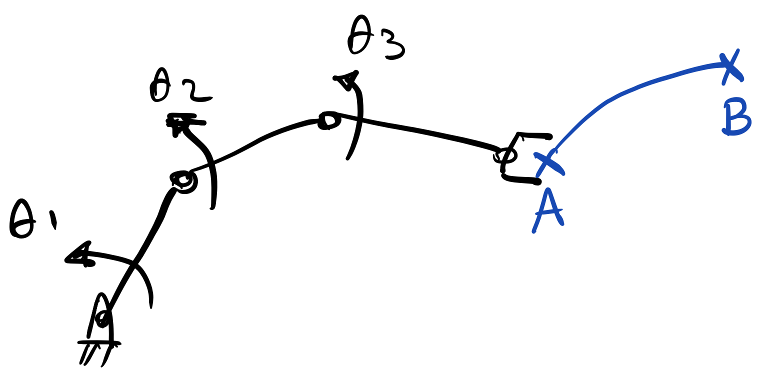
Third-order Polynomial Trajectory planning
(sec. 5.5.1 Book)

Joint-space Trajectory planning (5.5 Book)



In order to move the hand from the desired A location to B, we need to find the invese Kinematic of the robot.

For example for θ_1:

Find the initial joint angle corresponding to point A $\longrightarrow$ $\theta_i \xrightarrow{\text{same}} \theta(t_i)$

Find the final joint angle corresponding to point B $\longrightarrow \theta_f \xrightarrow[\text{same}]{} \theta(t_f)$

$$\theta(t_i) = \theta_i$$

$$\theta(t_f) = \theta_f$$

$$\dot{\theta}(t_i) = 0$$

$$\dot{\theta}(t_f) = 0 \qquad t_i = 0$$

$$\begin{cases} \theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 \\ \dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 \end{cases}$$

Eq. 5.4 Book

$$\begin{cases} \theta(t_i) = c_0 = \theta_i \\ \theta(t_f) = c_0 + c_1 t_f + c_2 t_f^2 + c_3 t_f^3 \\ \dot{\theta}(t_0) = c_1 = \theta_f \\ \dot{\theta}(t_f) = c_1 + 2c_2 t_f + 3c_3 t_f^2 = 0 \end{cases}$$

$$\begin{bmatrix} \theta_i \\ \dot{\theta}_i \\ \theta_f \\ \dot{\theta}_f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

Example (5.1 Book)

It is desired to have the first joint of a 6-axis robot go from intial angle of 30° to a final angle of 75° in 5 seconds. Using a third-order polynomial calculate the joint angle at 1, 2, 3 and 4 seconds.

substituting the boundary conditions into Equation 5.4 we get:

$$
\begin{cases}
\theta(t_i) = c_0 = 30 \\
\theta(t_f) = c_0 + c_1(5) + c_2(5^2) + c_3(5^3) = 75 \\
\dot{\theta}(t_i) = c_1 = 0 \\
\dot{\theta}(t_f) = c_1 + 2c_2(5) + 3c_3(5^2) = 0
\end{cases}
$$

$c_0 = 30$, $c_1 = 0$

$c_2 = 5.4$, $c_3 = -0.72$

$\theta(t) = 30 + 5.4t^2 - 0.72t^3$

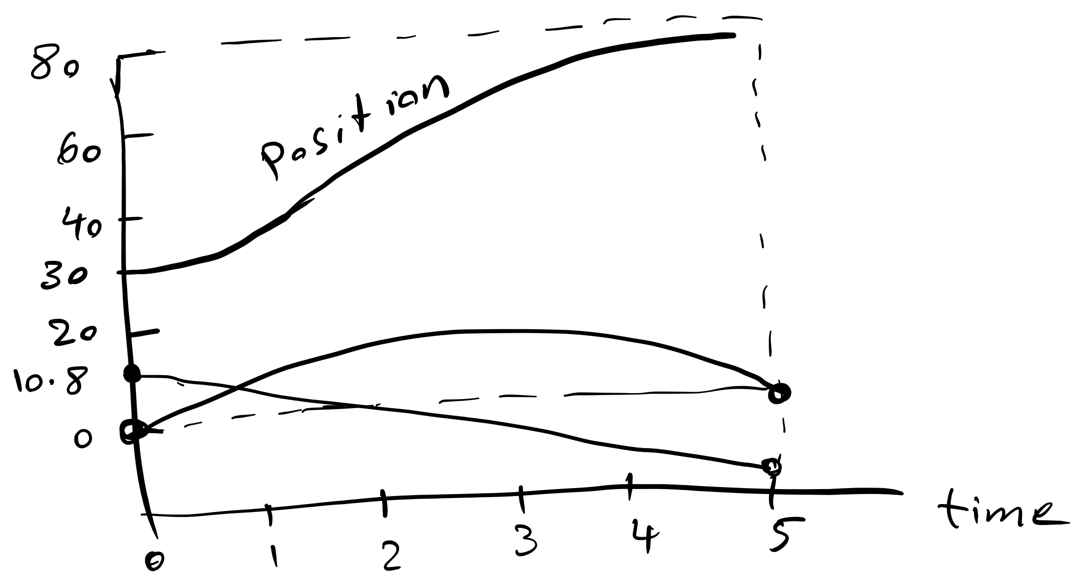
$\dot{\theta}(t) = 10.8t - 2.16t^2$

$\ddot{\theta}(t) = 10.8 - 4.32t$

Substituting the desired time intervals into the motion equation will result in:

$\theta(1) = 34.68°$ $\theta(2) = 45.84°$

$\theta(3) = 59.16°$ $\theta(4) = 70.32°$



Example (5.2 Book)

suppose the robot arm of Example 5.1 is to continue to the neat poit, where the joint is to reach 105° in another 3 seconds. Draw the position, velocity, and acceleration curves for the motion.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t$$

$t_i = 0 \quad \theta_i = 75^\circ \quad \dot{\theta}_i = 0$

$t_f = 3 \quad \theta_f = 105^\circ \quad \dot{\theta}_f = 0$

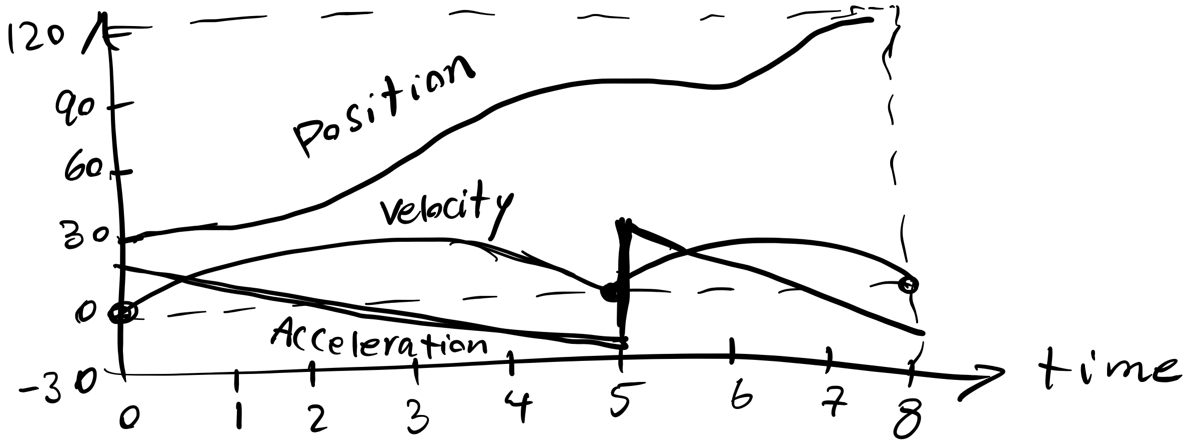
which will yield:

$c_0 = 75 \quad c_1 = 0 \quad c_2 = 10 \quad c_3 = -2.22$

$$\theta(t) = 75 + 10t^2 - 2.222\,t^3$$

$$\dot{\theta}(t) = 20t - 6.666\,t^2$$

$\ddot{\theta}(t) = 20 - 13.332\,t$



Fifth-order polynomial Trajectory planning (section 5.5.2)

Specifying the initial and ending positions, velocities, and acceleration of a segment yields six pieces of information, enabling us to use a fifth-order polynomial to plan a trajectory, as follows.

$$\begin{cases} \theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 \\ \dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4 \\ \ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3 \end{cases}$$

Example (5.3)

Repeat example 5.1 but assume the initial acceleration and final acceleration will be $5\ °/sec^2$.

$\theta_i = 30°$ $\quad \dot{\theta}_i = 0\ °/sec$ $\quad \ddot{\theta} = 5\ °/sec^2$

$\theta_f = 75°$ $\quad \dot{\theta}_f = 0\ °/sec$ $\quad \ddot{\theta}_f = -5\ °/sec^2$

using Equations (5.6) through (5.8) with the given initial and final boundary conditions.

$c_0 = 30$ $\quad c_1 = 0$ $\quad c_2 = 2.5$

$c_3 = 1.6$ $\quad c_4 = -0.58$ $\quad c_5 = 0.0464$

This results in the following motion equations:

$$\theta(t) = 30 + 2.5t^2 + 1.6t^3 - 0.58t^4 + 0.0464t^5$$

$$\dot{\theta}(t) = 5t + 4.8t^2 - 2.32t^3 + 0.232t^4$$

$$\ddot{\theta}(t) = 5 + 9.6t - 6.96t^2 + 0.928t^3$$

Linear segments with parabolic blends 5.5.3