Vectors

Equal vectors:

$$\vec{a} = \vec{b}$$

For $\vec{a} = \vec{b}$, all components of $\vec{a}$ must be equal to $\vec{b}$:

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

IF $\vec{a} = \vec{b}$ then

$$\begin{cases} a_x = b_x \\ a_y = b_y \\ a_z = b_z \end{cases}$$

Example

Find x, y, z in the following vector equation.

$$x\,\hat{i} + (x+y)\,\hat{j} + (x+y+2z)\,\hat{k} = \hat{i} + 2\hat{k}$$

$$\downarrow$$

$$1\times\hat{i} + 0\times\hat{j}$$

$x = ?$ $\quad y = ?$ $\quad z = ?$

$$\begin{cases} x = 1 & (1) \\ x + y = 0 & (2) \\ x + y + 2z = 2 & (3) \end{cases}$$

$(1), (2) \Rightarrow 1 + y = 0$

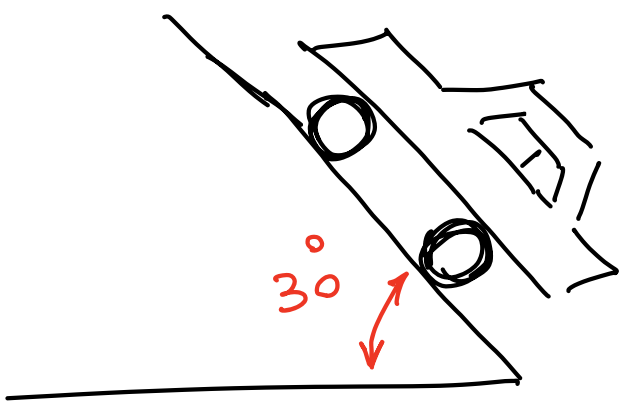
$y = -1$

$(3) \Rightarrow 1 - 1 + 2z = 2$

$z = 1$

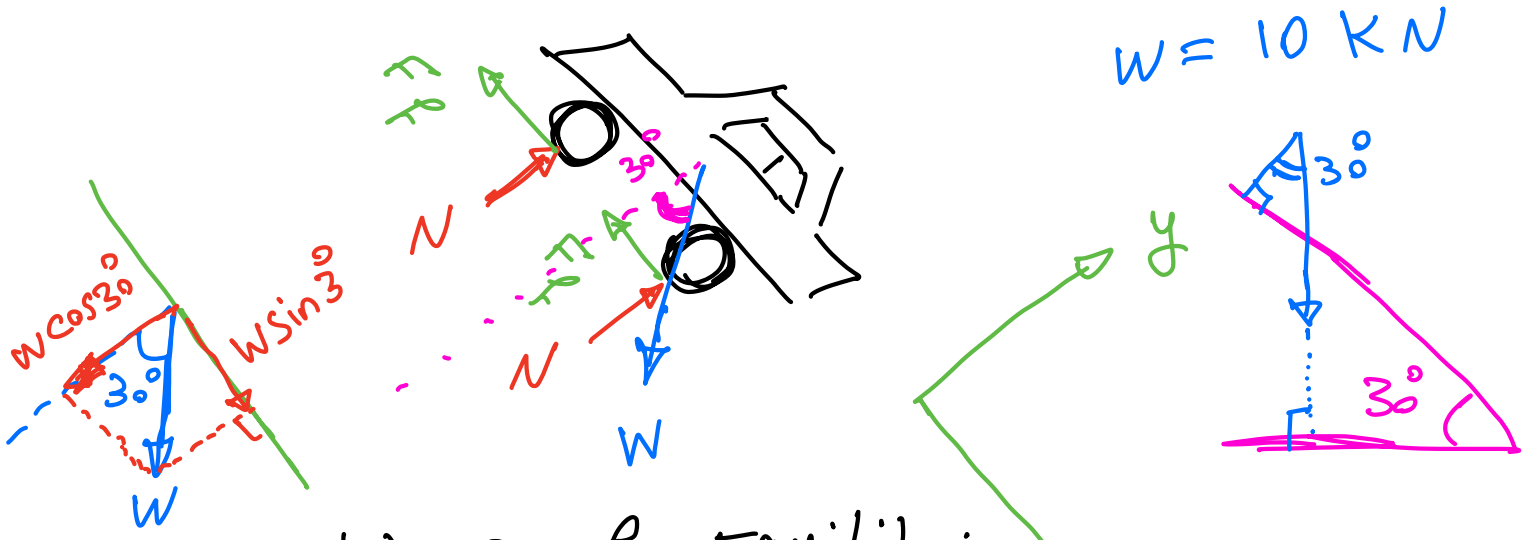
Example

The vehicle is not moving in the figure below. Find the Friction force f_b. The weight of the vehicle is $W = 10\ kN$



Always start with Free-body-diagram

F.B.D.



Equations of Equilibrium

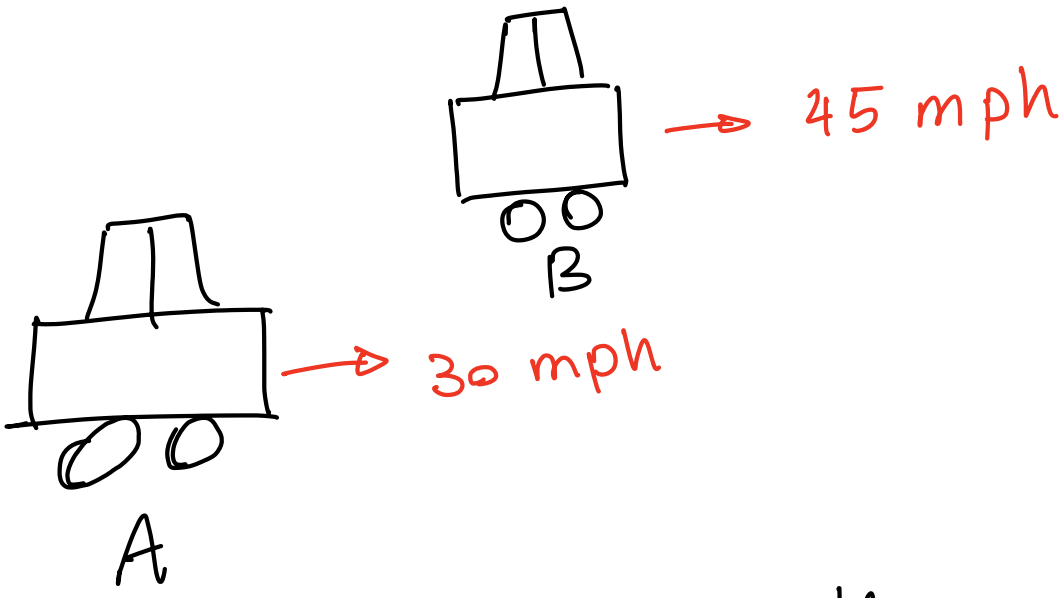
$$\begin{cases} \Sigma F_x = 0 \implies +W\sin 30^\circ - 2F_f = 0 \\ \Sigma F_y = 0 \implies +2N - W\cos 30^\circ = 0 \end{cases}$$

Alternatively:

Equations in vector form:

$$\Sigma \vec{F} = 0 \implies +W\sin 30^\circ \hat{i} - 2f\hat{i} + 2N\hat{j} - W\cos 30^\circ \hat{j} = 0$$

Relative velocity



The speed of B with respect to A is $v_{B/A} = 45 \text{ mph} - 30 \text{ mph}$

$= 15 \text{ mph}$

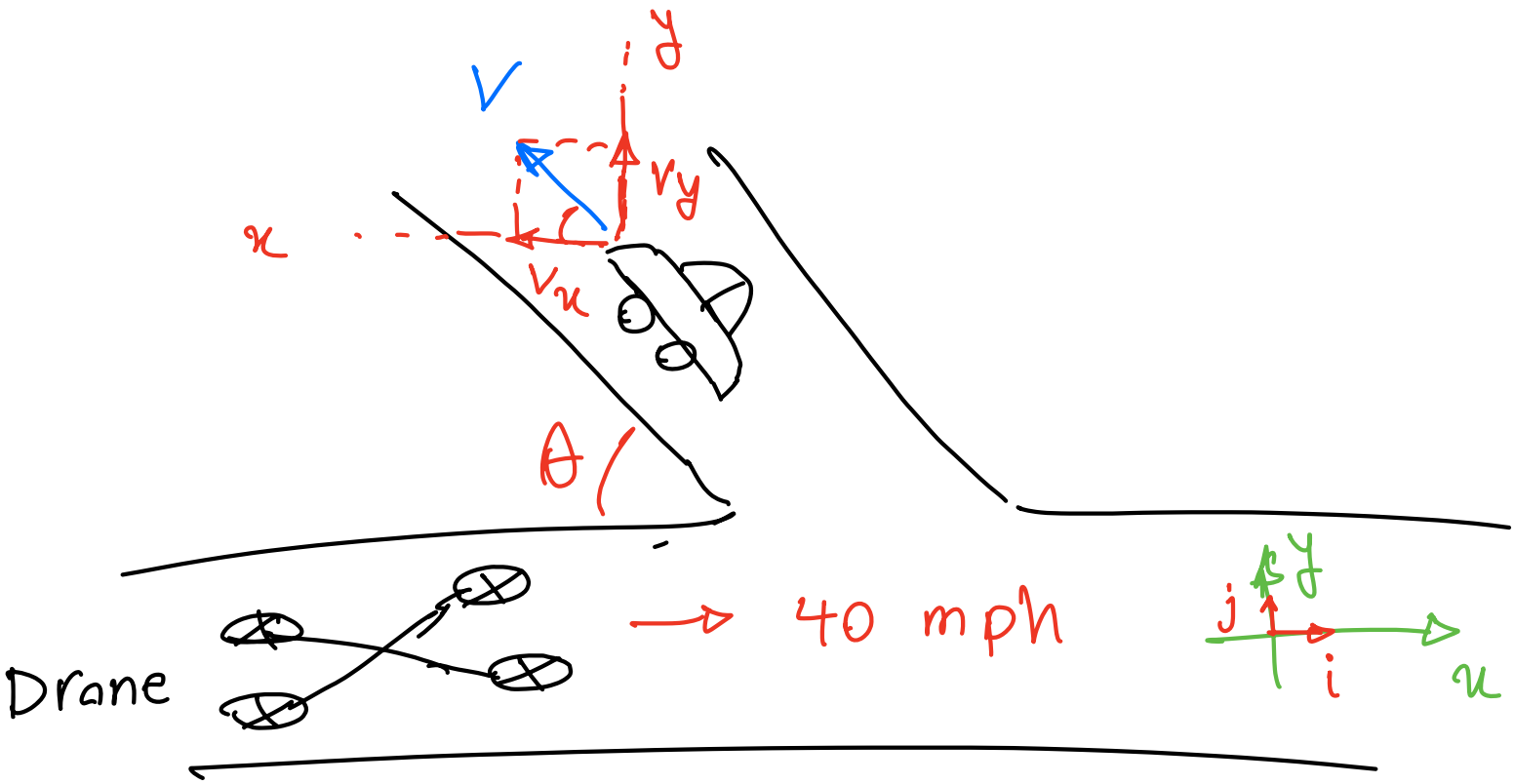
In general the velocity of B relative to A can be written as:

Relative Absolute Absolute

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

Example

A drone flying at 40 mph is measuring the speed of a vehicle as shown below.



If the drone measures the speed of the vehicle as

$$\vec{V}_{\text{vehicle}/\text{Drone}} = -20 \text{ mph } \hat{i} + 5 \text{ mph } \hat{j}$$

Find the speed of the vehicle (Absolute)

$$\vec{V}_{vehicle} = -V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$$

$$\vec{V}_{\frac{vehicle}{Drone}} = \vec{V}_{vehicle} - \vec{V}_{Drone}$$

$$\vec{V}_{Drone} = 40\,\hat{i}$$

$$\vec{V}_{\frac{vehicle}{Drone}} = -V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$$

$$-40\,\hat{i} = -20\,\hat{i} + 5\,\hat{j}$$

$$\begin{cases} V\cos\theta + 40 = 20 \rightarrow V\cos\theta = -20 \\ V\sin\theta = 5 \end{cases}$$

$$\Rightarrow V \cong 21$$