Representation of a frame at the origin of a fixed reference frame

A frame is generally represented by three mutually orthogonal axes (such as x, y, z).

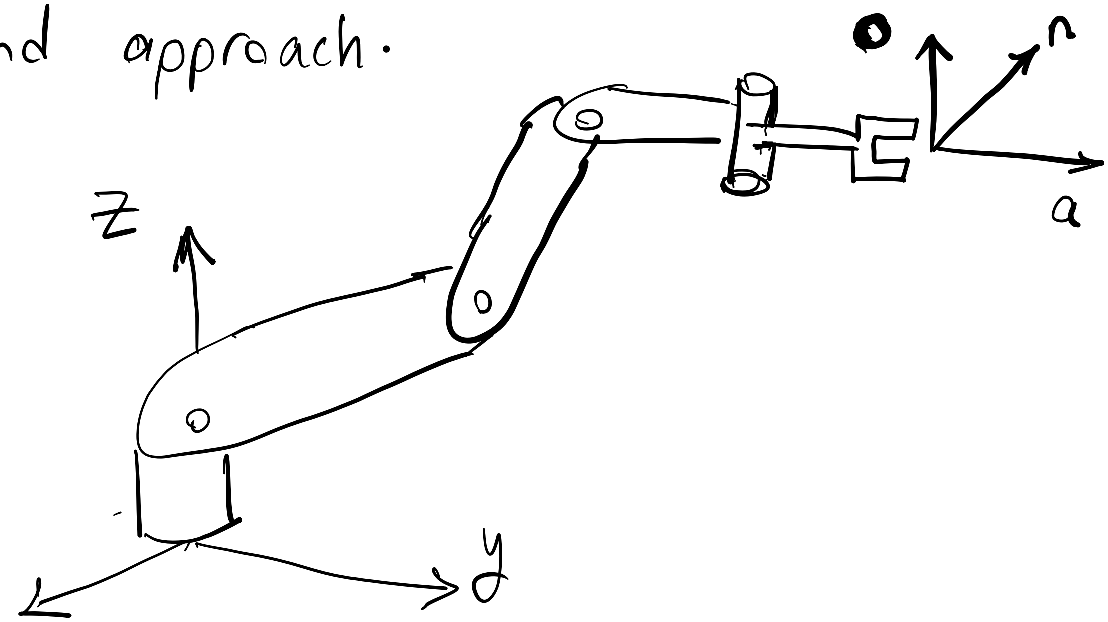
Fixed Universe reference frame

$$F_{x,y,z}$$

and a set of axes n, o, a to represent another (moving) frame

$$F_{n,o,a}$$

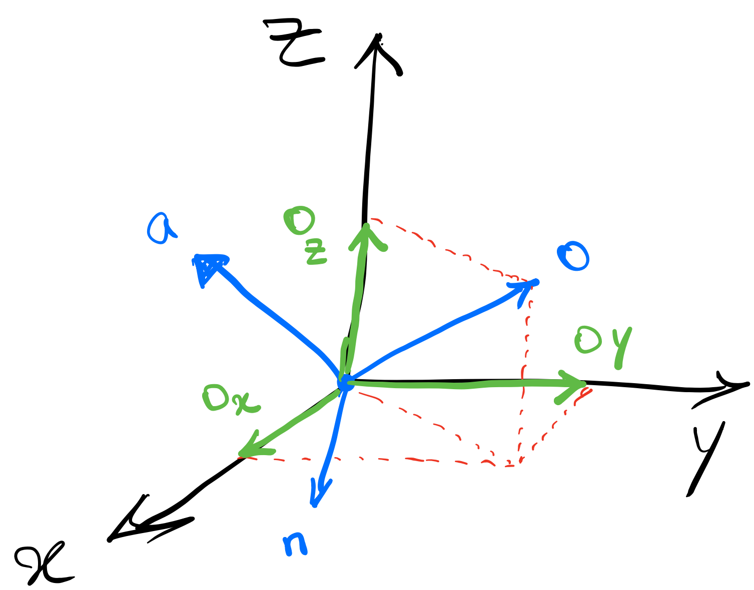
The letters n, o, a are derived from the words normal, orientation, and approach.



it should be clear that in order to avoid hitting the part while trying to pick it up, the robot would have to approach it along the a-axis of the gripper (or approach axis or a-axis).

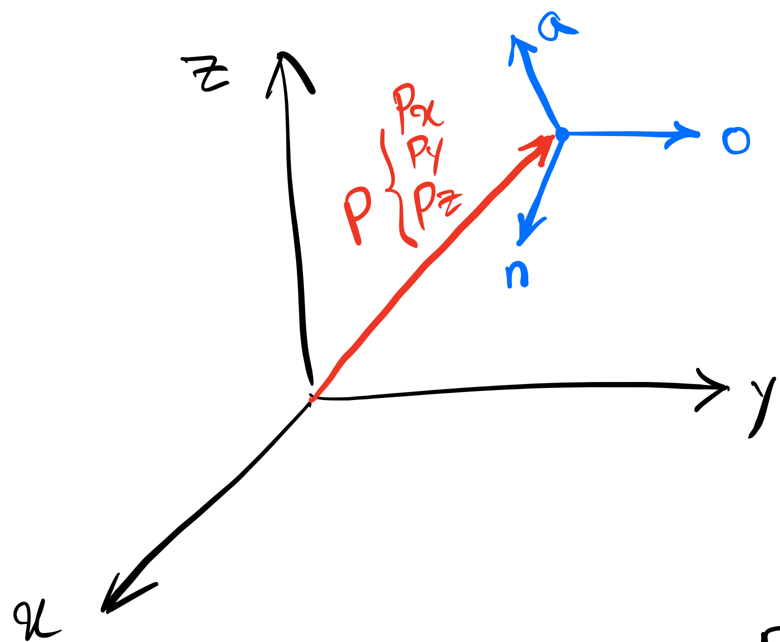
The orientation with which the gripper frame approaches the part is called orientation axis and it is referred to as o-axis.

n-axis is normal to both o-axis and a-axis. The moving frame is referred to as $F_{n,o,a}$.



$$F = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$

Representation of a frame relative to a fixed reference frame

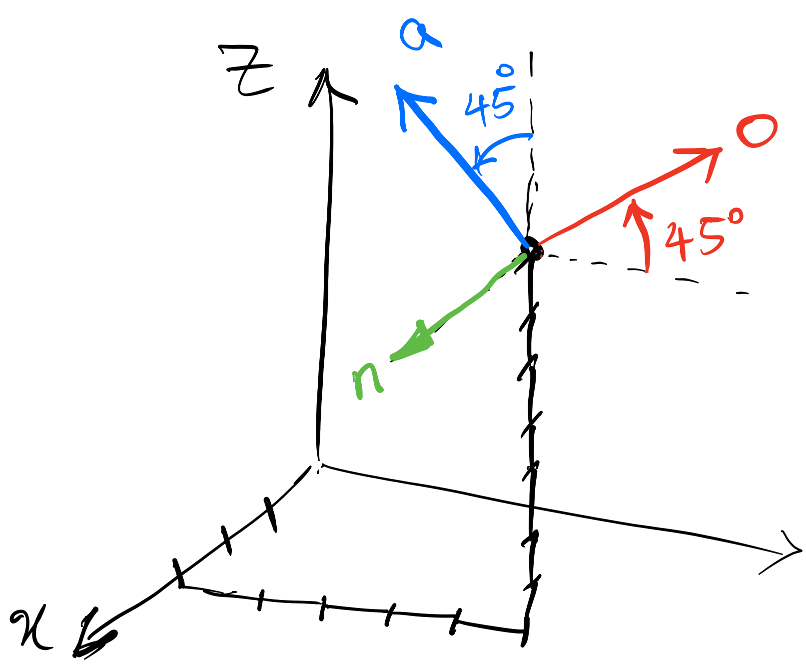


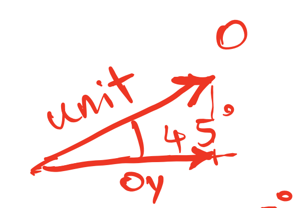
$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4}$$

(Section 2.4.4 of the book)

Example

The frame F shown below is located at 3, 5, 7 units, with it's n-axis parallel to x, its o-axis at 45° relative to the y-axis, and its a-axis at 45° relative to the z-axis.

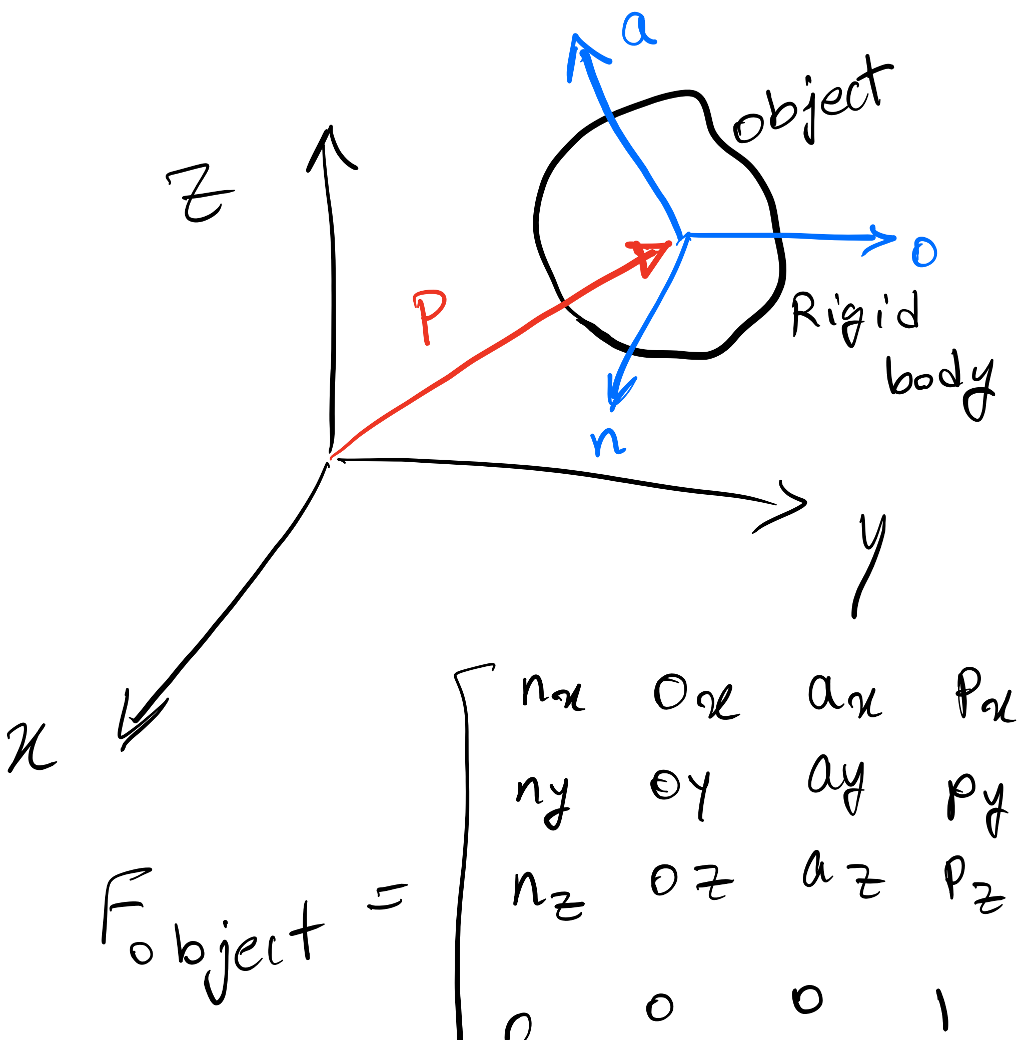




$o_y = \cos 45°$
$= 0.707$

$$F = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0.707 & -0.707 & 5 \\ 0 & 0.707 & 0.707 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Representation a Rigid body



$$F_{object} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

section 2.4.5.

A rigid body in space has six degrees of freedom, meaning it can move along x-, y-, z-axes, it can rotate about thease three axes.

All that is needed to completely define an object in space, six pieces of information.

However, twelve pieces of information are given in the frame matrix: nine for orientation, and three for position.

There must be some constraints

to limit the above to six.

- The three unit vectors $\vec{n}$, $\vec{o}$, $\vec{a}$ are mutually perpendicular.
- each unit vector's length must be equal to 1.

These constraints translate into the following six constraint equations.

1. $\vec{n} \cdot \vec{o} = 0$

(the dot-product of $\vec{n}$ and $\vec{o}$ vectors must be zero).

$$\vec{n} \cdot \vec{o} = |\vec{a}|\,|\vec{o}| \cos(\hat{n,o})$$

$$\cos(\hat{n,o}) = 0 \qquad \cos 90^\circ = 0$$

$$\vec{n} \cdot \vec{o} = 0$$

$$\vec{n} \cdot \vec{o} = (n_x \vec{i} + n_y \vec{j} + n_z \vec{k}) \cdot (o_x \vec{i} + o_y \vec{j} + o_z \vec{k})$$

$$\vec{n} \cdot \vec{o} = n_x o_x + n_y o_y + n_z o_z$$

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2. $\vec{n} \cdot \vec{a} = 0$

3. $\vec{a} \cdot \vec{o} = 0$

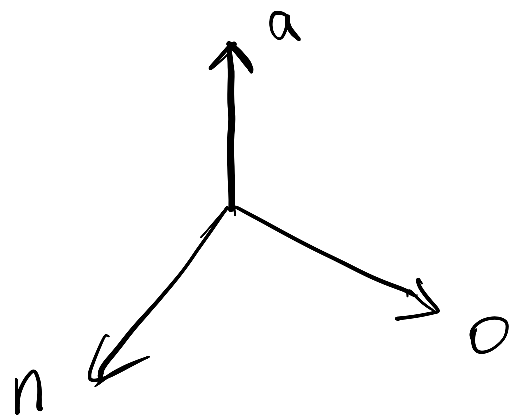
4. $|\vec{n}| = 1$ (the magnitude of the length of the vector must be 1)

5. $|\vec{o}| = 1$

6. $|\vec{a}| = 1$

The first three equations can be replaced by a cross product of the three vectors as:

$$\vec{n} \times \vec{o} = \vec{a}$$



This equation includes the correct right-hand-rule relationship too. it is recomended that this equation be used to determine the correct relationship between the three vectors.

Example 2.4

For the following frame, find the values of the missing elements and complete the matrix representation of the frame.

$$F = \begin{bmatrix} ? & 0 & ? & 5 \\ 0.707 & ? & ? & 3 \\ ? & ? & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$