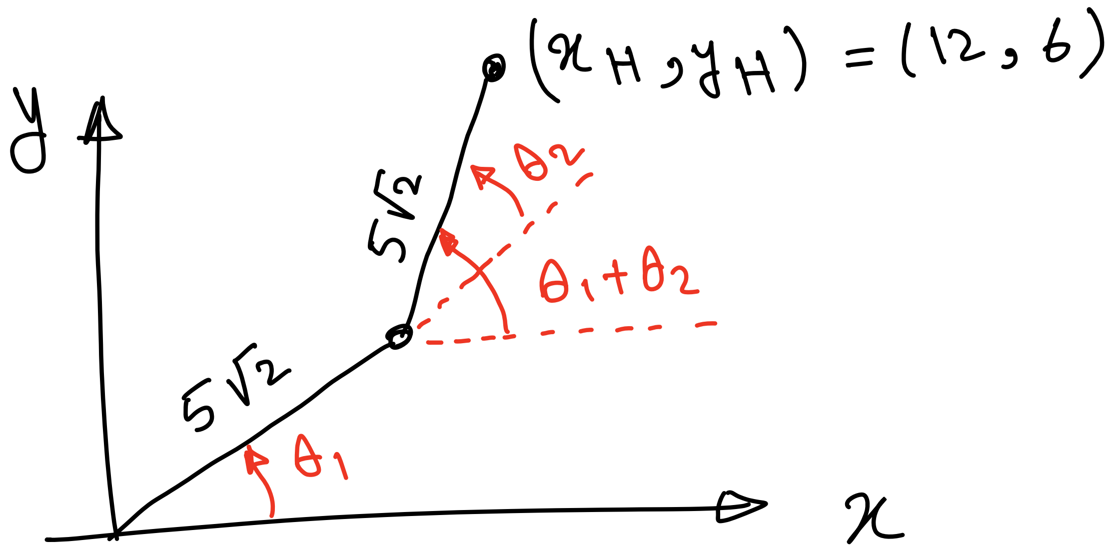
# Inverse Kinematic problem




$x_H$ and $y_H$ are given. Find $\theta_1$ and $\theta_2$.

$$\begin{cases} 12 = 5\sqrt{2}\cos\theta_1 + 5\sqrt{2}\cos(\theta_1+\theta_2) \\ 6 = 5\sqrt{2}\sin\theta_1 + 5\sqrt{2}\sin(\theta_1+\theta_2) \end{cases}$$

$$\theta_1+\theta_2 \rightarrow \Phi$$

$$\begin{cases} 12 = 5\sqrt{2}\cos\theta_1 + 5\sqrt{2}\cos\Phi \\ 6 = 5\sqrt{2}\sin\theta_1 + 5\sqrt{2}\sin\Phi \end{cases}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Rearrange the equations:

$$\begin{cases} 12 - 5\sqrt{2}\cos\theta_1 = 5\sqrt{2}\cos\phi & (a) \\ 6 - 5\sqrt{2}\sin\theta_1 = 5\sqrt{2}\sin\phi & (b) \end{cases}$$

$$(a)^2 + (b)^2 \Rightarrow 144 + 36 + 50\underbrace{(\cos^2\theta_1 + \sin^2\theta_1)}_{1} - 120\sqrt{2}\cos\theta_1 - 60\sqrt{2}\sin\theta_1 = 50\underbrace{(\cos^2\phi + \sin^2\phi)}_{1}$$

$$180 - 60\sqrt{2}(\sin\theta_1 + 2\cos\theta_1) = 0$$

$$\sqrt{2}\sin\theta_1 + 2\sqrt{2}\cos\theta_1 = 3$$

$$\sin^2\theta_1 + \cos^2\theta_1 = 1 \xrightarrow{\text{or}} \cos\theta_1 = \pm\sqrt{1 - \sin^2\theta_1}$$

Let's choose $S \to \sin\theta_1$ and substitute in the equation:

$$\sqrt{2}\,S \pm 2\sqrt{2}\sqrt{1 - S^2} = 3$$

$$\sqrt{2}\,s - 3 = \pm 2\sqrt{2}\sqrt{1-s^2}$$

$$(\sqrt{2}\,s - 3)^2 = (\pm 2\sqrt{2}\sqrt{1-s^2})^2$$

$$10s^2 - 6\sqrt{2}\,s + 1 = 0$$

$$s = \frac{6\sqrt{2} \pm \sqrt{72-40}}{20}$$

$$\sin\theta_1 = s = \frac{1}{\sqrt{2}} \quad \text{or} \quad \frac{1}{5\sqrt{2}}$$

$$\theta_1 = 45^\circ \quad \text{or} \quad 8.13^\circ \quad (*)$$

substitute (*) into (b) to solve for $\phi$:

$$6\sqrt{2} - 5\sqrt{2}\sin\theta_1 = 5\sqrt{2}\sin\phi$$

$$\sin\phi = \frac{6\sqrt{2} - 5\sqrt{2}\sin\theta_1}{5\sqrt{2}}$$

For $\sin\theta_1 = \frac{1}{\sqrt{2}} \Rightarrow \sin\phi = \frac{1}{5\sqrt{2}}$

For $\sin\theta_1 = \frac{1}{5\sqrt{2}} \Rightarrow \sin\Phi = \frac{1}{\sqrt{2}}$

$\Phi = \theta_1 + \theta_2$

For $\theta_1 = 45^\circ \Rightarrow \Phi = 8.13^\circ$

For $\theta_1 = 8.13^\circ \Rightarrow \Phi = 45^\circ$

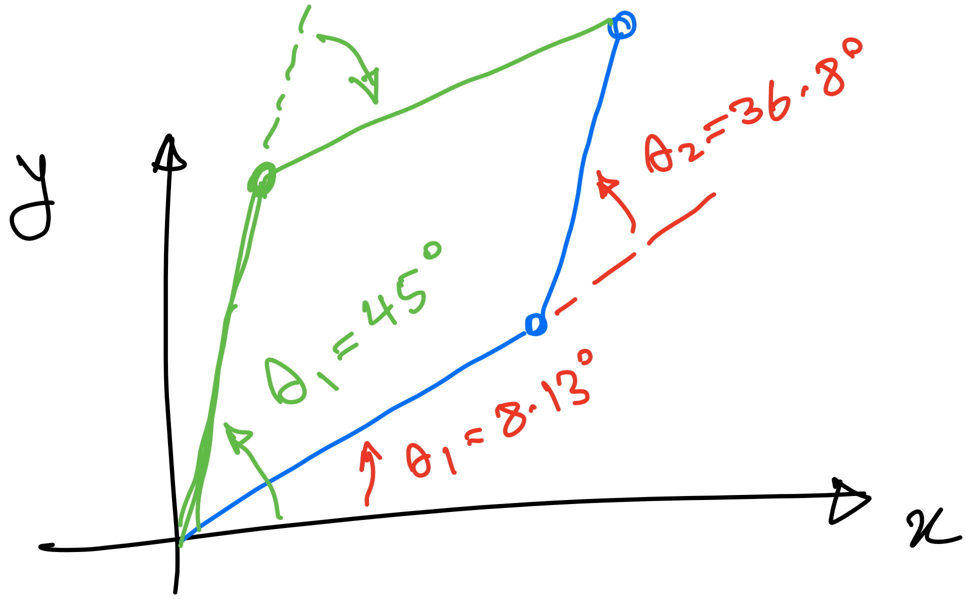
$\theta_2 = \Phi - \theta_1$

For $\theta_1 = 45^\circ \Rightarrow \theta_2 = -36.8^\circ$

For $\theta_1 = 8.13^\circ \Rightarrow \theta_2 = 36.8^\circ$

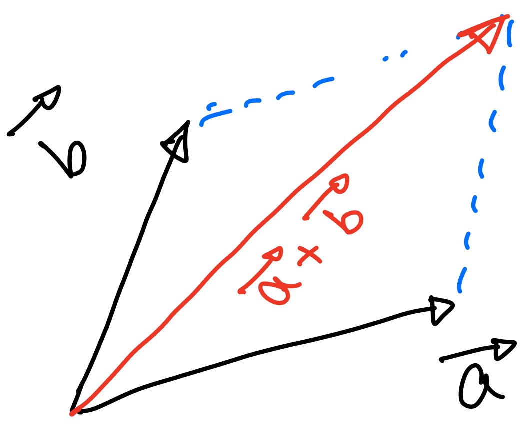
$$\begin{cases} \theta_1 = 45^\circ \\ \theta_2 = -36.8^\circ \end{cases} \qquad \begin{cases} \theta_1 = 8.13^\circ \\ \theta_2 = 36.8^\circ \end{cases}$$

$\theta_2 = -36.8^\circ$




## Vectors:

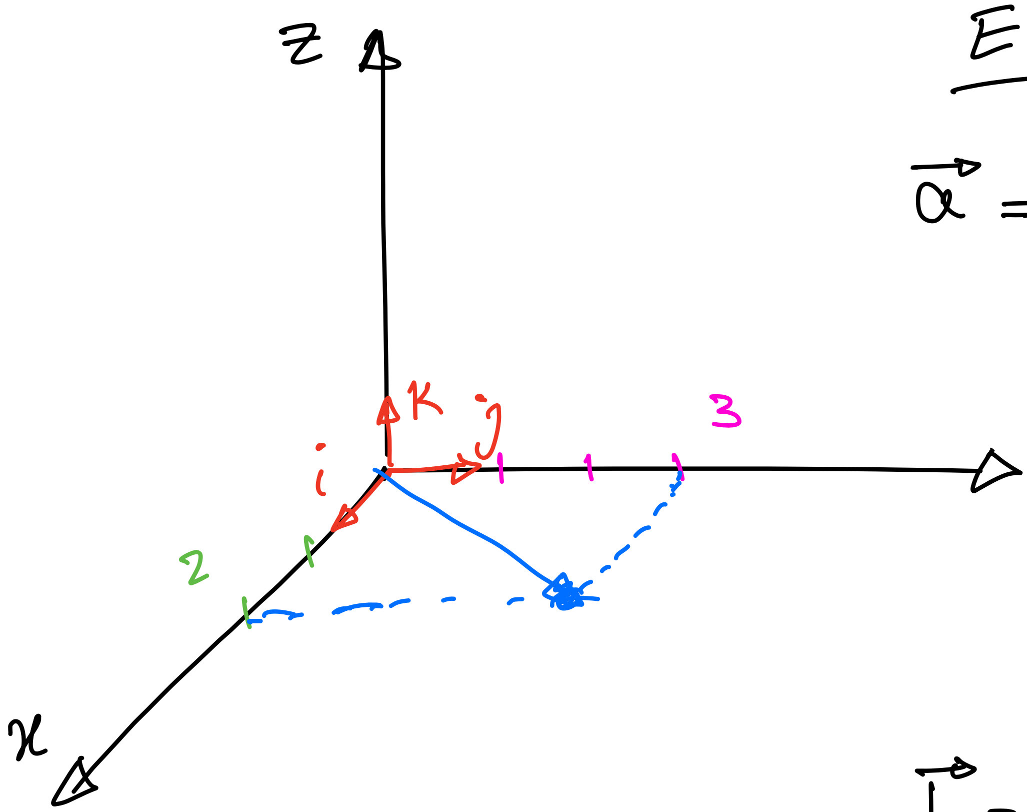
Adding and subtracting vectors:



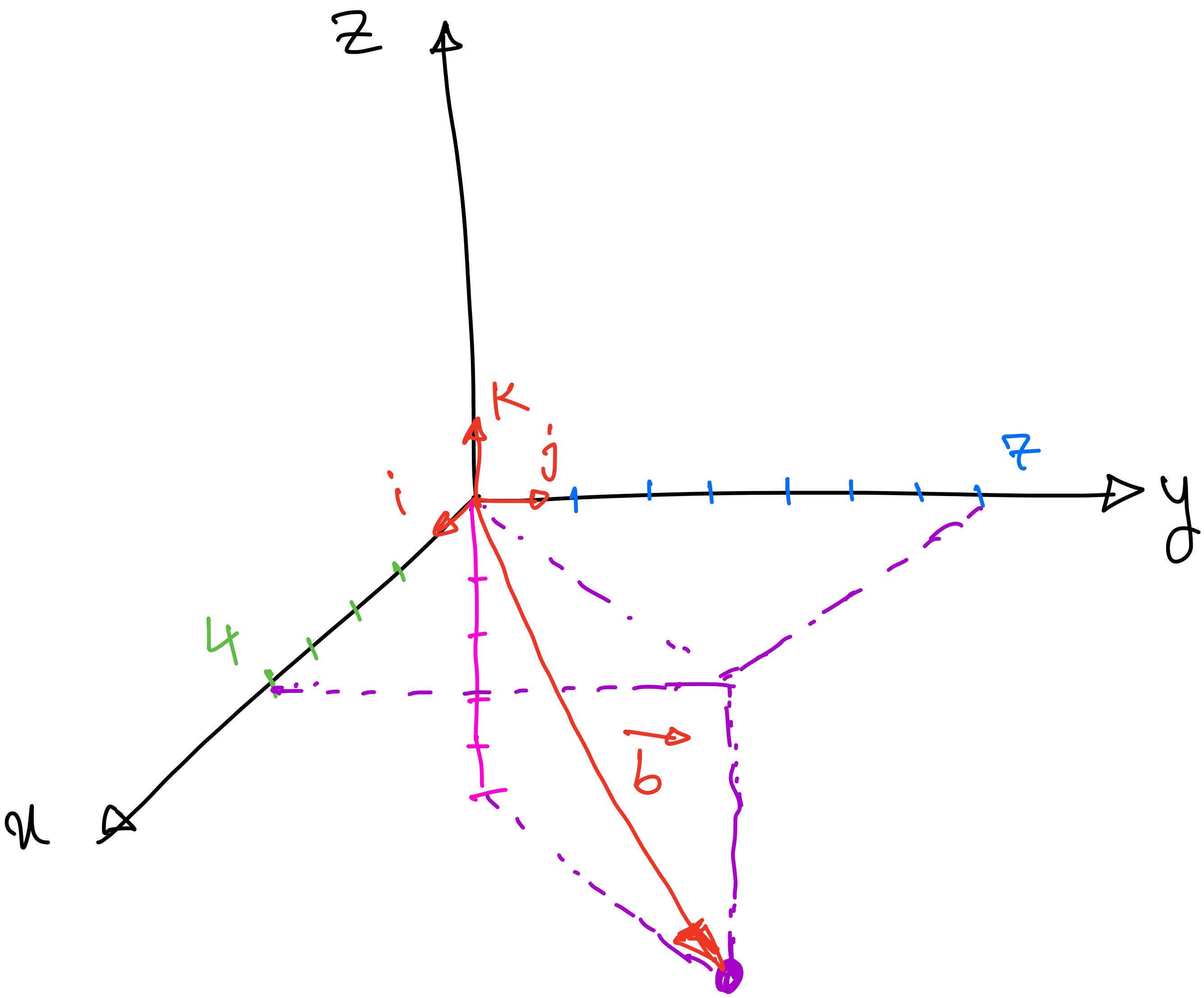

$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

# Example

$$\vec{a} = \underline{2}\,\hat{i} + \underline{3}\,\hat{j} + 0\,\hat{k}$$



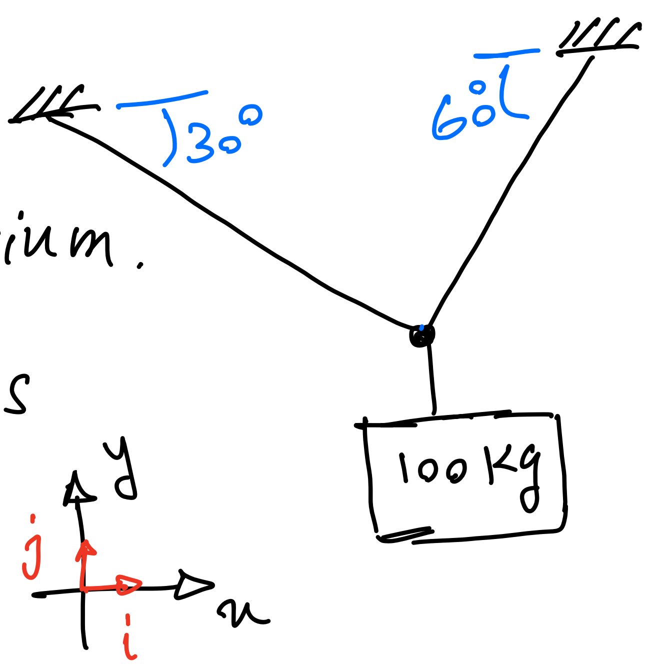

$$\vec{b} = 4\,\hat{i} + 7\,\hat{j} - 5\,\hat{k}$$



## Example

In static equilibrium.

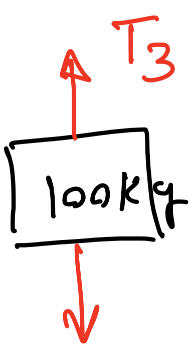
Find the tensions in the ropes




Free-body-diagram (F.B.D)

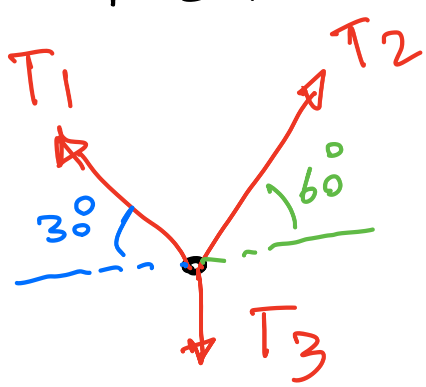
(show the mass and all the forces applied to it)

FBD




Tension in the rope

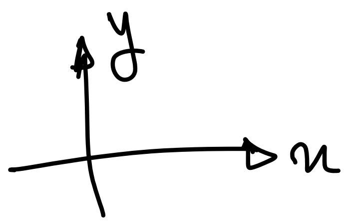
FBD




$W = mg = 100 \times 10$ N

$\sum F_y = 0 \Rightarrow T_3 - W = 0$

$T_3 = W$



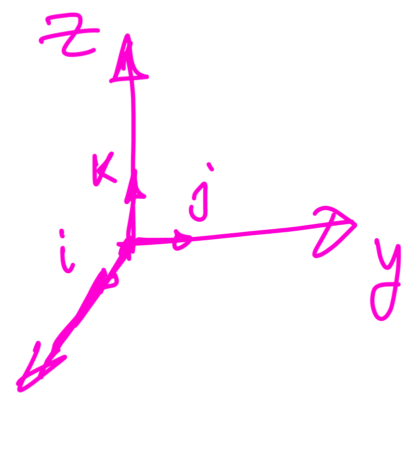

$$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -T_1 \cos 30^\circ + T_2 \cos 60^\circ = 0$$

$+\uparrow \sum F_y = 0 \implies -T_3 + T_1 \sin 30^\circ + T_2 \sin 60^\circ = 0$

Alternative: $\sum F = 0$

$$-T_1 \cos 30^\circ\, \hat{i} + T_2 \cos 60^\circ\, \hat{i}$$
$$-T_3\, \hat{j} + T_1 \sin 30^\circ\, \hat{j} + T_2 \sin 60^\circ\, \hat{j} = \vec{0}$$

$$0 = 0\,\vec{i} + 0\,\vec{j}$$




Note $\vec{F} = 0$

Each component in x, y, z directions is equal to zero

$$\vec{F} = F_x\, \hat{i} + F_y\, \hat{j} + F_z\, \hat{k} = 0$$

$$F_x = 0$$

$$\begin{cases} F_y = 0 \\ F_z = 0 \end{cases}$$