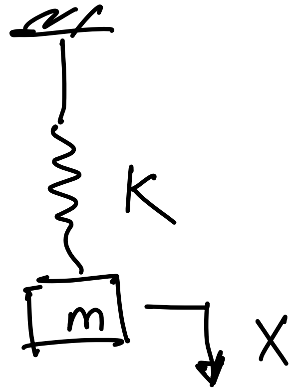
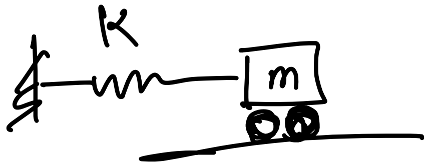
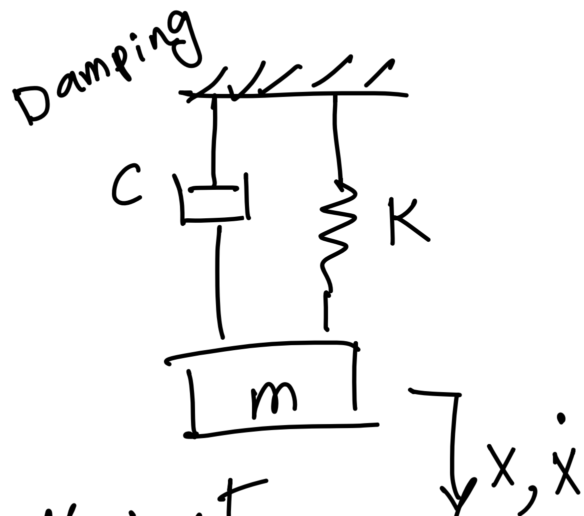
Free vibration

No Force applied to the mass





Vibration model



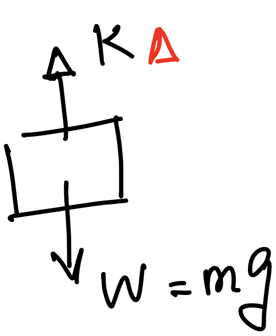
$$F\,(\text{spring}) = KX$$

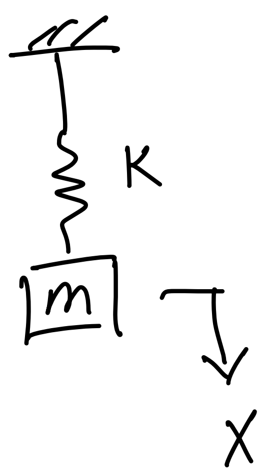
$$F\,(\text{Damper}) = C\dot{X}$$

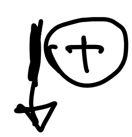
Damping coefficient

Equations of Motion

F.B.D.







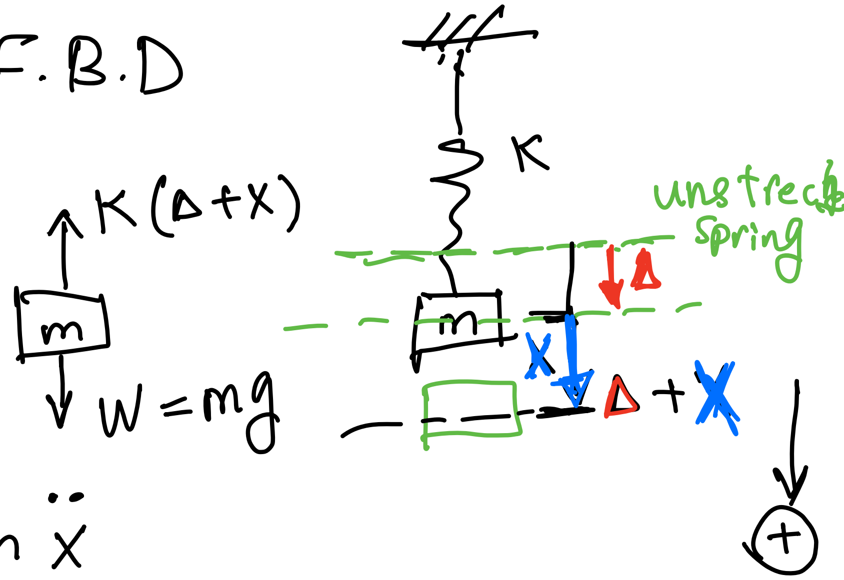
Statics:

$$\overset{+}{\downarrow} \sum F_x = 0$$

$$\boxed{mg - K\Delta = 0}$$

⊕ Dynamics :

F.B.D



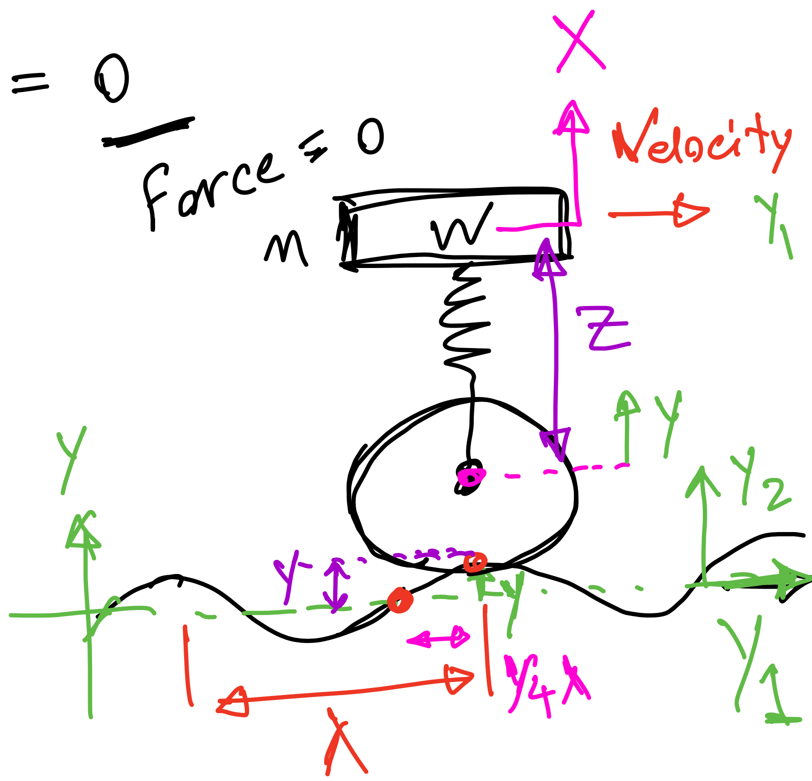
$\downarrow \sum F_X = m\ddot{X}$

$$W - K(\Delta + X) = m\ddot{X}$$

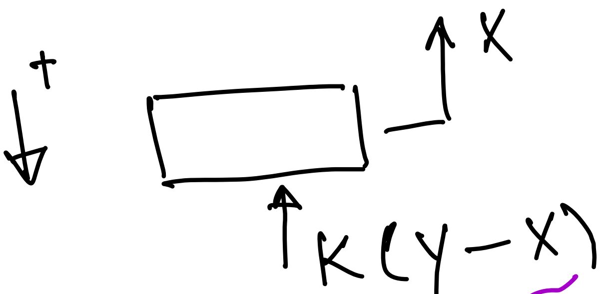
$$\underbrace{W - K\Delta}_{\approx 0} - KX = m\ddot{X}$$

$$m\ddot{X} + KX = 0$$

Force ≈ 0



F.B.D. $\quad X < Y$



$$\sum F = M\ddot{X}$$

$$K(Y-X) = M\ddot{X}$$

$$KZ = M(\ddot{Y} - \ddot{Z})$$

$$M\ddot{Z} + KZ = M\ddot{Y}$$

$$\begin{cases} Z = Y - X \\ \ddot{Z} = \ddot{Y} - \ddot{X} \\ \ddot{X} = \ddot{Y} - \ddot{Z} \end{cases}$$

$$\text{velocity}_{Y_1} = \frac{\text{Displacement}}{\text{Time}} \Rightarrow \text{Time}$$

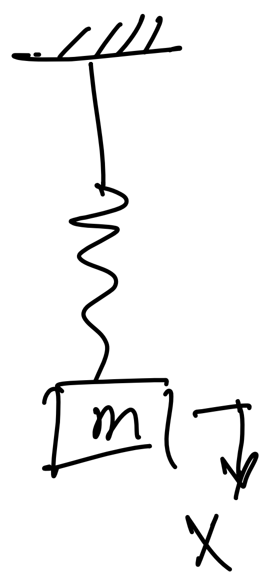
$$\text{Velocity}_{\cancel{Y}_2} = \frac{Y}{\text{Time}}$$

$$\ddot{Y} = \frac{\text{velocity } Y_2}{\text{Time}}$$

$$m\ddot{x} + Kx = 0$$

Standard form of the equation of motion in vibration:

(No coefficient for $\ddot{x}$)



standard $\ddot{x} + \underbrace{\frac{K}{m}}_{\omega_n^2 = \frac{K}{m}} x = 0$

$\omega_n = \sqrt{\frac{K}{m}}$ Natural frequency

$$\ddot{x} + \omega_n^2 x = 0$$

Salution:

$$x = \underline{A} \sin \omega_n t + \underline{B} \cos \omega_n t$$

?   ?

Initial canditions:

$x(t=0) = x(0)$ Displacement at time = 0

$\dot{x}(t=0) = \dot{x}(0)$ velocity

$$\dot{x} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$\ddot{x} = -A\omega_n^2 \sin \omega_n t - B\omega_n^2 \cos \omega_n t$$

$$\ddot{x} + \omega_n^2 x = 0 \qquad (1)$$

$$x = A \sin \omega_n t + B \cos \omega_n t \qquad (2)$$

$$\ddot{x} = -A \omega_n^2 \sin \omega_n t - B \omega_n^2 \cos \omega_n t \qquad (3)$$

(1), (2), (3) $\rightarrow$ proof

$$-A \omega_n^2 \sin \omega_n t - B \omega_n^2 \cos \omega_n t$$
$$+ A\omega_n^2 \sin \omega_n t + \omega_n^2 B \cos \omega_n t = 0$$

valid ✓

$$x = \frac{\dot{x}(0)}{\omega_n} \sin \omega_n t + x(0) \cos \omega_n t$$

$\omega_n \left(\frac{rad}{s}\right) \longrightarrow f\,(Hz) = \frac{\omega_n}{2\pi}$

natural frequency

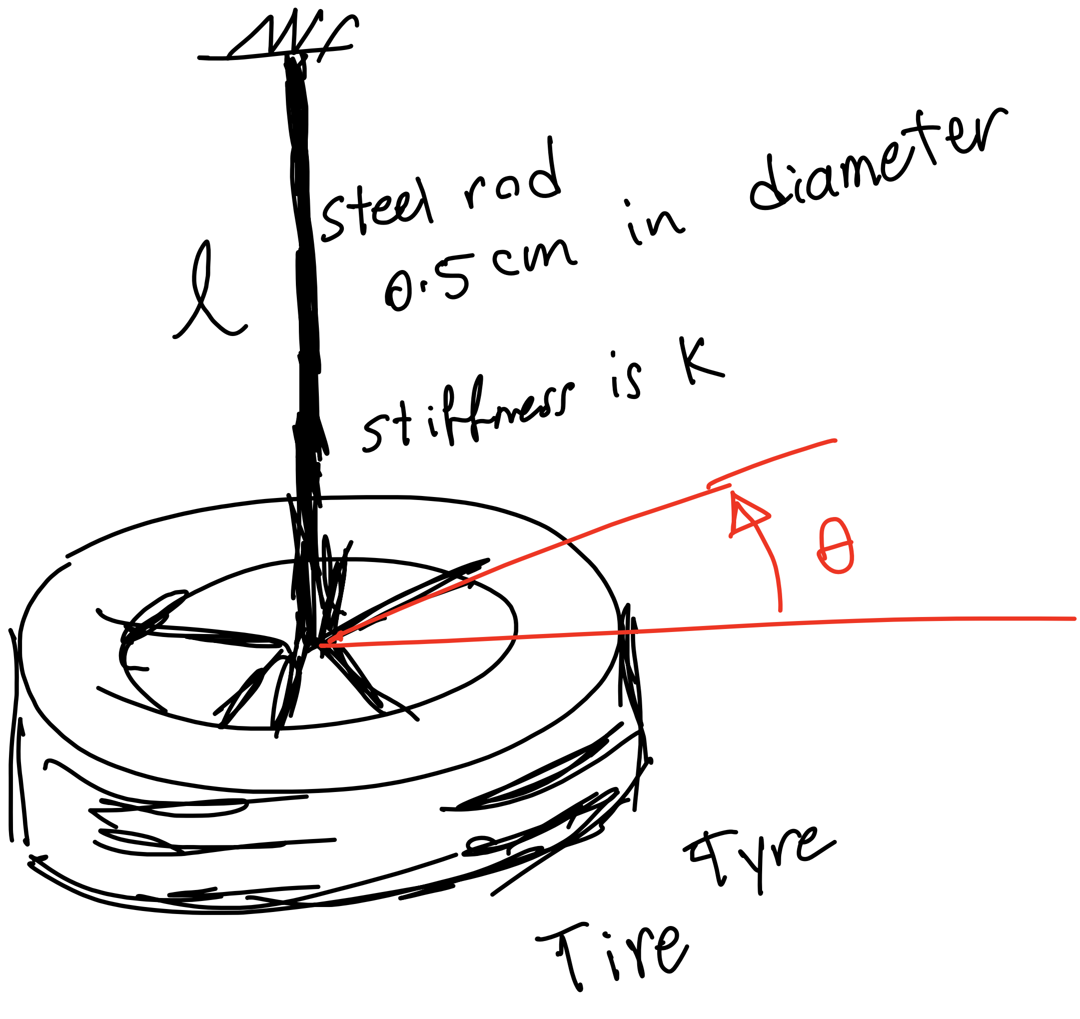
angular frequency

freqency

τ (period of oscillation) $= \frac{1}{f}$

$$\tau = 2\pi\sqrt{\frac{m}{K}} \leftarrow \boxed{\tau = \frac{2\pi}{\omega_n}}$$

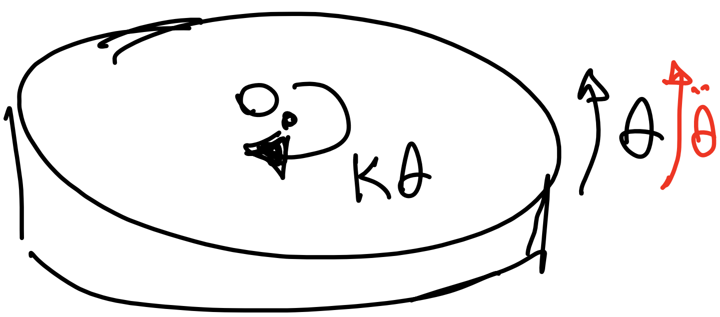
Example 2.2-3 (Book)



$F_{spring} = KX$

$F_{\text{torsional spring}} = K\theta$

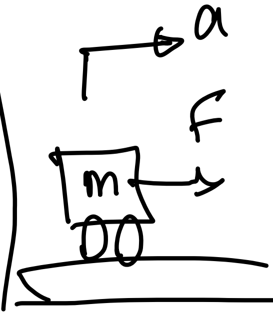
F.B.D.:



+↺ $\sum M_0 = J\ddot{\theta}$

$-K\theta = J\ddot{\theta}$

$\ddot{\theta} + \frac{K}{J}\theta = 0$ standard $w_n = \sqrt{\frac{K}{J}}$



Newton's Second law

$F = ma$

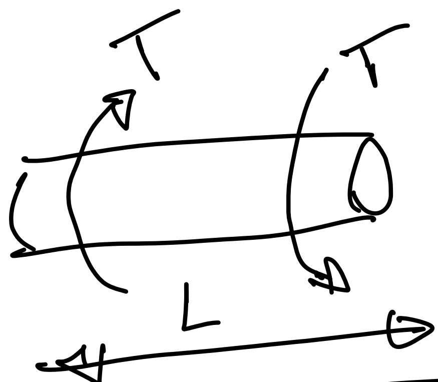
It make 10 oscillations in 30.2 s

$\tau = \frac{30.2}{10}$ s

$w_n = \frac{2\pi}{\tau} = 2.081$ rad/s

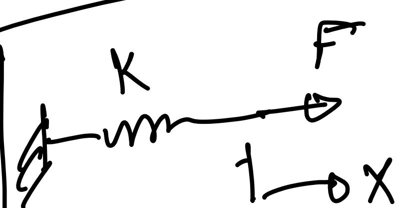
Torsional stiffness of the rod:

$\theta = \frac{TL}{GJ}$



$T = K\theta$

$K = \frac{GJ}{L}$



$F = KX$

J = polar moment of inertia

$$J = \frac{\pi d^4}{32}$$

G = shear modulus $= 80 \times 10^9 \ \frac{N}{m^2}$

$K = 2.455 \ Nm/rad$

calculated for the rad

$$J = \frac{K}{w_n^2} = 0.567 \ kg\,m^2$$

Experiment