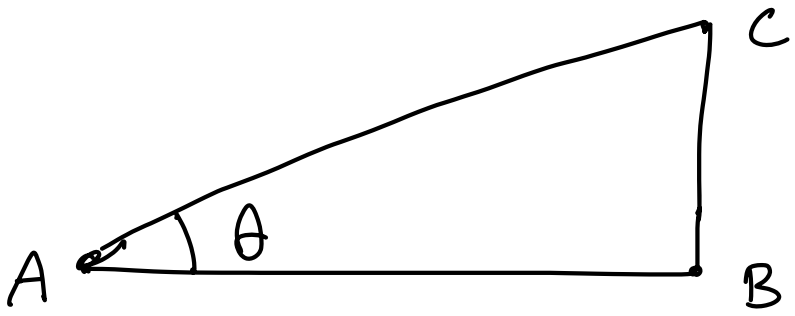
# Trigonometry

## position analysis

Review




$$\cos\theta = \frac{AB}{AC} \qquad \sin\theta = \frac{BC}{AC}$$

$$AC^2 = AB^2 + BC^2$$

$$AB = AC\cos\theta$$

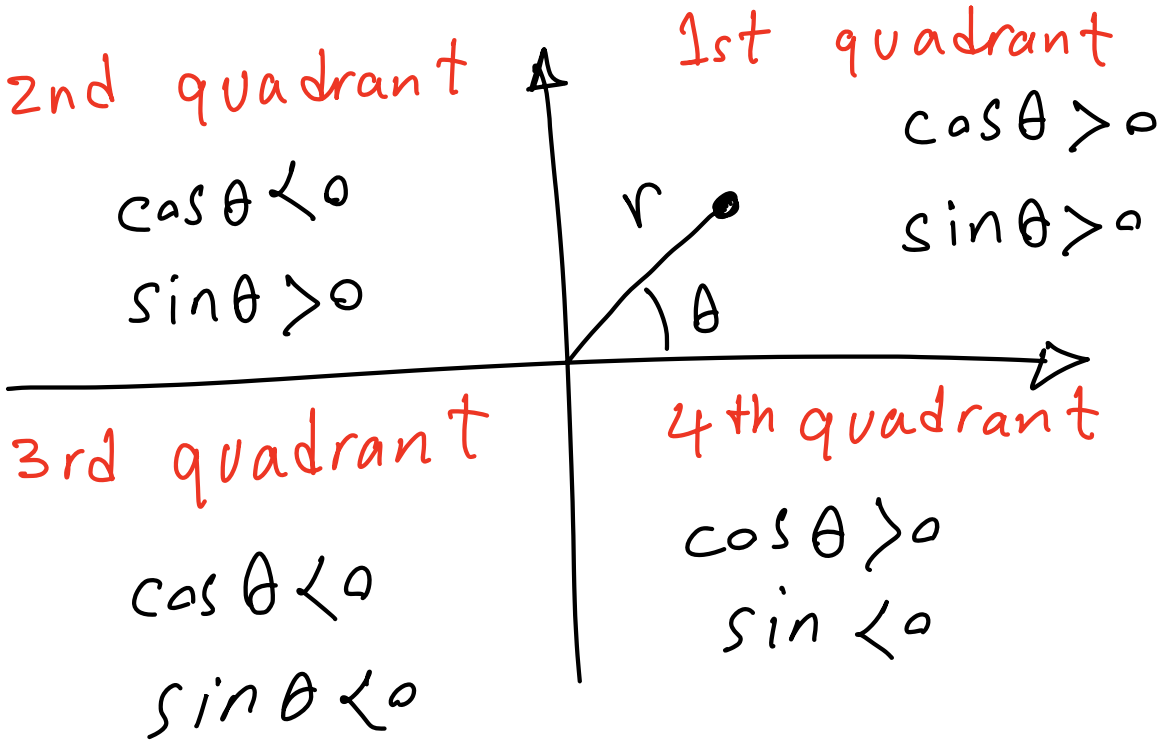
$$BC = AC\sin\theta$$

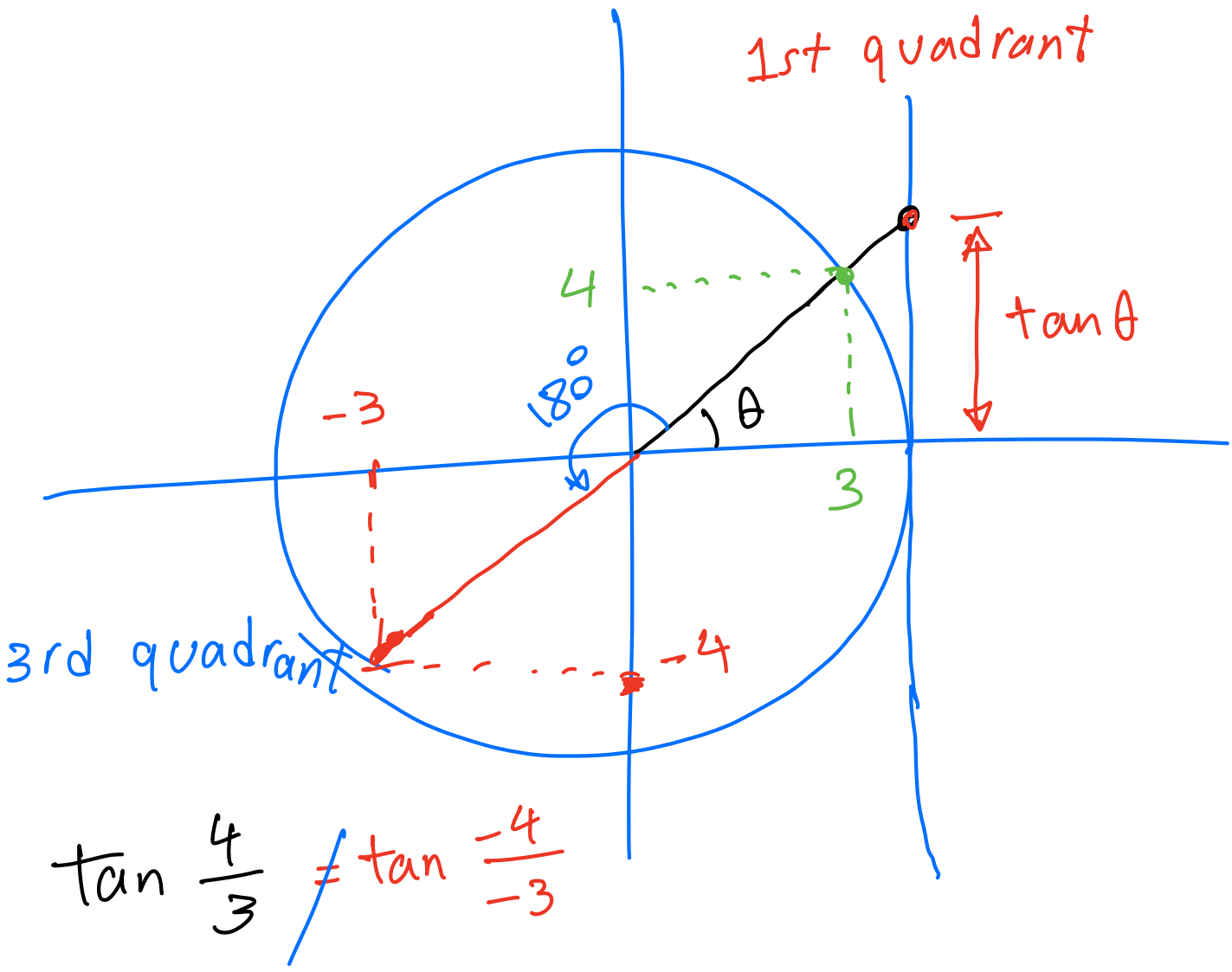
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{BC}{AB}$$

$$AC^2 = (AC\cos\theta)^2 + (AC\sin\theta)^2$$

$$AC^2 = AC^2\cos^2\theta + AC^2\sin^2\theta$$

$$1 = \cos^2\theta + \sin^2\theta$$




$$\tan\frac{4}{3} \neq \tan\frac{-4}{-3}$$

Common values: $\sin(0) = 0$ $\cos(0) = 1$

$\sin(30^\circ) = 1/2$ $\cos(60^\circ) = 1/2$

$\sin(90^\circ) = 1$ $\cos(90^\circ) = 0$

$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$

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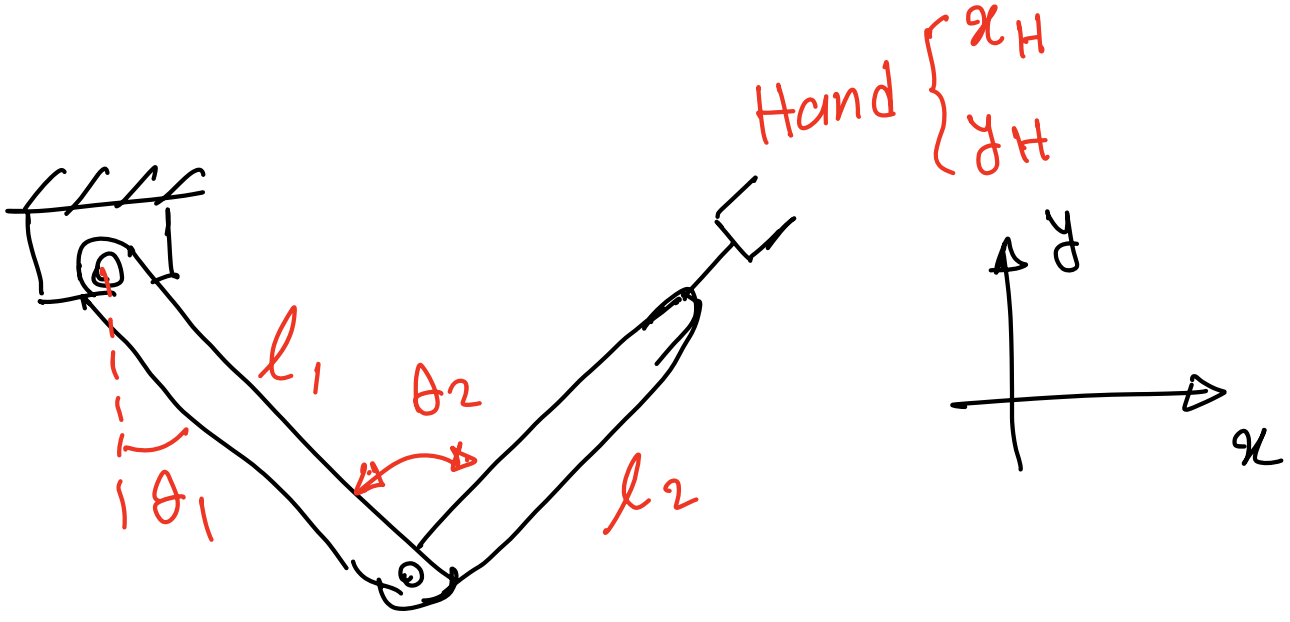
## Example

Motivation example




1) Forward Kinematics

$\theta_1$ and $\theta_2$ are given

$\Rightarrow$ Find the position of

the hand $(x_H, y_H)$

## 2) Inverse Kinematics

A desired $(x_H, y_H)$ is given

$\Rightarrow$ Find $\theta_1$ and $\theta_2$

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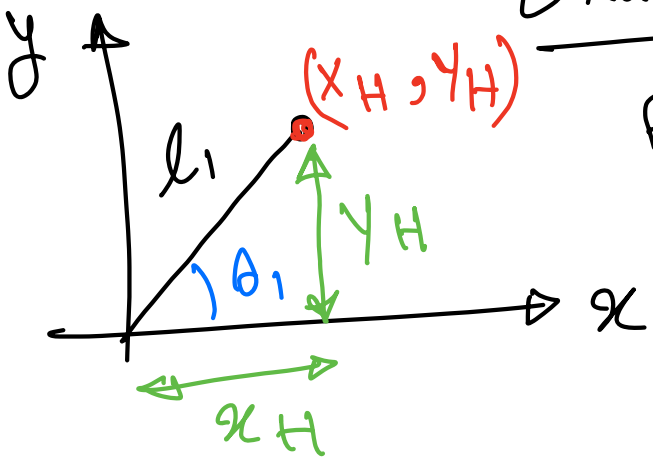
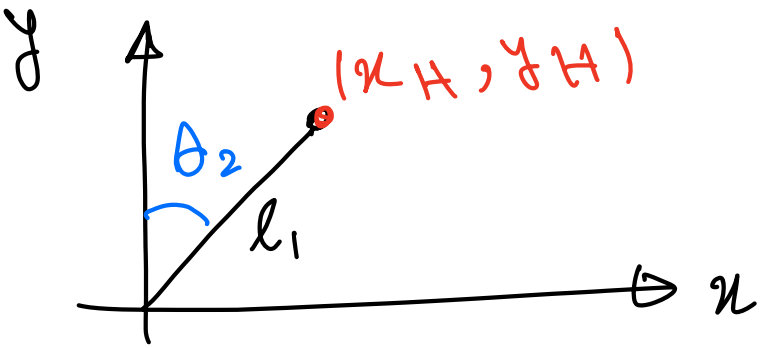
Example




Find $x_H$ and $y_H$ if $l_1$ and $\theta_1$ are given.

$$\begin{cases} x_H = l_1 \cos\theta_1 \\ y_H = l_1 \sin\theta_1 \end{cases}$$




$$\begin{cases} x_H = l_1 \sin\theta_2 \\ y_H = l_1 \cos\theta_2 \end{cases}$$

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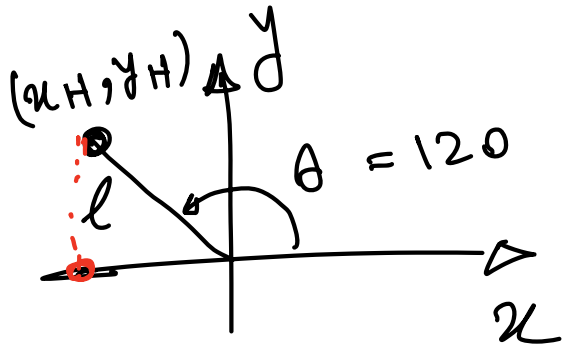
## Example

$l = 1$ ft $\quad \theta = 120^\circ$




Find $x_H$ and $y_H$

$$x_H = l_1 \cos 120^\circ = -1/2$$

$$y_H = l_1 \sin 120^\circ = \sqrt{3}/2$$

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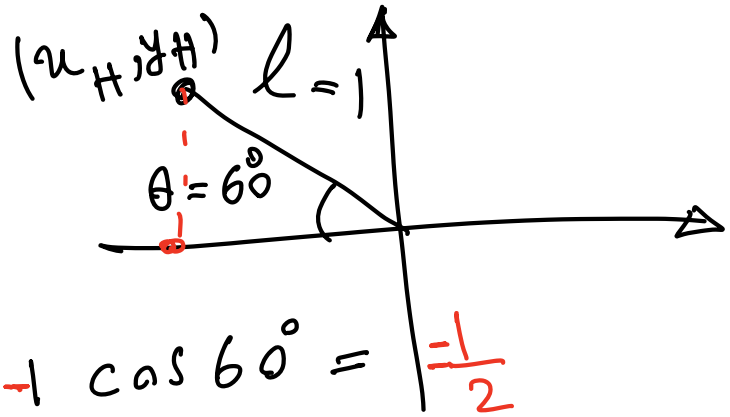
## Example




Find $x_H$ and $y_H$

$$x_H = -l \cos\theta = -1 \cos 60^\circ = -\frac{1}{2}$$

$$y_H = l \sin\theta = 1 \sin 60^\circ = \sqrt{3}/2$$

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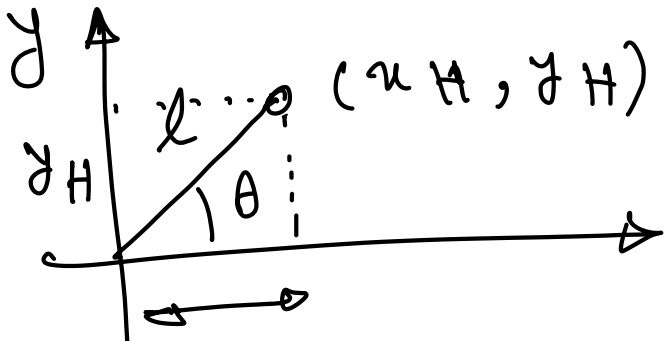
## Example

Inverse Kinematics




$(x_H, y_H)$ are given

$x_H$

$$\begin{cases} x_H = 2 \text{ ft} \\ y_H = 3 \text{ ft} \end{cases}$$

Find $\theta$

$$l = \sqrt{x_H^2 + y_H^2} = \sqrt{13} \text{ ft}$$

$$\begin{cases} x_H = l \cos\theta \\ y_H = l \sin\theta \end{cases} \Rightarrow \begin{cases} \cos\theta = 2/\sqrt{13} \\ \sin\theta = 3/\sqrt{13} \end{cases}$$

$$\theta = 0.98 \text{ rad} = 56.3^\circ$$

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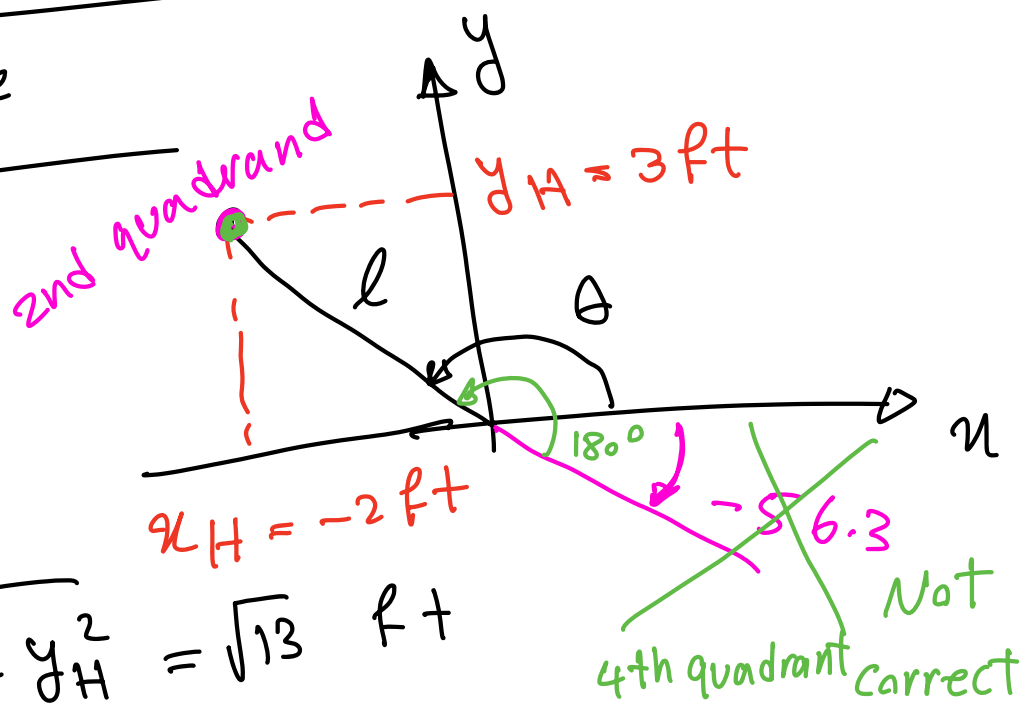
Example

$\theta = ?$




$$l = \sqrt{x_H^2 + y_H^2} = \sqrt{13} \text{ ft}$$

$x_H = l \cos\theta$ $\xrightarrow{\text{divide}}$ $\tan\theta = \frac{3}{-2}$

$y_H = l \sin\theta$

$\theta = ?$

$\theta = -56.3$ ?

$\theta = -56.3^\circ + 180^\circ = 123.7^\circ$

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