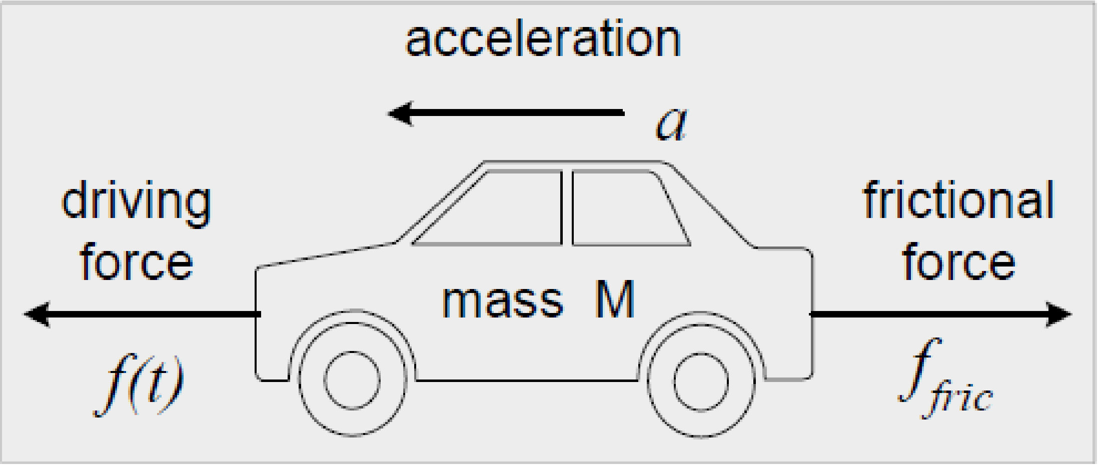
Instrumentation and Controls
ETM 3301

Lecture 3

Instructor
Dr. Farbod Khoshnoud

Cruise Control System Modelling



$f(t)$: driving force

f_{fric}: frictional force (e.g. air resistance)

Newton's 2nd law:

$$f(t) - f_{fric} = Ma$$

Assume the frictional force is proportional to velocity (viscous friction)

$$f_{fric} = Bv(t)$$

Remember that

$$a = \frac{dv(t)}{dt}$$

Cruise Control System Modelling

$$f(t) - Bv(t) = M\frac{dv(t)}{dt}$$

Or

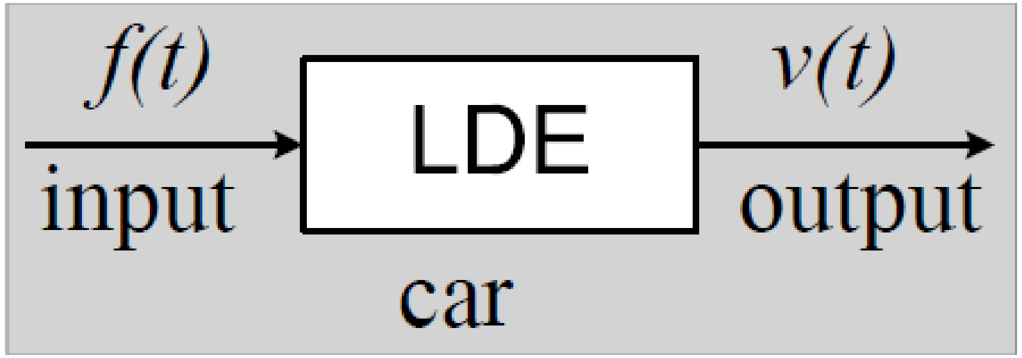
$$\frac{dv(t)}{dt} + \frac{B}{M}v(t) = \frac{1}{M}f(t)$$

Substitute: $M = 1000kg\ ; \quad B = 50Ns/m$

$$\frac{dv(t)}{dt} + 0.05v(t) = 0.001f(t)$$

Model

- The input-output relationship is described by a linear differential equation.



Cruise Control System Solution (1)

$f(t) = 1000N \quad ; \quad v(0) = 0$

- Find $v(t)$

$$\frac{dv(t)}{dt} + 0.05v(t) = 0.001f(t)$$

Applying Laplace Transform of both sides of LDE.

$$\mathcal{L}\left[\frac{dv(t)}{dt}\right] + 0.05\mathcal{L}[v(t)] = \mathcal{L}[0.001 \times 1000]$$

$$\text{Define} \quad V(s) = \mathcal{L}[v(t)]$$

$$[sV(s) - v(0)] + 0.05V(s) = \frac{1}{s}$$

Cruise Control System Solution (2)

$$V(s) = \frac{1}{s(s+0.05)} = \frac{20}{s} - \frac{20}{s+0.05}$$

$$v(t) = \mathcal{L}^{-1}\left[V(s)\right] = 20 - 20e^{-0.05t} \quad \text{when } f(t) = 1000$$

- How to find the response *v(t)* for other form of control input *f(t)*?
  - Solve a LDE for each form of input?
- We need to find a generalised solution!!!

$$f(t) = 10t$$

$$f(t) = 1000(1 - e^{-10t})$$

$$f(t) = 500(1 - \sin 2t)$$

$$\ldots\ldots\ldots\ldots$$

https://www.mathsisfun.com/algebra/partial-fractions.html

Cruise Control System General Solution

$$\frac{dv(t)}{dt} + 0.05v(t) = 0.001f(t) \qquad v(0) = 0$$

- We can assume that initial condition v(0) is zero because we are investigating input-output relationship.
- Applying Laplace Transform of both sides of LDE.

$$\mathcal{L}\left[\frac{dv(t)}{dt}\right] + 0.05\mathcal{L}[v(t)] = 0.001\mathcal{L}[f(t)]$$

Define $\mathrm{V}(s) = \mathcal{L}\left[v(t)\right] \qquad \mathrm{F}(s) = \mathcal{L}\left[f(t)\right]$

$$[sV(s) - v(0)] + 0.05V(s) = 0.001F(s)$$

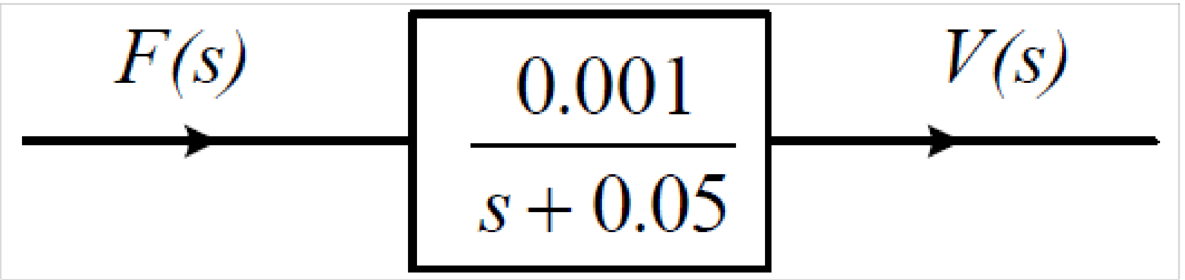
$$(s + 0.05)V(s) = 0.001F(s)$$

Cruise Control System Transfer Function (TF)

$$V(s) = \frac{0.001}{s+0.05} F(s) = G(s)F(s)$$

$$G(s) = \frac{V(s)}{F(s)} = \frac{0.001}{s+0.05}$$

- The function *G(s)* relates the output (car's velocity, *V(s)*) to the input (driving force, *F(s)*) .
  - it is known as the **transfer function**.



Cruise Control System Solution using TF

$$f(t) = 10t \quad \text{find } v(t)$$

$$F(s) = \frac{10}{s^2}$$

$$V(s) = G(s)F(s) = \frac{0.01}{s^2(s+0.05)}$$

$$= \frac{0.2}{s^2} - \frac{4}{s} + \frac{4}{s+0.05}$$

$$v(t) = \mathcal{L}^{-1}\{V(s)\} = 0.2t - 4 + 4e^{-0.05t}$$