$$C_c = 2\sqrt{km}$$

$$\zeta = \frac{c}{c_c} \quad \text{Damping ratio}$$

$$\frac{c}{2m} = \zeta\left(\frac{c_c}{2m}\right) = \zeta\omega_n$$

Solution to the differential equation of motion:

$$x = e^{-(c/2m)t}\left(Ae^{\left(\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right)t} + Be^{-\left(\sqrt{\left(\frac{c}{2m}\right)^2 - k/m}\right)t}\right)$$

$$x = A\,e^{s_1 t} + Be^{s_2 t}$$

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$\frac{c}{2m} = \zeta\omega_n \qquad \frac{k}{m} = \omega_n^2$$

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{(\zeta^2 - 1)\omega_n^2}$$

$$s_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$$

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}F(t)$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 = \frac{1}{m}F(t)$$

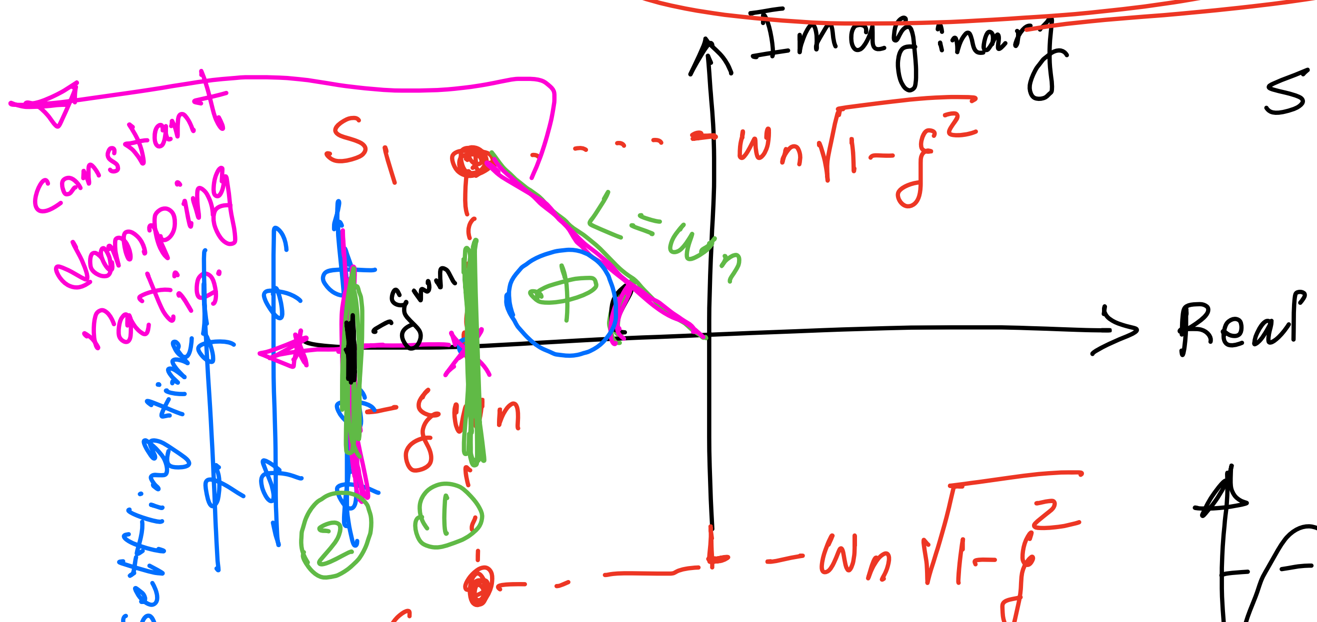
standard form of a second order system

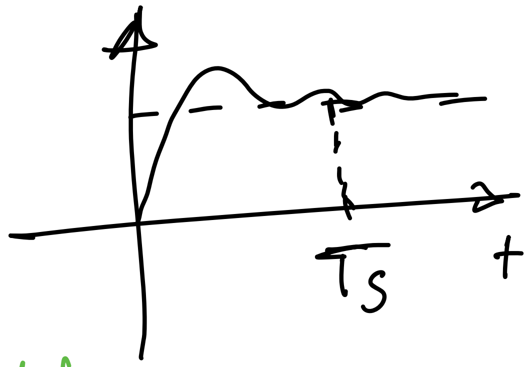
$$\boxed{\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2 = \frac{1}{I}T(t)}$$

For angular displacement

$$s_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$$

$$0 < \zeta < 1 \implies s_{1,2} = \underbrace{-\zeta\omega_n}_{\text{Real}} \pm \underbrace{i\,\omega_n\sqrt{1-\zeta^2}}_{\text{Imaginary}}$$

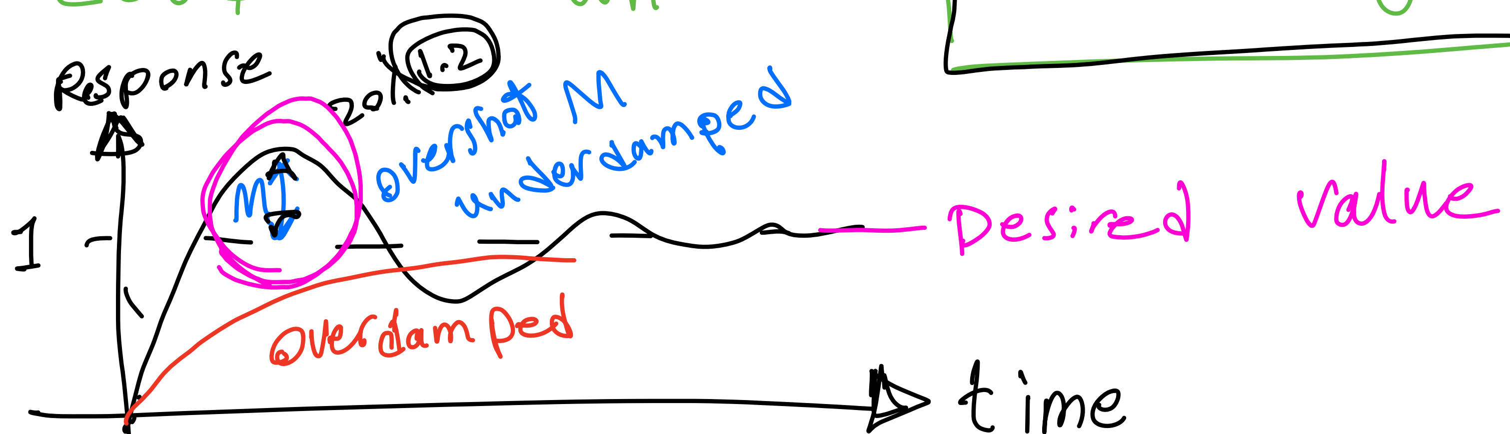




$$\boxed{T_s=\frac{4}{\xi\omega_n}}$$

$$L=\sqrt{(\xi\omega_n)^2+\omega_n^2(1-\xi^2)} = \omega_n$$

$$\cos\Phi = \frac{-\xi\omega_n}{\omega_n} \Rightarrow \boxed{\Phi = \cos^{-1}\xi}$$

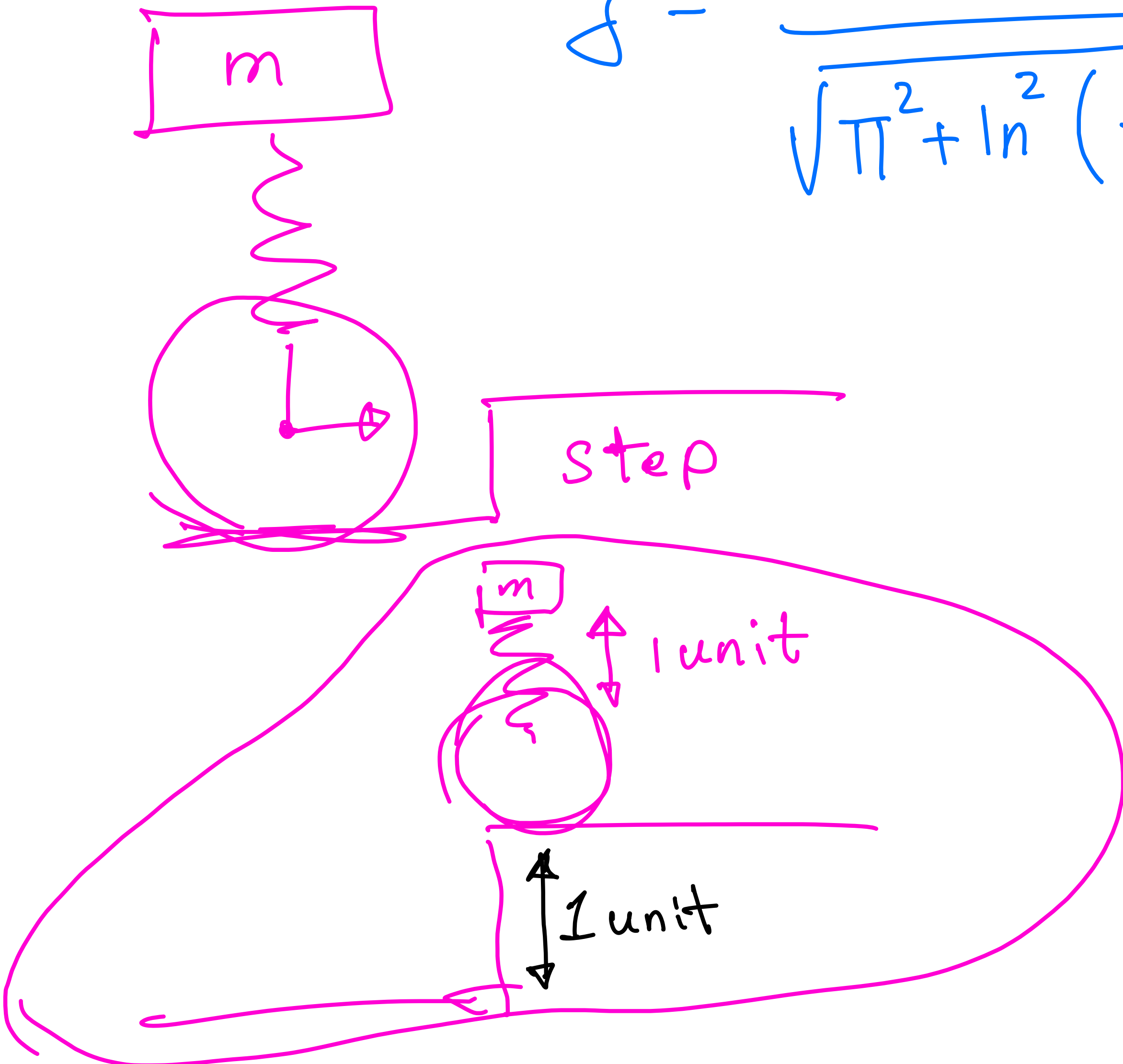


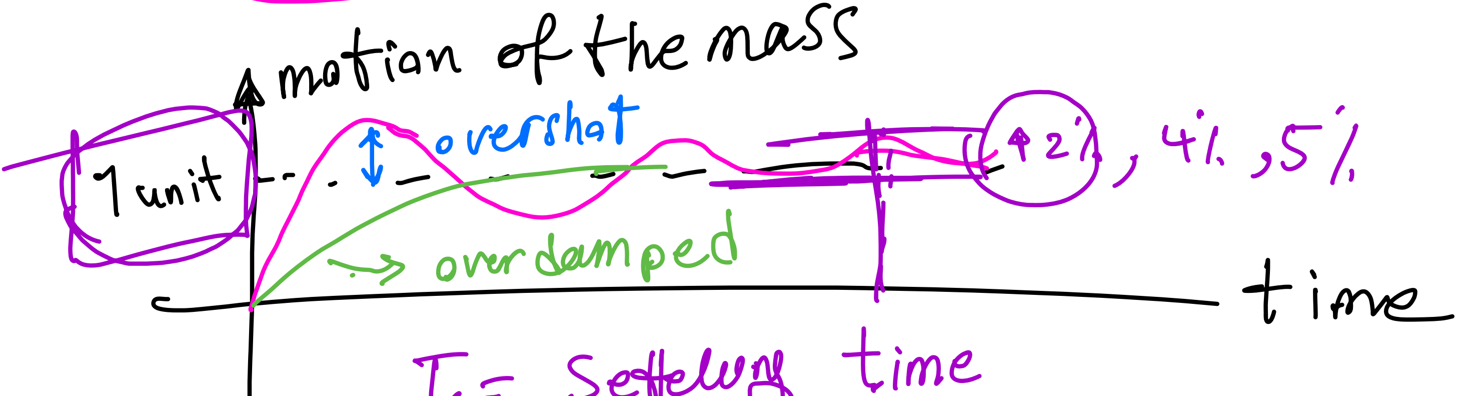
$$\boxed{M = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)}}$$

$$PO = M \times 100$$

overshot is only a function of ξ (Damping ratio)

$$\zeta = \frac{\ln\left|\frac{PO}{100}\right|}{\sqrt{\pi^2 + \ln^2\left(\frac{PO}{100}\right)}}$$



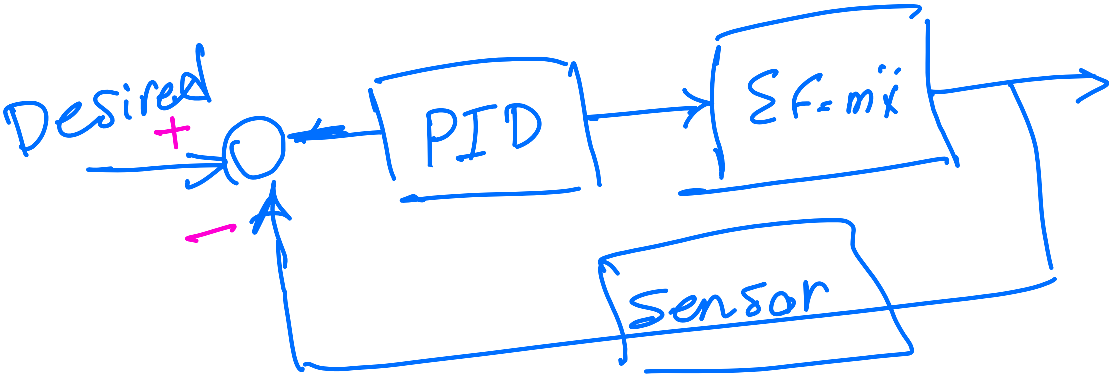


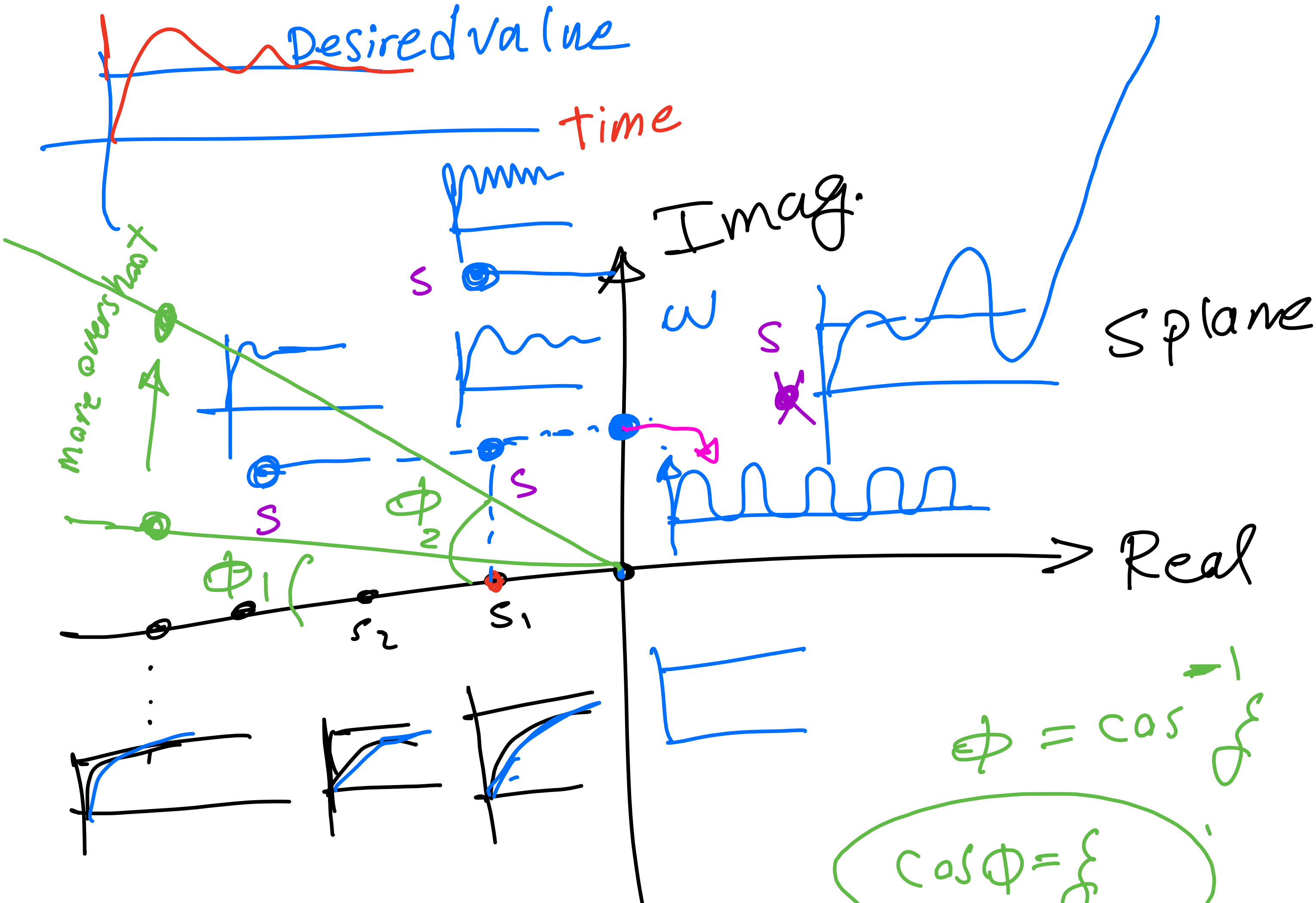
T_s = Settling time

2% of the error:

$$T_s = \frac{4}{\zeta \omega_n}$$

$\zeta \omega_n$



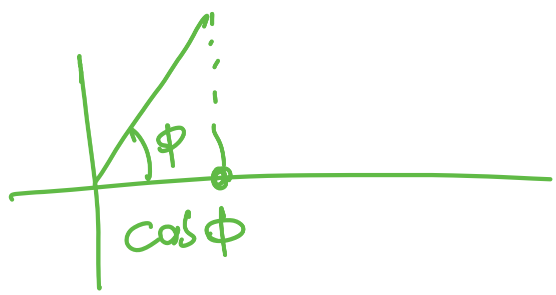


$$\Phi = \cos^{-1} \zeta$$

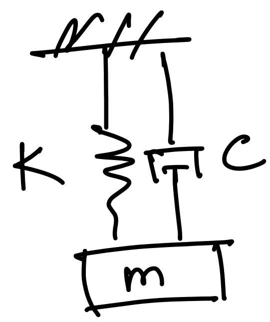
$$\cos \Phi = \zeta$$

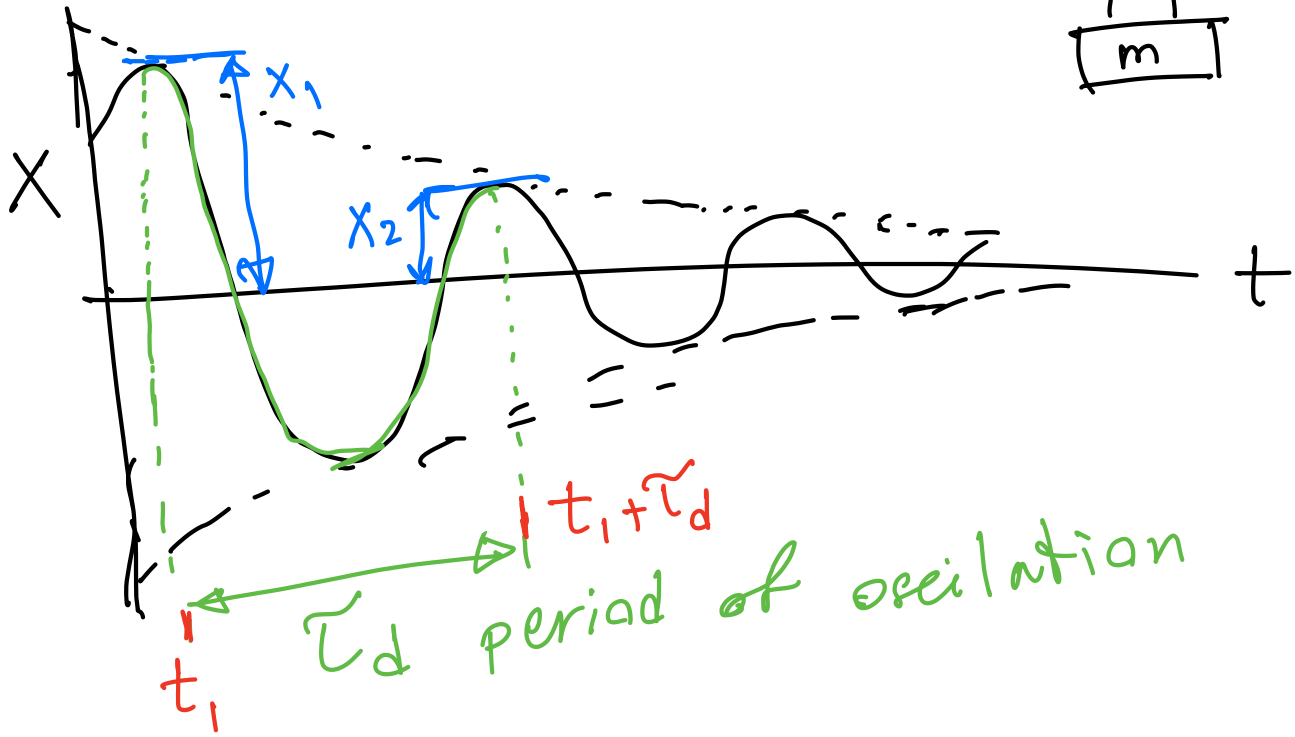
$0 < \zeta < 1$

$\zeta > 1$



Logarithmic Decrement





The logarithmic decrement is

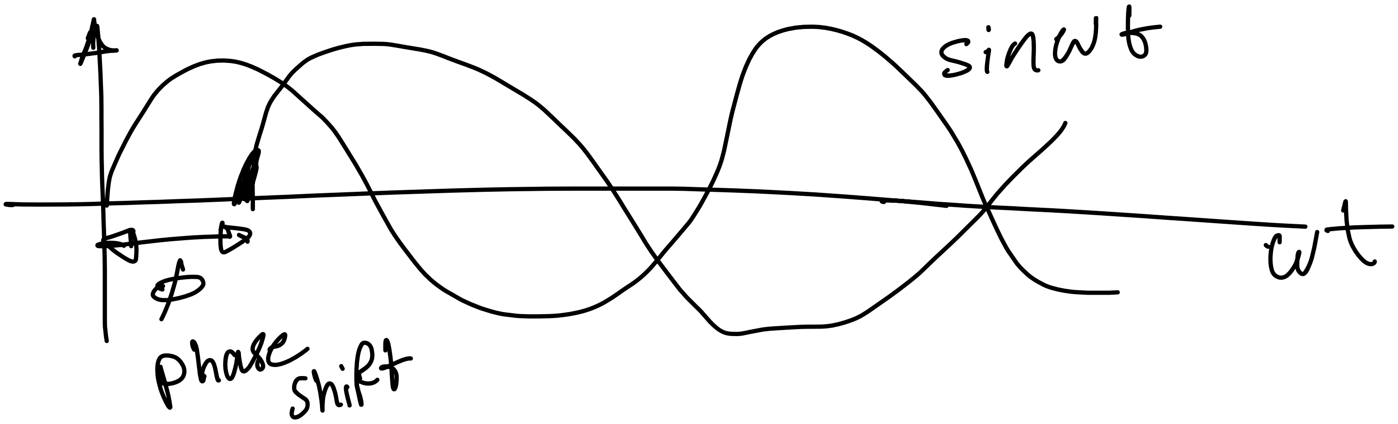
$$\delta = \ln \frac{x_1}{x_2}$$

Different ϕ than above

$$\delta = \ln \frac{e^{-\xi \omega_n t_1} \sin\left(\sqrt{1-\xi^2}\,\omega_n t_1 + \phi\right)}{e^{-\xi \omega_n (t_1+\tau_d)} \sin\left(\sqrt{1-\xi^2}\,\omega_n (t_1+\tau_d) + \phi\right)}$$

Book (2.7-1)

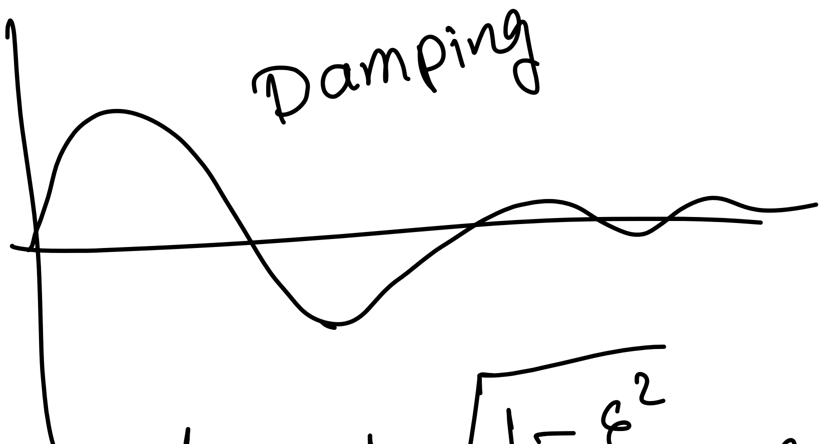
phase shift



$$\delta = \ln e^{\xi \omega_n \tau_d} = \xi \omega_n \tau_d$$

Response from the experiment

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \qquad \tau_d = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$$



$\tau_{period} = \frac{2\pi}{\omega}$

ω → angular frequency

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

d natural frequency

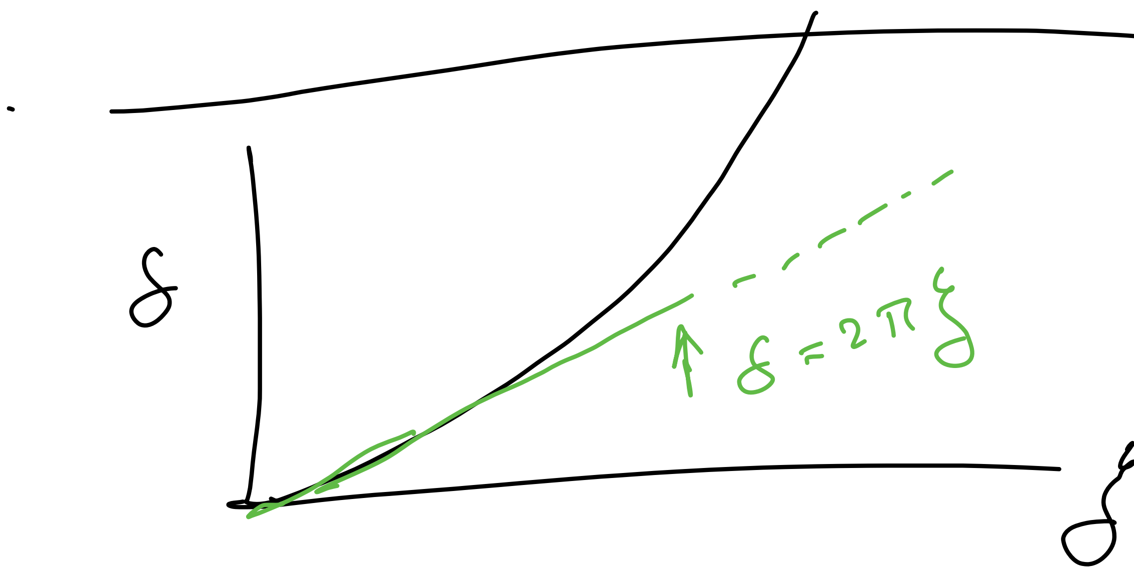
Damped

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

for small damping (ξ) $\sqrt{1-\xi^2} \cong 1$

$$\delta = 2\pi\xi$$

Equation 2.7-4 Book



Damping ratio

Damping ratio of a car suspension

30%

Damping ratio of a building

structure

5%