2+2s+5=0$$

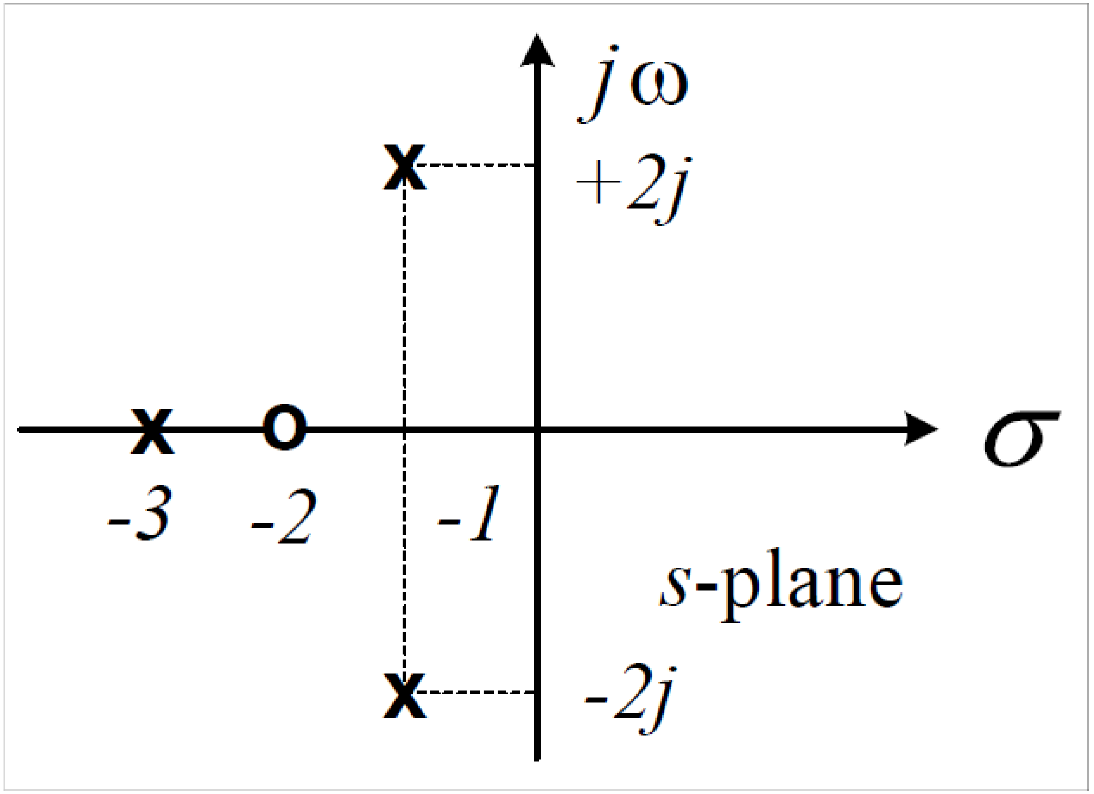
$$s_1 = -3, \;\; s_{2,3} = \frac{-2 \pm \sqrt{2^2 - 4\times 5}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2j$$

Transfer Function Example (4)

Zeros: solution to $s+2=0$ $z=-2$

$s_1=-3,\ \ s_2=-1-2j,\ \ s_3=-1+2j$

Pole/Zero Plot

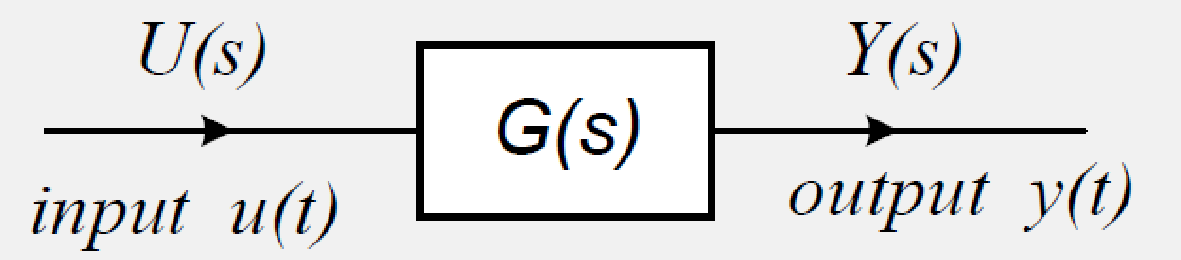


Chapter 3: System Time Responses, First and Second Order System Step Responses

- Time response concept
- First order system step response
- Standard second system transfer function, damping ratio and undamped natural frequency.
- Main characteristics of second order system step responses.
- Percentage overshoot and settling time.

Input-Output Method & Transfer Function (TF)

- The transfer function of a system is defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all initial conditions assumed to be zero.

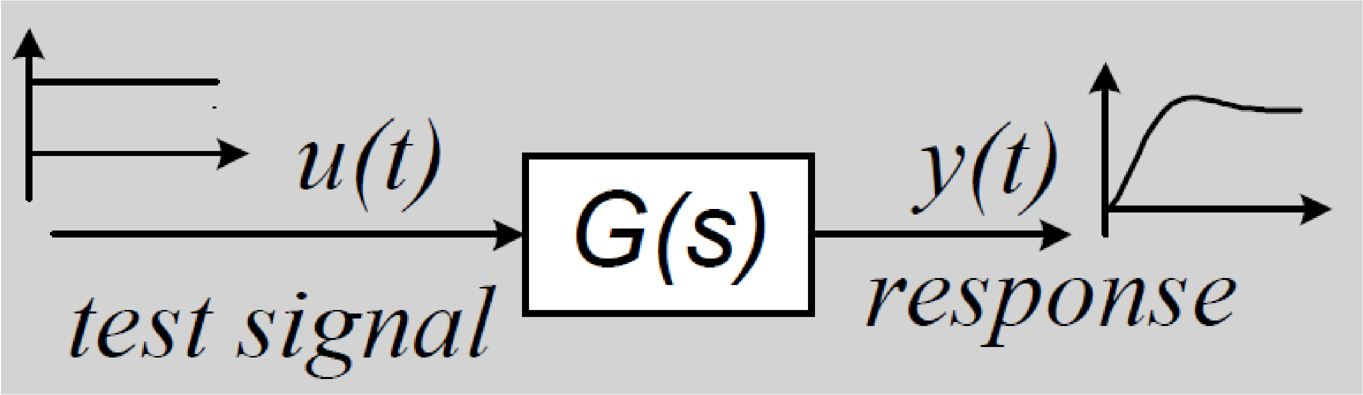


$$G(s)=\frac{Y(s)}{U(s)}=\frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]}$$

Input	Output
Action	Consequence
Cause	Effect
Command	Response

Time Responses

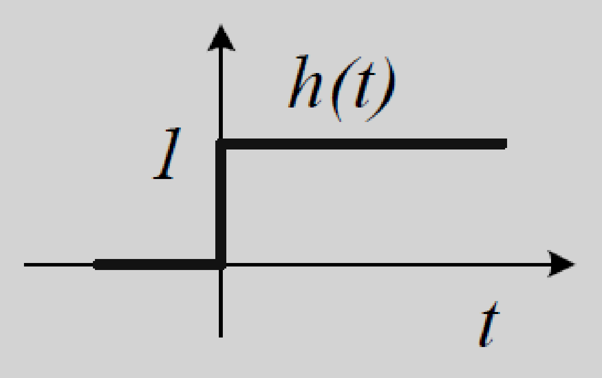
- It is important to know how a system responds to an input signal?
  - The output response is a function of time – time response.



- The system response is normally evaluated using standard test input signals.

Standard Test Signals (1)

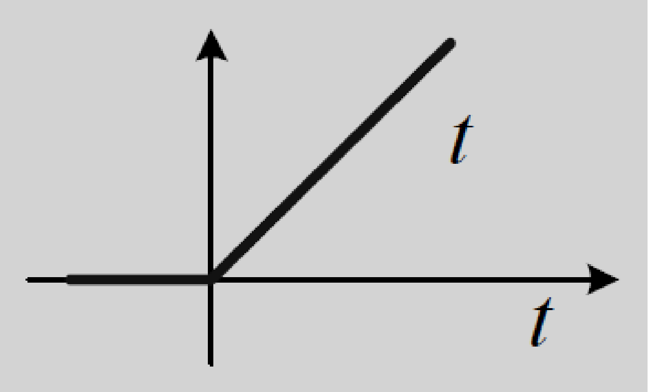
- **Step input:**



$$u(t) = h(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\mathcal{L}[u(t)] = \frac{1}{s}$$

- **Rump input:**

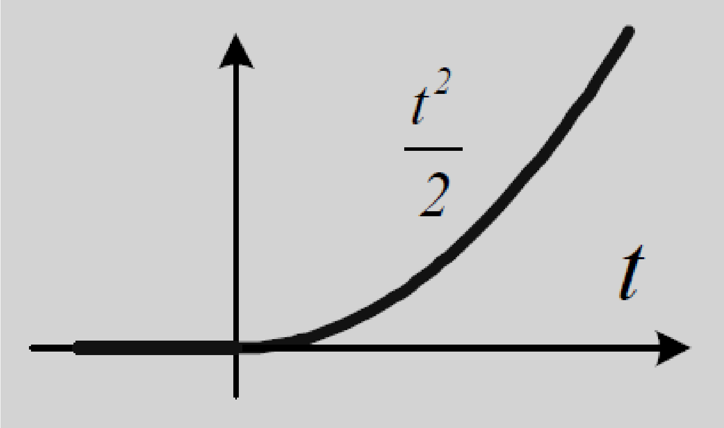


$$u(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\mathcal{L}[u(t)] = \frac{1}{s^2}$$

Standard Test Signals (2)

- **Parabolic input :**



$$u(t) = \begin{cases} \frac{t^2}{2}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

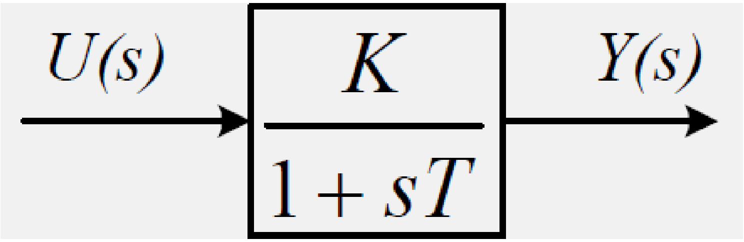
$$\mathcal{L}[u(t)] = \frac{1}{s^3}$$

First order systems

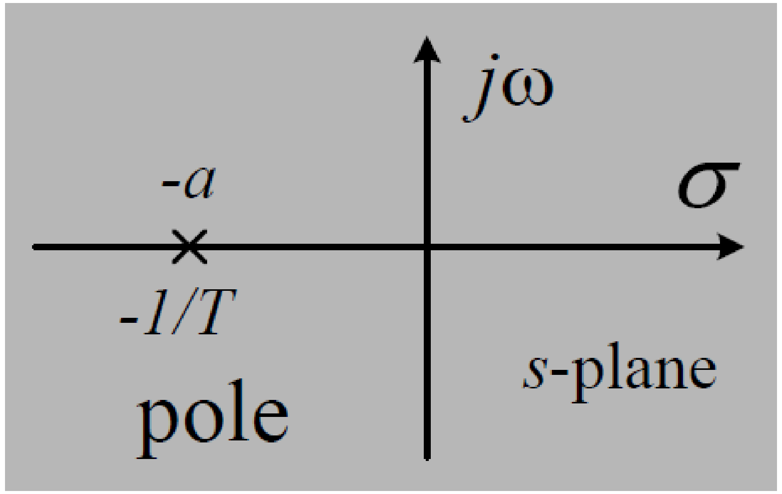
- A first order system is described by the TF:

$$G(s)=\frac{K}{1+sT} \quad \text{Or} \quad G(s)=K\frac{a}{s+a} \qquad a=\frac{1}{T}$$

- We only consider the case when $T>0$, $a>0$.



Pole $-\frac{1}{T}$ or $-a$



- T: time constant; K: gain

First order system response to a step input, 1

- Unit step input $u(t) = 1$ $U(s) = \dfrac{1}{s}$

- Output response

$$Y(s) = G(s)U(s) = \frac{K}{s(1+sT)} = K\frac{1}{s(1+sT)}$$

$$= K\frac{(1+sT)-sT}{s(1+sT)} = K\left\{\frac{1}{s} - \frac{T}{1+sT}\right\}$$

$$= K\left\{\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right\}$$

$$\frac{A-B}{C} = \frac{A}{C} - \frac{B}{C}$$

First order system response to a step input, 2

$$Y(s) = K\left\{\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right\}$$

$$y(t) = \mathcal{L}^{-1}\left[Y(s)\right] = K\left(1 - e^{-\frac{t}{T}}\right)$$