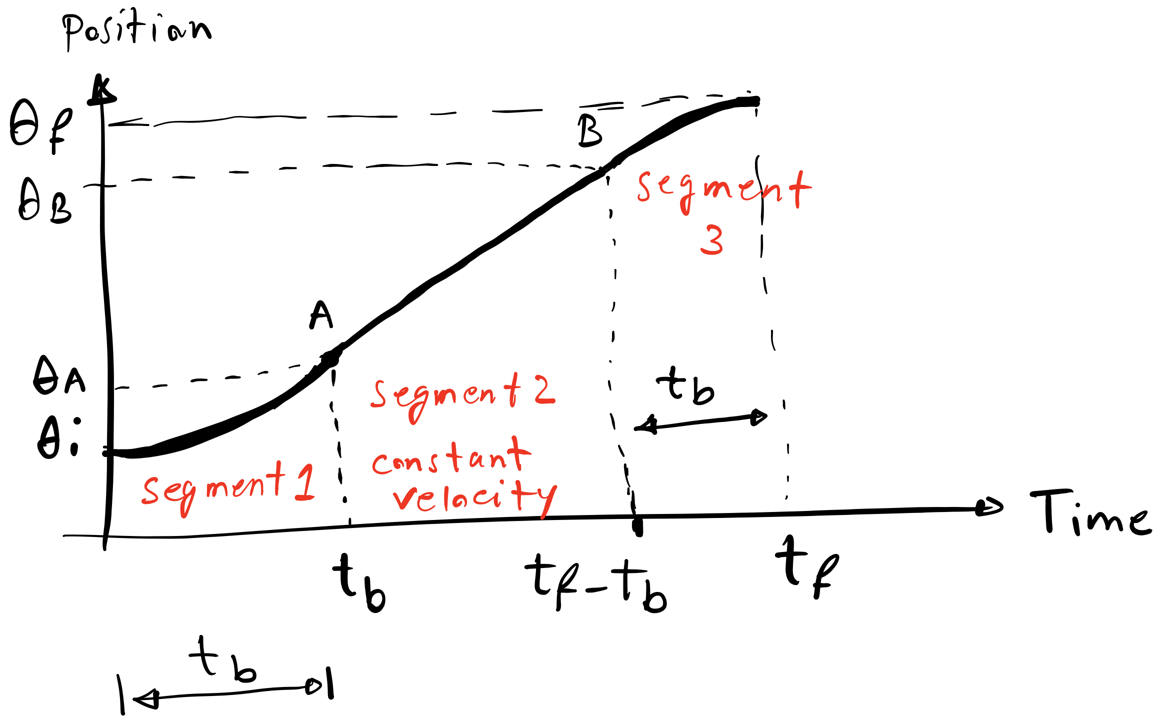
linear segments with parabolic Blends (section 5.5.3 Book)



Segment 1:

$$\theta(t) = c_0 + c_1 t + \frac{1}{2} c_2 t^2$$

$$\dot{\theta}(t) = c_1 + c_2 t$$

$$\ddot{\theta}(t) = c_2$$

Substituting the boundary conditions:

$$\theta(t=0) = \theta_i = c_0$$

$$\dot{\theta}(t=0) = 0 = c_1 \quad \longrightarrow \quad \begin{cases} c_0 = \theta_i \\ c_1 = 0 \\ c_2 = \ddot{\theta} \end{cases}$$

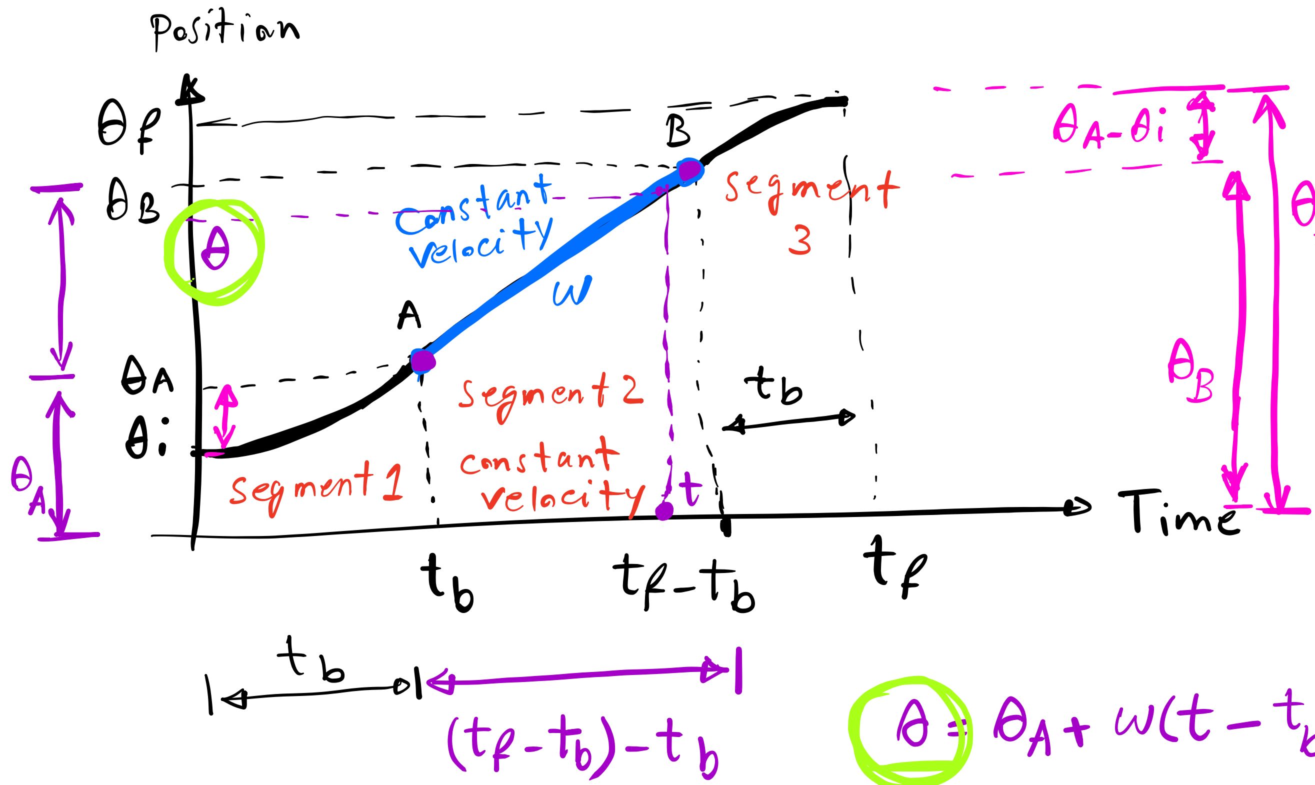
$$\ddot{\theta}(t) = c_2$$

This will result in parabolic segment in the form:

$$\theta(t) = \theta_i + \frac{1}{2} c_2 t^2$$

$$\dot{\theta}(t) = c_2 t$$

$$\ddot{\theta}(t) = c_2$$



$$\theta = \theta_A + \omega(t - t_b)$$

Segment 2:

$$\theta_A = \theta_i + \tfrac{1}{2} c_2 t_b^2$$

$$\dot{\theta}_A = c_2 t_b = \omega \longrightarrow c_2 = \frac{\omega}{t_b}$$

$$\theta_B = \theta_A + \omega\left[(t_f - t_b) - t_b\right]$$

$$= \theta_A + \omega\left[t_f - 2t_b\right]$$

$$\dot{\theta}_B = \dot{\theta}_A = \omega$$

$$\theta_f = \theta_B + (\theta_A - \theta_i)$$

$$\dot{\theta}_f = 0$$

The necessary blending time t_b can be found from Equation above

$$c_2 = \frac{w}{t_b}$$

$$\left\{\begin{array}{l} \theta_f = \theta_B + (\theta_A - \theta_i) \\ \theta_B = \theta_A + w[t_f - 2t_b] \\ \theta_A = \theta_i + \frac{1}{2} c_2 t_b^2 \end{array}\right\}$$ from above

$$\theta_f = \theta_i + c_2 t_b^2 + w(t_f - 2t_b)$$

$$\theta_f = \theta_i + \left(\frac{w}{t_b}\right) t_b^2 + w(t_f - 2t_b)$$

We can calculate the blending time:

$$t_b = \frac{\theta_i - \theta_f + w t_f}{w}$$

The maximum velocity is when $t_b = \frac{t_f}{2}$

$$\frac{t_f}{2} = \frac{\theta_i - \theta_f + w t_f}{w}$$

$$w t_f \underbrace{\left(\frac{1}{2} - 1\right)}_{-1/2} = \theta_i - \theta_f$$

$$w_{max} = \frac{2(\theta_f - \theta_i)}{t_f}$$

The final segment:

$$\theta(t) = \theta_f - \frac{1}{2} c_2 (t_f - t)^2 \qquad c_2 = \frac{\omega}{t_b}$$

$$\begin{cases} \theta(t) = \theta_f - \dfrac{\omega}{2t_b} (t_f - t)^2 \\ \dot{\theta}(t) = \dfrac{\omega}{t_b} (t_f - t) \\ \ddot{\theta}(t) = -\dfrac{\omega}{t_b} \end{cases}$$

Example (5.4 Book)

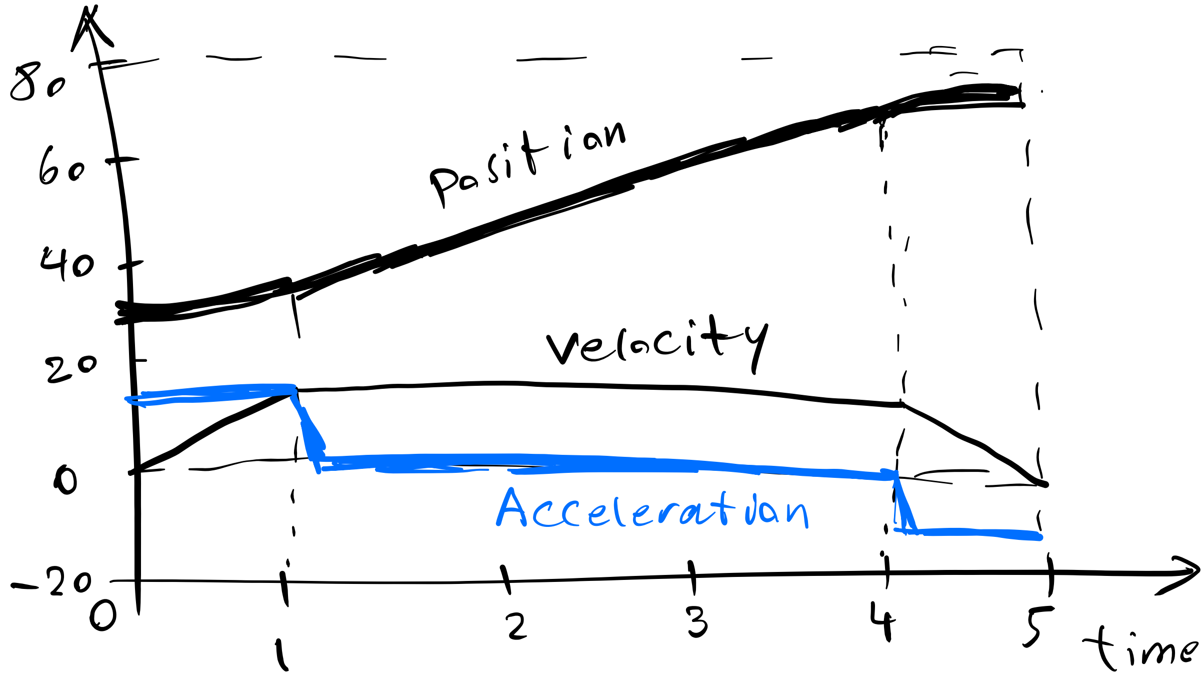
Joint 1 of the 6-axis robot of Example 5.1 is to go from initial angle of $\theta_i = 30°$ to the final angle of $\theta_f = 70°$ in 5 seconds, with a cruising velocity of $\omega_1 = 10°/\text{sec}$.

Find the necessary time for blending and plot the joint positions, velocities, and accelerations:

From Equations (5.10) throng (5.12), (5.15) and (5.16) we have:

$$t_b = \frac{\theta_i - \theta_f + \omega_1 t_f}{\omega_1} = \frac{30 - 70 + 10(5)}{10} = 1 \text{ sec}$$

For $\theta = \theta_i$ to θ_A	For $\theta = \theta_A$ to θ_B	For $\theta = \theta_B$ to t_f
$\theta = 30 + 5t^2$	$\theta = \theta_A + 10(t-1)$	$\theta = 70 - 5(5-t)^2$
$\dot{\theta} = 10t$	$\dot{\theta} = 10$	$\dot{\theta} = 10(5-t)$
$\ddot{\theta} = 10$	$\ddot{\theta} = 0$	$\ddot{\theta} = -10$



Linear segments with parabolic Blends and via points (5.5.4)

Higer-order trajectories (5.5.5)

In addition to the initial and final points, other via points can be specified to match the desired positions.

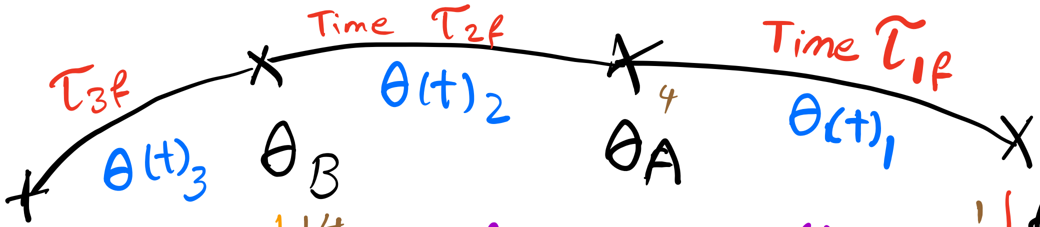
$$\theta(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_{n-1} t^{n-1} + c_n t^n$$

But not practical because very computationaly intesive.

The alternative is to use combination of lower-order polynomials for different segments of the trajectory and blend them together to satify all required boundry conditions, including 4-3-4 trajectory, or 3-5-3 trajectory, ...

For the 4-3-4 trajetory:

$$\begin{cases} \theta(t)_1 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \\ \theta(t)_2 = b_0 + b_1 t + b_2 t^2 + b_3 t^3 \\ \theta(t)_3 = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 \end{cases}$$



1 θ_i

2 $\ddot{\theta}_i = 0$

3 $\ddot{\theta}_i$

5 $\theta(0)_2 = \theta(\tau_1 f)_1$

6 $\dot{\theta}(0)_2 = \theta(\tau_1 f)_1$

7 $\ddot{\theta}(0)_2 = \theta(\tau_1 f)_1$

8 $\theta(0)_3 = \theta(\tau_2 f)_2$

9 $\dot{\theta}(0)_3 = \theta(\tau_2 f)_2$

10 $\ddot{\theta}(0)_3 = \theta(\tau_2 f)_2$

11 θ_f

12 $\dot{\theta}_f = 0$

13 $\ddot{\theta}_f$

See page 192