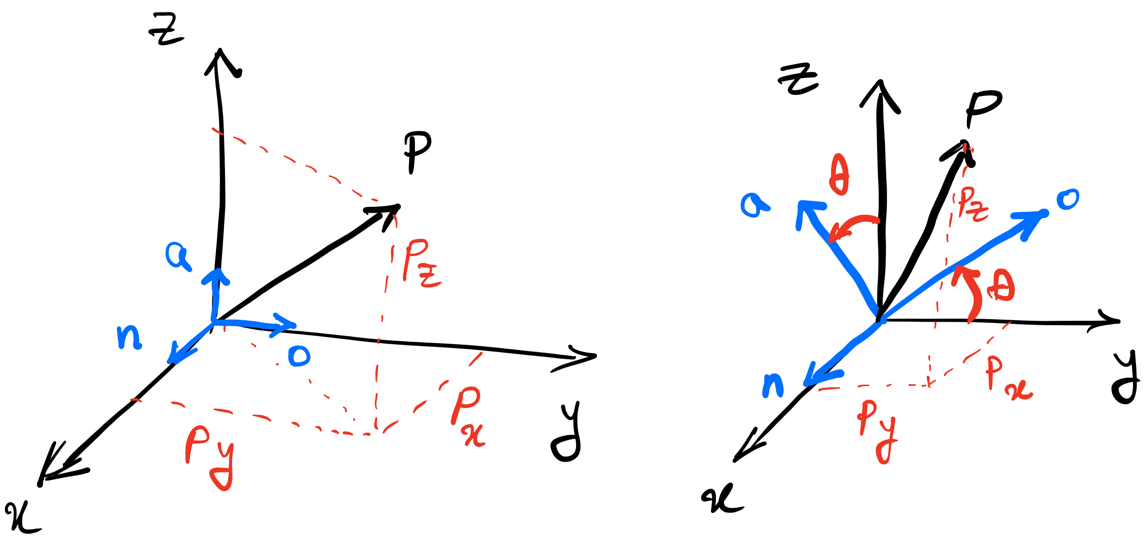
(2.6.2 of The Book)

Representation of a pure Rotation about an axis.



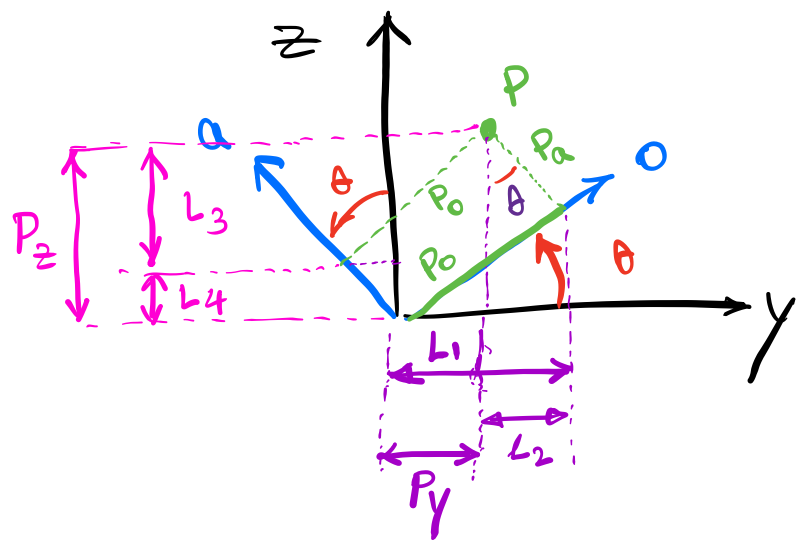
Vector P is in the moving frame F_{noa} (attached to the moving frame) and moves with the moving frame.

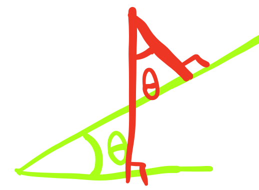
As an example frame F_{noa} rotates about x-axis.

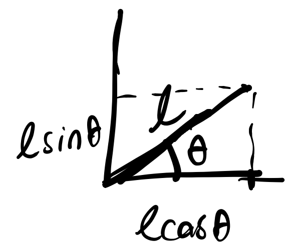
Drawing the same rotation in 2D:

Coordinates of a point relative to

the reference frame and rotrating frame as viewed from the x-axis.







$$P_x = P_n$$

$$P_y = L_1 - L_2 = P_o\cos\theta - P_a\sin\theta$$

$$P_z = L_3 + L_4 = P_o\sin\theta + P_a\cos\theta$$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

This means that the coordinates of point P (or vector $\vec{P}$) in the rotated frame must be pre-multiplied by the rotation matrix, as above, to get the coordinates in the reference frame. This rotation matrix is only for a pure rotation about the x-axis of the reference frame and is denoted as:

$$P_{xyz} = Rot(x, \theta) \times \vec{P}_{noa}$$

$$\begin{cases} \cos\theta \longrightarrow c\theta \\ \sin\theta \longrightarrow s\theta \end{cases}$$

$$Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$$

You may want to do the same for the rotation of a frame about the y-axis and z-axis of the reference frame.

$$Rot(y,\theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$$Rot(z,\theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Denoting the transformation as ${}^{U}T_{R}$ (and reading it as the transformation of frame R relative to frame U (for universe)).

denoting P_{noa} a ${}^{R}P$ (P relative to frame R), and denoting P_{xyz} as ${}^{U}P$ (P relative to frame U).

$$^{U}P = {}^{U}T_R \times {}^{R}P$$

Example (Book 2.7)

A point $p(2,3,4)^T$ is attached to

$$\boxed{(2,3,4)^T = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}}$$

a rotating frame. The frame rotates 90° about the x-axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation.

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix} \begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 90^\circ & -\sin 90^\circ \\ 0 & \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$$

Representation of Combined Transformations. (section 2.6.3 Book)

To see how combined transformations are handled, let's assume that frame F_{noa} is subjected to the following three successive transformations relative to the reference frame F_{xyz}.

1. Rotation of α degrees about the x-axis.

2. Followed by a translation of $[L_1, L_2, L_3]$ (relative to the x-, y- and z-axes respectively).

3. Followed by a rotation of β degrees about the y-axis.

Also, let's say that a point P_{noa} is attached to the rotating frame at the origin of the reference frame.

As frame F_{noa} rotates or translates relative to the reference frame, point P within the frame moves as well, and the coordinates of the point relative to the reference frame.

$$P_{1,xyz} = Rot(x, \alpha) \times P_{noa}$$

$P_{1,xyz}$ is the coordinate of the point after the first transformation.

After the second transformation.

$$P_{2,xyz} = Trans(L_1, L_2, L_3) \times P_{1,xyz}$$

After the third transformation

$$P_{xyz} = P_{3,xyz} = Rot(y, \beta) \times P_{2,xyz}$$

The coordinates of the point relative to the reference frame

$$\begin{array}{c} \text{step3} \times \text{step2} \times \text{step1} \\ P_{xyz} = Rot(Y, \beta) \times Trans(L_1, L_2 L_3) \times Rot(x, \alpha) \times P_{noa} \end{array}$$

Important: The order of matrices cannot be changed.

You notice that for each transformation relative to the reference frame, the matrix is pre-multiplied.

Example (Example 2.8 from the Book)