Example

Kinetic Energy

Newton's second law of Motion:

$$\Sigma F = m a$$

ΣF: sum of all forces

m: mass

a: acceleration

Acceleration is defined by:

$$a = \frac{dv}{dt}$$

The time derivative of velocity

work done by a force, F :

$$W = \text{Force} \times \text{displacement}$$

A force causing a small displacement dx :

→ Force

$$dw = F \times dx$$

small differential work

differential displacement (small displacement)

$$dw = (ma)\,dx$$

$\leftarrow F = ma$

$$dw = m\frac{dv}{dt}\,dx$$

$\leftarrow a = \frac{dv}{dt}$

$$dw = m\,dv\,\frac{dx}{dt}$$

$\leftarrow \frac{dx}{dt} = v$

$$dw = m\,v\,dv$$

$$\int dw = \int m\,v\,dv$$

$\leftarrow$ Assume mass m is constant

$$w = m\int v\,dv$$

$$w = m\,\frac{1}{2}\,v^2$$

Energy = $W = \frac{1}{2}mv^2$

KE for kinetic energy:

kinetic energy of a mass m moving with the velocity of v:

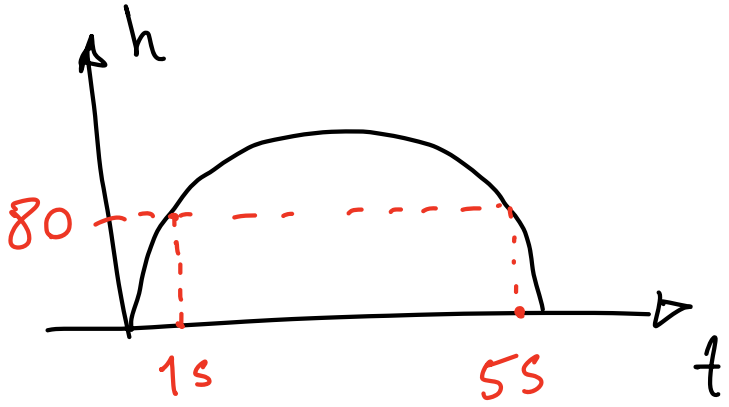
$$KE = \frac{1}{2}mv^2$$

Example

A ball is thrown upwards.

The equation and the plot of the change in the elevation of the ball as a function of time are given as below.

$$h(t) = 96t - 16t^2$$



Find the time at which $h = 80$ ft

$$80 = 96t - 16t^2$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{96 \pm \sqrt{96^2 - 4 \times 16 \times 80}}{32}$$

$$t = 3 \pm 2 = \{5 \text{ s} \quad \text{and} \quad 1 \text{ s}\}$$

What is the maximum height of the ball, and the corresponding time to max.

To find the maximum of function, take the derivative of the function and then equate it to zero.

$$\frac{dh(t)}{dt} = 0 \Longrightarrow h_{max}$$

$$h(t) = 96t - 16t^2$$

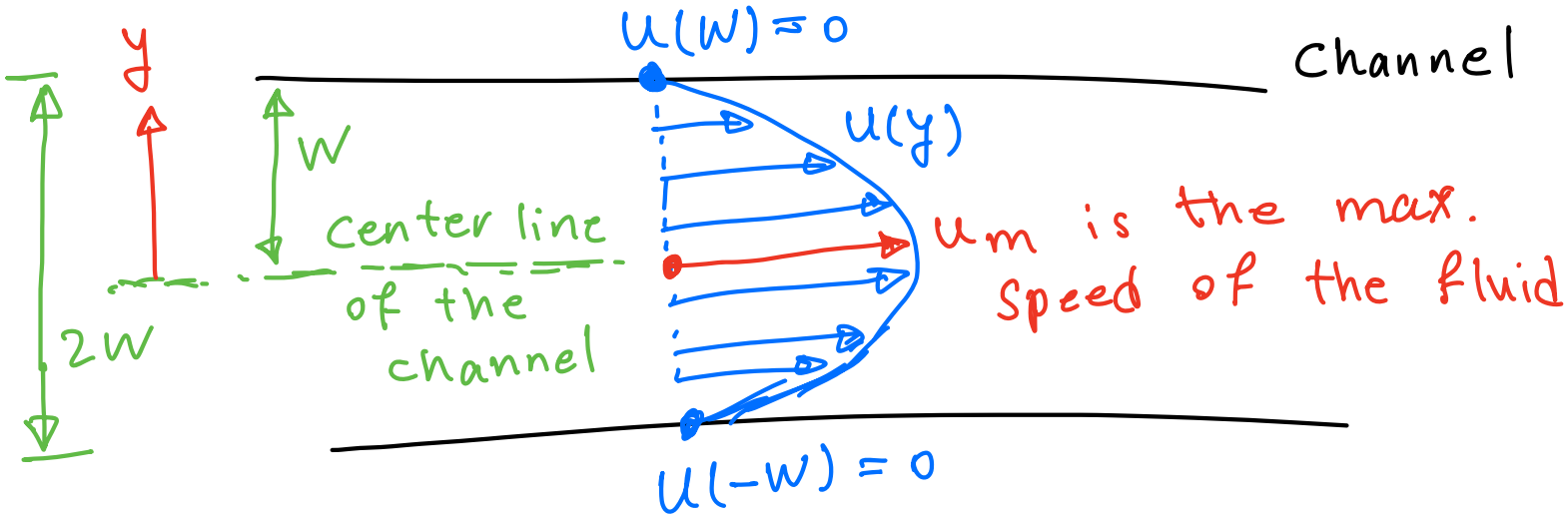
$$\frac{dh}{dt} = 96 - 32t = 0$$

$$t = 3 \text{ seconds}$$

$$h(3) = 144 \text{ ft}$$

Example

The speed profile for the fluid flow, u, in a channel is given in the figure below as a function of y, as $u(y)$. Find the equation for $u(y)$ if the profile is a quadratic equation.



W and u_m are given.

Find $u(y)$.

$$u(y) = ay^2 + by + c$$

Find a, b, c.

$$\begin{cases} y = 0 \\ u(0) = u_m \end{cases} \Rightarrow u_m = 0 + 0 + c \Rightarrow \boxed{c = u_m}$$

$$\begin{cases} y = W \\ u(W) = 0 \end{cases} \Rightarrow 0 = aW^2 + bW + u_m \quad (1)$$

$$\begin{cases} y = -W \\ u(W) = 0 \end{cases} \Rightarrow 0 = aW^2 - bW + u_m \quad (2)$$

Add Eq. ① and Eq. ②

$$0 = 2aw^2 + 2um \Rightarrow a = \frac{-um}{w^2}$$

subtract Eq. ① from Eq ②

$$0 = 0 - 2bw - 0$$

$$\Rightarrow b = 0$$

Therefore the quadratic equations for the speed profile is:

$$u(y) = ay^2 + by + c$$

$$u(y) = -\frac{um}{w^2} y^2 + um$$

Trigonometry