Instrumentation and Controls
ETM 3301

Lecture 4

Instructor

Dr. Farbod Khoshnoud

Cruise Control System General Solution

$$\frac{dv(t)}{dt} + 0.05v(t) = 0.001f(t) \qquad v(0) = 0$$

- We can assume that initial condition v(0) is zero because we are investigating input-output relationship.
- Applying Laplace Transform of both sides of LDE.

$$\mathcal{L}\left[\frac{dv(t)}{dt}\right] + 0.05\mathcal{L}[v(t)] = 0.001\mathcal{L}[f(t)]$$

$$\text{Define} \quad \mathrm{V}(s) = \mathcal{L}\left[v(t)\right] \qquad \mathrm{F}(s) = \mathcal{L}\left[f(t)\right]$$

$$[sV(s) - v(0)] + 0.05V(s) = 0.001F(s)$$

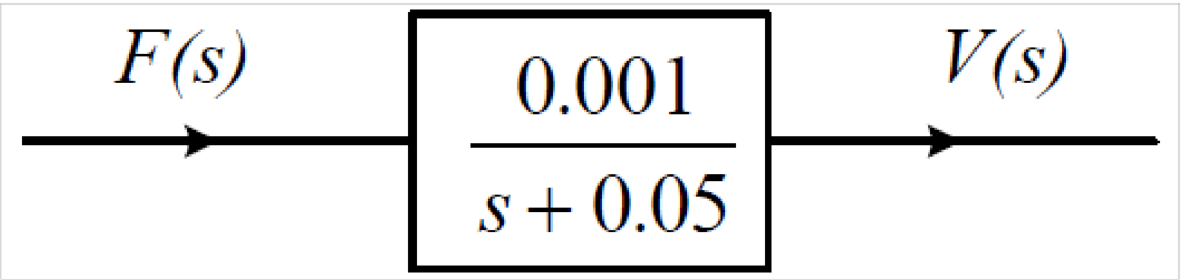
$$(s + 0.05)V(s) = 0.001F(s)$$

Cruise Control System Transfer Function (TF)

$$V(s) = \frac{0.001}{s + 0.05} F(s) = G(s)F(s)$$

$$G(s) = \frac{V(s)}{F(s)} = \frac{0.001}{s + 0.05}$$

- The function $G(s)$ relates the output (car's velocity, $V(s)$) to the input (driving force, $F(s)$) .
  - it is known as the **transfer function**.



Cruise Control System Solution using TF

$$f(t) = 10t \quad \text{find } v(t)$$

$$F(s) = \frac{10}{s^2}$$

$$V(s) = G(s)F(s) = \frac{0.01}{s^2(s+0.05)}$$

$$= \frac{0.2}{s^2} - \frac{4}{s} + \frac{4}{s+0.05}$$

$$v(t) = \mathcal{L}^{-1}\left\{V(s)\right\} = 0.2t - 4 + 4e^{-0.05t}$$

Cruise Control System Solution using TF

$$V(s) = \frac{0.001}{s+0.05} F(s) = G(s)F(s)$$

$$f(t) = 1000(1 - e^{-10t}) \quad F(s) = 1000\left(\frac{1}{s} - \frac{1}{s+10}\right) = \frac{10000}{s(s+10)}$$

$$V(s) = \frac{10}{s(s+10)(s+0.05)} = \frac{20}{s} + \frac{0.1005}{s+10} - \frac{20.101}{s+0.05}$$

$$v(t) = 20 - 0.1005e^{-10t} + 20.101e^{-0.05t}$$

Cruise Control System Solution using TF

$$V(s)=\frac{0.001}{s+0.05}F(s)=G(s)F(s)$$

$$f(t)=500(1-\sin 2t)$$

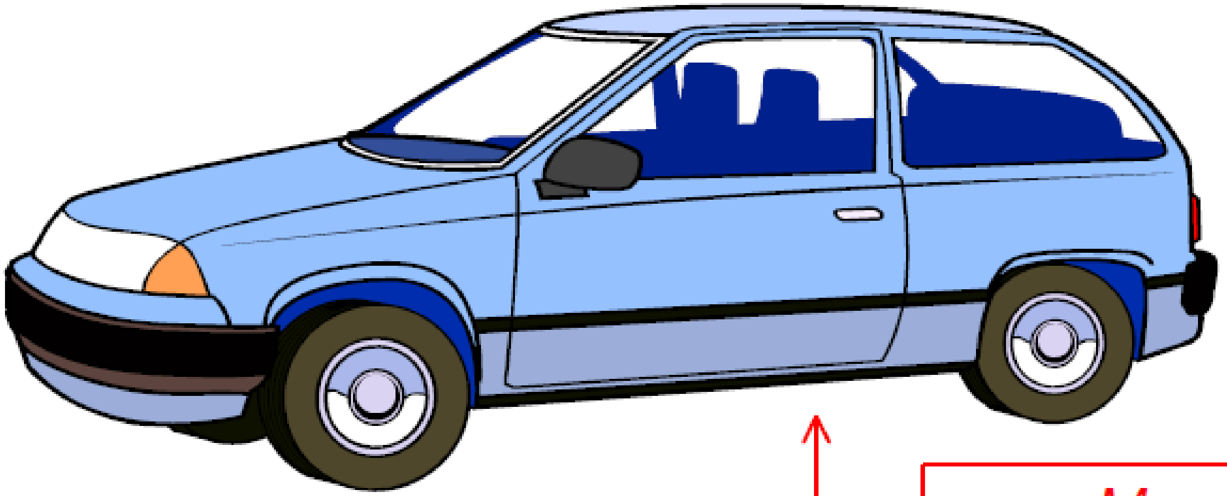
$$F(s)=500\left(\frac{1}{s}-\frac{2}{s^2+4}\right)=\frac{500(s^2-2s+4)}{s(s^2+4)}$$

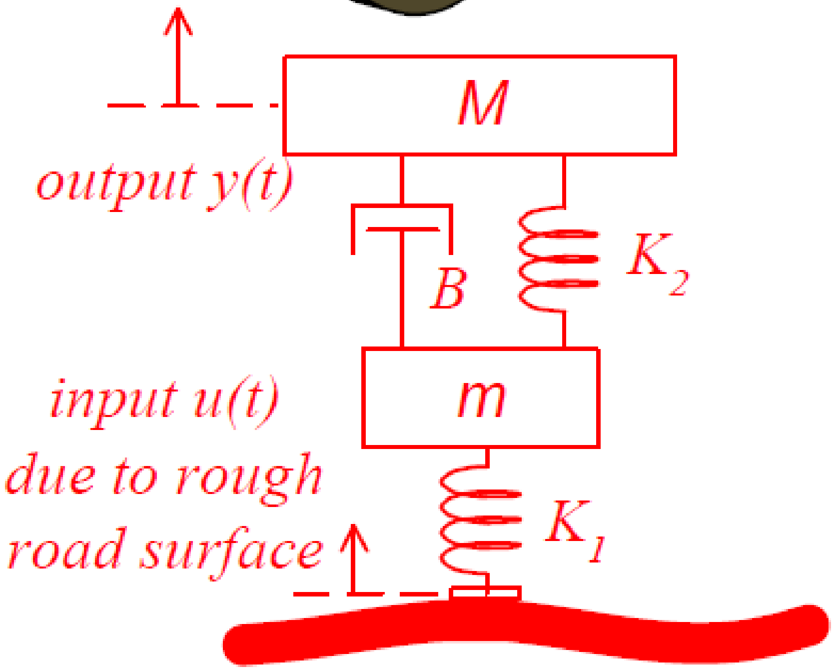
$$V(s)=\frac{0.5(s^2-2s+4)}{s(s^2+4)(s+0.05)}=\frac{10}{s}-\frac{10.25}{s+0.05}+\frac{0.25s-0.013}{s^2+4}$$

$$=\frac{10}{s}-\frac{10.25}{s+0.05}+0.25\frac{s}{s^2+4}-0.006\frac{2}{s^2+4}$$

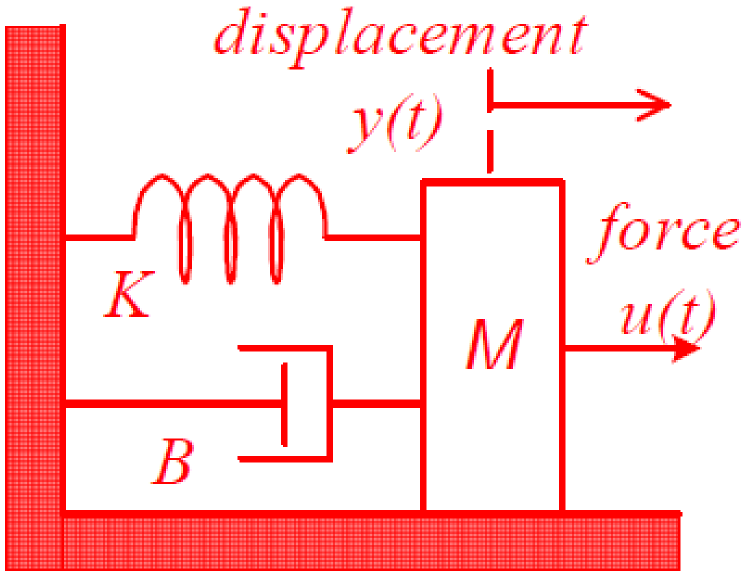
$$v(t)=10-10.25e^{-0.05t}+0.25\cos 2t-0.006\sin 2t$$

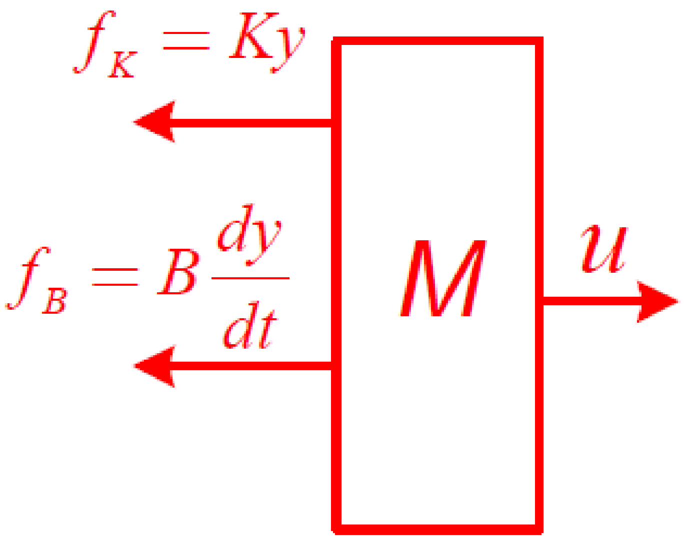
Car Suspension System Example





Car Suspension System Modelling
Simplified Case: *Mass-Spring-Damper System*





- To investigate the displacement *y(t)* caused by the force *u(t)*.

Newton's 2nd law:
$$M\frac{d^2y(t)}{dt^2} = u(t) - Ky(t) - B\frac{dy(t)}{dt}$$

Mass-Spring-Damper System Modelling

$$M\frac{d^2y(t)}{dt^2} = u(t) - Ky(t) - B\frac{dy(t)}{dt}$$

Rearrange the equation:

$$M\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{d(t)} + Ky(t) = u(t)$$

- Taking the Laplace transform of the both sides and assuming zero initial conditions.

$$Ms^2Y(s) + BsY(s) + KY(s) = U(s)$$

$$(Ms^2 + Bs + K)Y(s) = U(s)$$

$$Y(s) = \frac{1}{Ms^2 + Bs + K}U(s)$$

Mass-Spring-Damper System TF

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{Ms^2 + Bs + K}$$

Substituting parameters

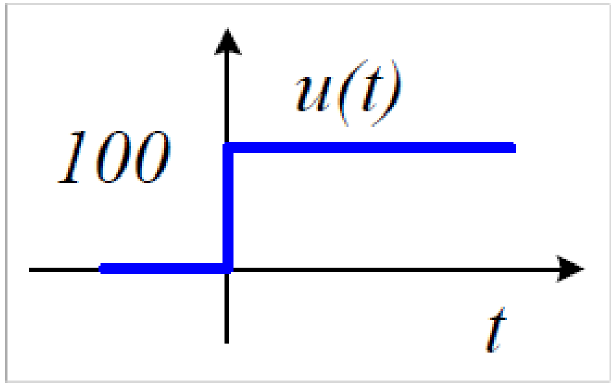
$M = 800$ kg; $K = 400$ N/m

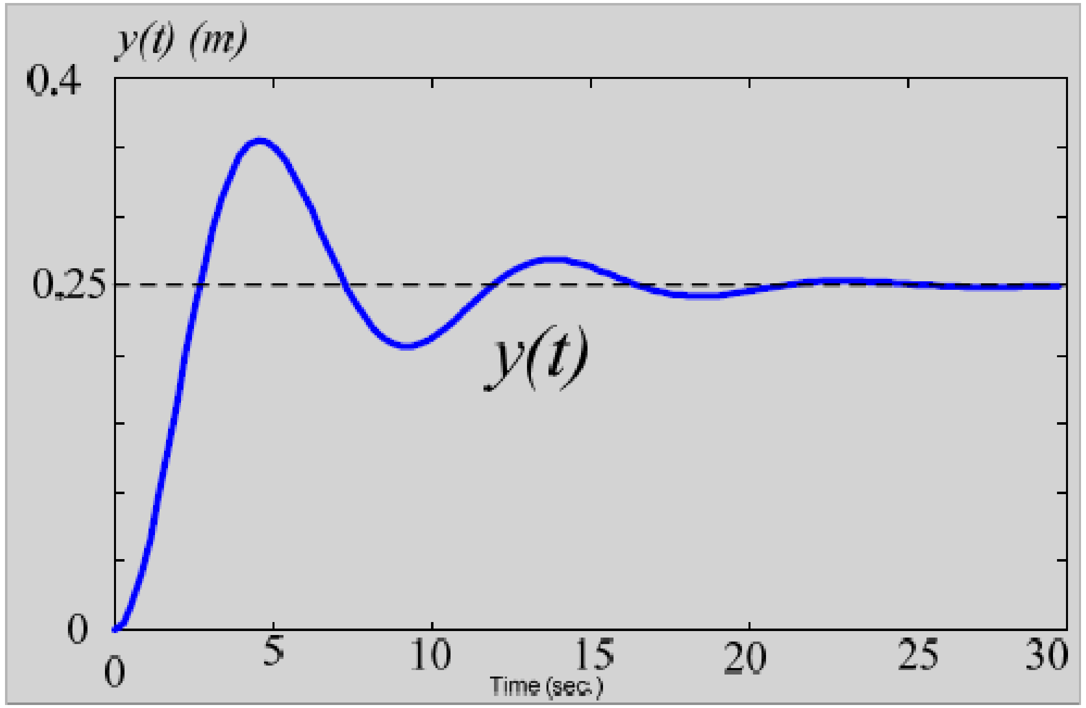
$B = 300$ Ns/m

- Transfer function: $$G(s) = \frac{0.125}{s^2 + 0.375s + 0.5}$$

Mass-Spring-Damper System Response

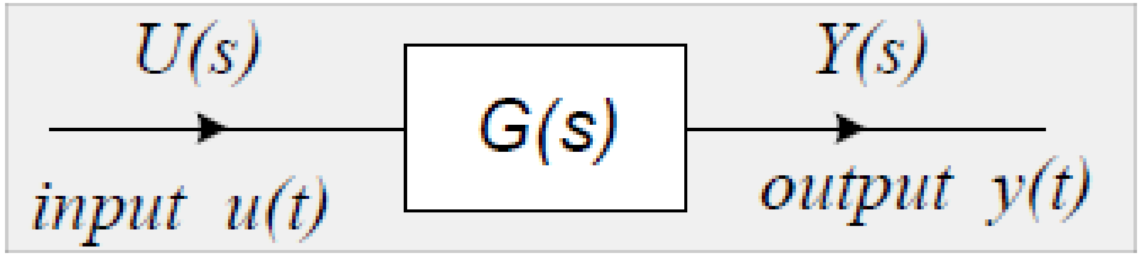
When $u = 100N$ (step input), system response:





Transfer Function (TF)

- The transfer function of a dynamic system is defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all initial conditions assumed to be zero.



$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]}$$