Instrumentation and Controls

ETM 3301

Lecture 21

Instructor

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Frequency Response Example, 1

$$M(\omega) = \frac{2}{\sqrt{1+\omega^2}} \qquad \phi(\omega) = -\tan^{-1}\omega$$

ω (rad/s): nature (angular) frequency, normally called as frequency

ω (rad/s)	Real	Im	Gain	Phase
0	2	0	2	0^o
1	1	-1	1.414	-45^o
2	0.4	-0.8	0.894	-63.4^o
3	0.2	-0.6	0.633	-71.6^o
5	0.0769	-0.3846	0.392	-78.7^o
10	0.0198	-0.198	0.199	-84.3^o
30	0.005	-0.0998	0.067	-87.1^o
∞	0	0	0	-90^o

Frequency Response Example, 1

ω	Gain	Phase
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3	0.633	-71.6^o
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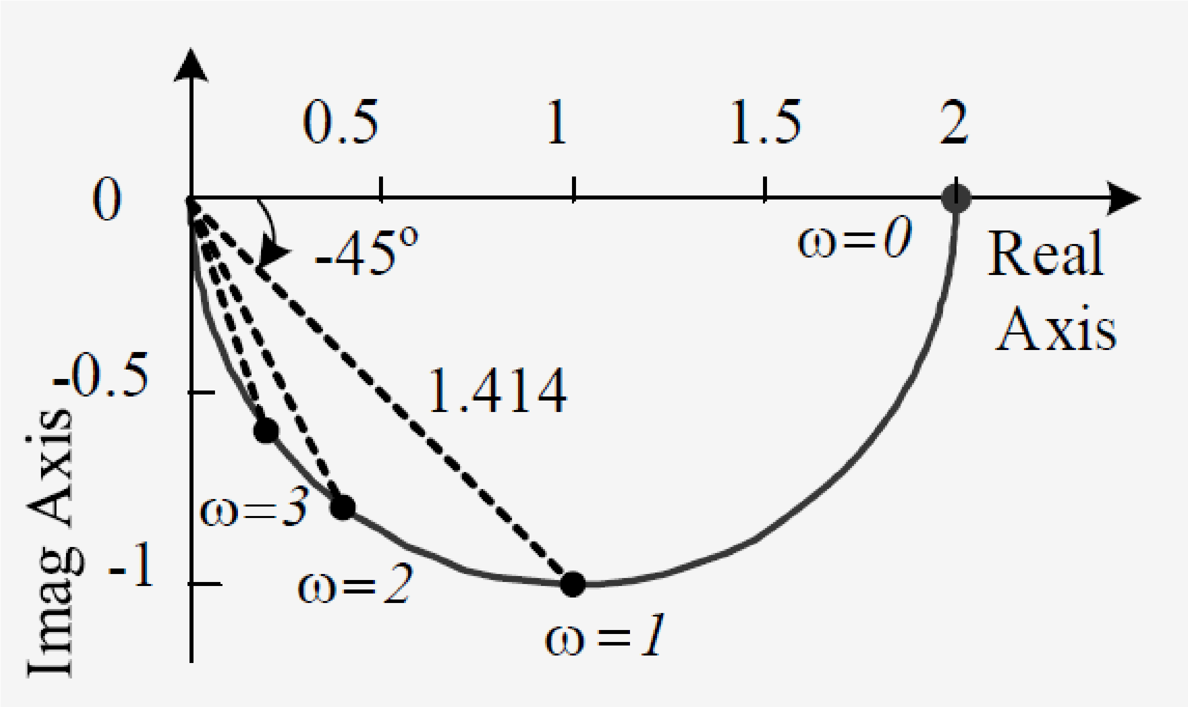
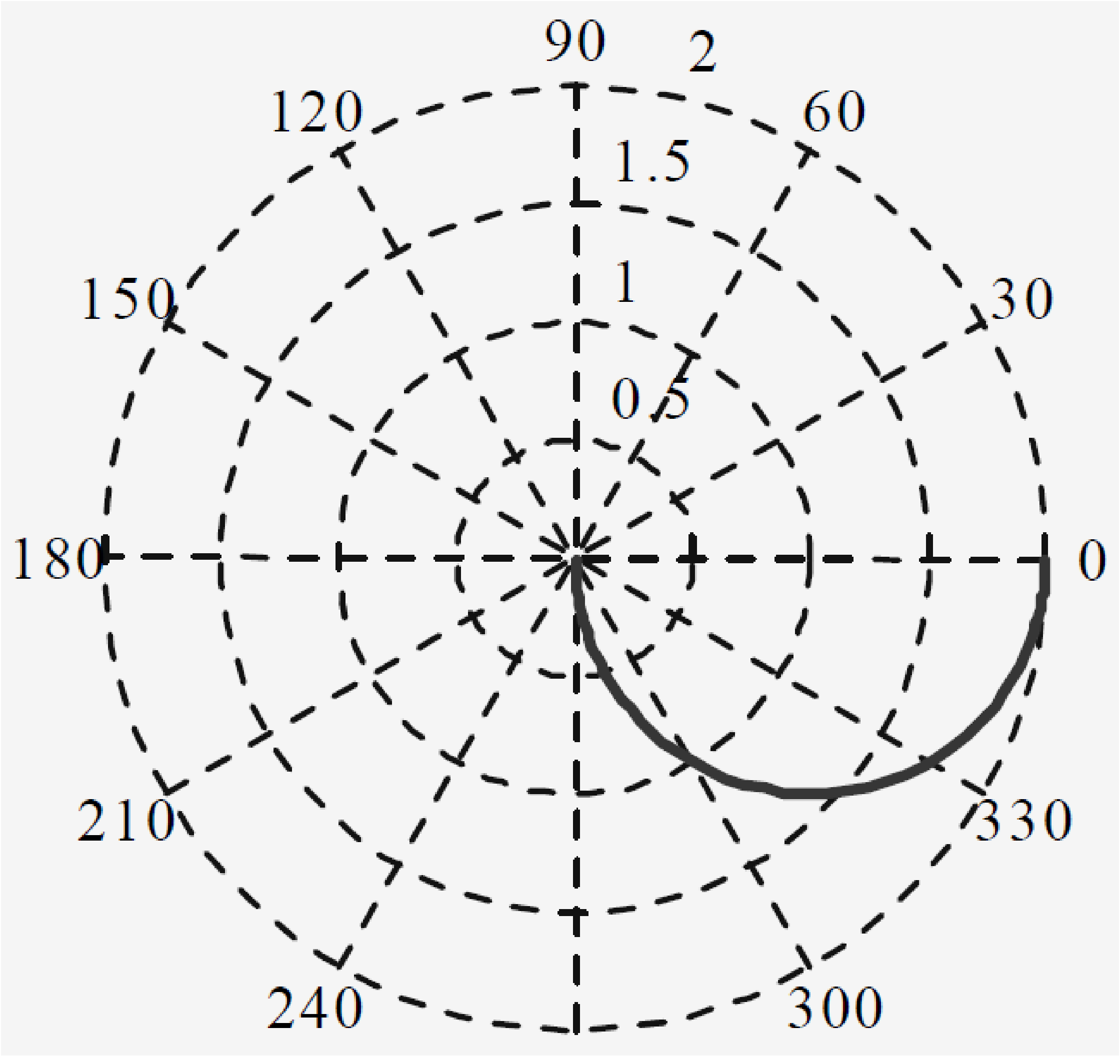
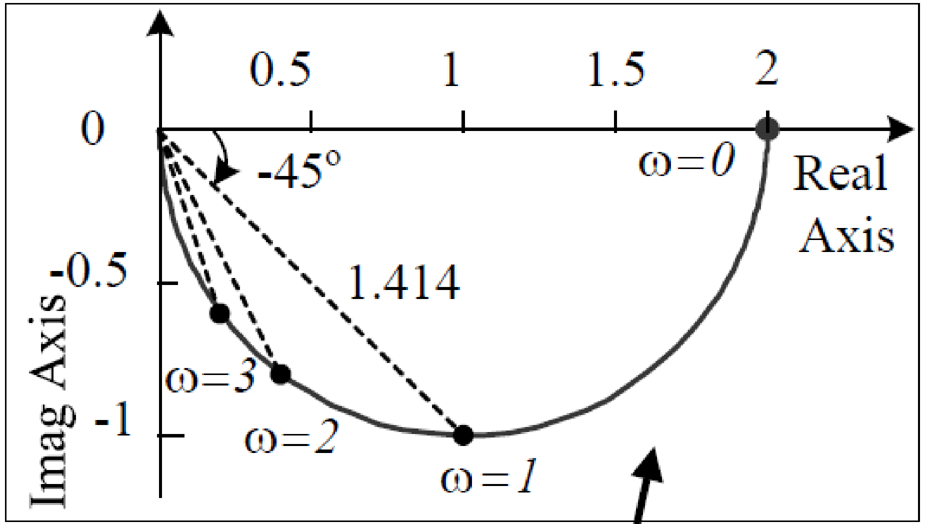
30	0.067	-87.1^o
∞	0	-90^o

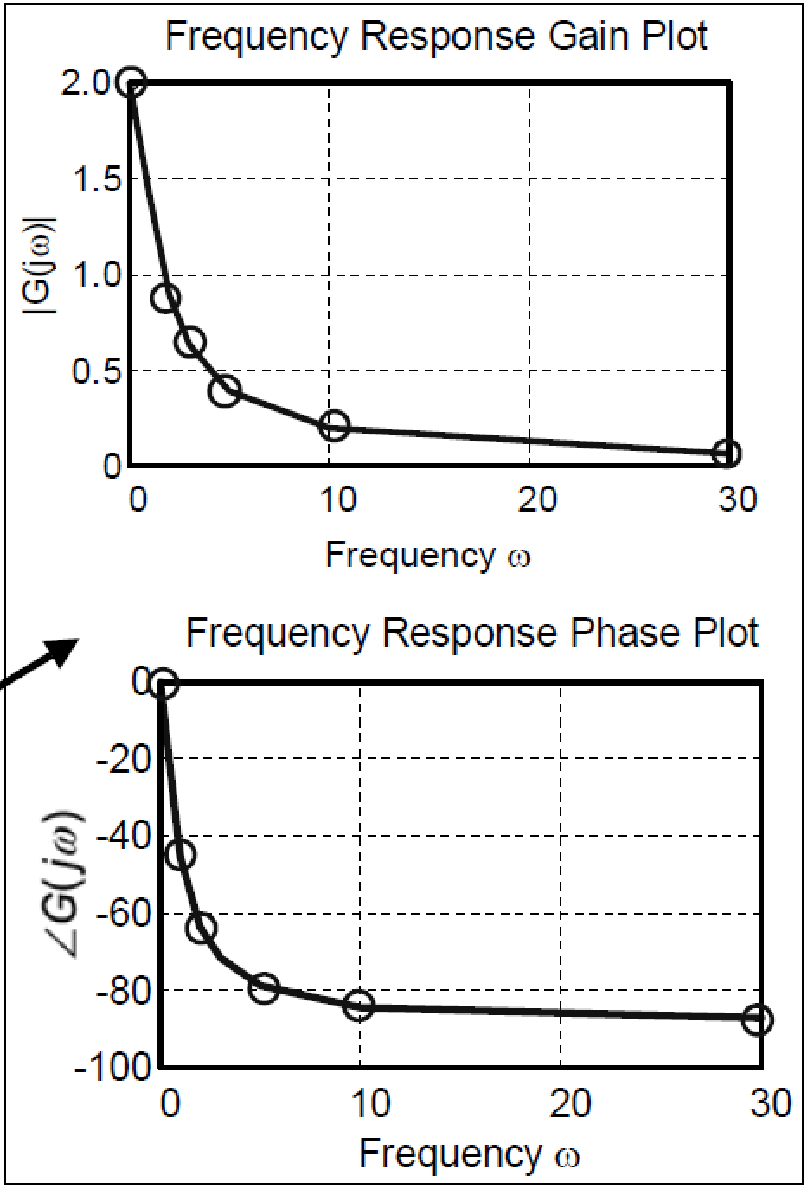


Nyquist plot on polar graph paper



Three Types of Frequency Response Plots





Type 1: Nyquist Plot (usually plot on Polar graph paper)

Gain and Phase plot

Bode Plot (see next page)

Bode Plots

- The *Bode plot* consists of two graphs, one of the gain against frequency and another the phase against frequency.
- The gain and frequency are plotted using logarithmic scale.

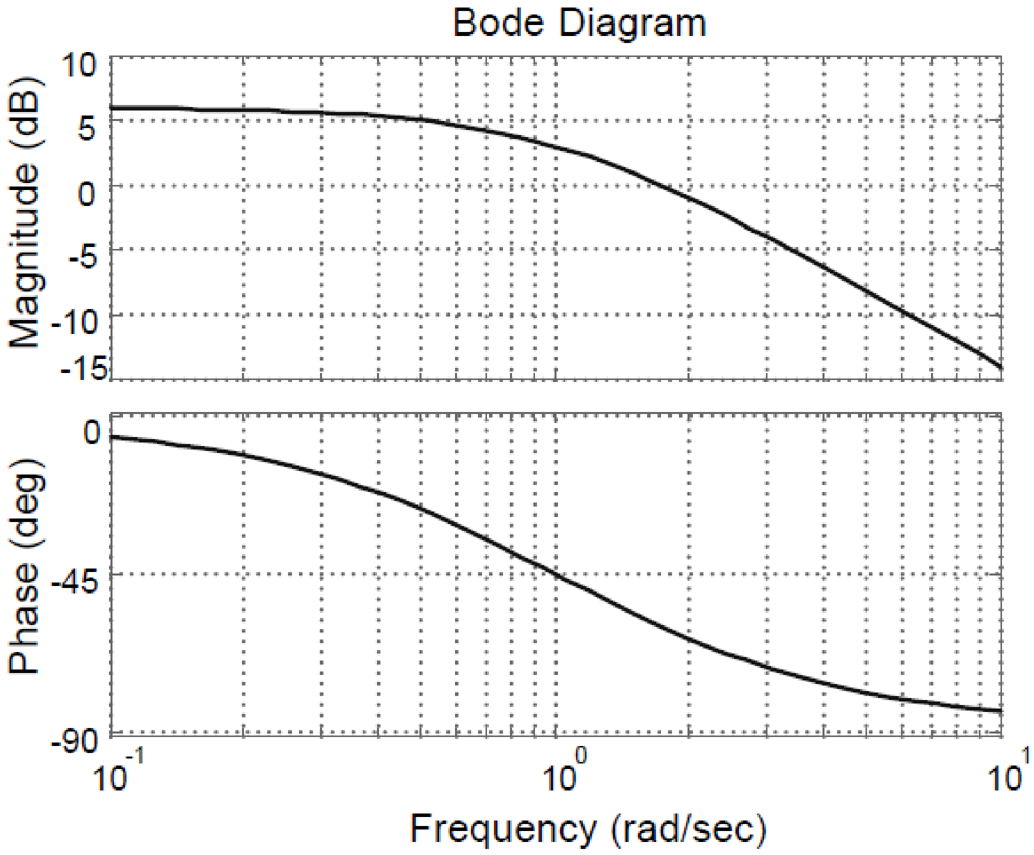
Gain: $20\log_{10}|G(j\omega)|$ *dB* *(decibel)*

Frequency $\log_{10}\omega$

Phase: degree (no change) $\angle G(j\omega)$

Frequency Response Example

ω	Gain	Phase	20log|G|
0	2	0^o	6.02
1	1.414	-45^o	3.01
2	0.894	-63.4^o	-0.97
3	0.633	-71.6^o	-3.97
5	0.392	-78.7^o	-8.13
10	0.199	-84.3^o	-14.02
30	0.067	-87.1^o	-23.47
∞	0	-90^o	$-\infty$



$$G(s) = \frac{2}{s+1}$$

$$G(j\omega) = \frac{2}{j\omega+1}$$

$$M(\omega) = \frac{2}{\sqrt{1+\omega^2}}$$

$$\phi(\omega) = -\tan^{-1}\omega$$

Bode Plot Example

Transfer function $G(s) = \frac{K}{Ts + 1}$

Frequency response $G(j\omega) = \frac{K}{jT\omega + 1}$

Gain $M(\omega) = \frac{K}{\sqrt{1 + (T\omega)^2}}$

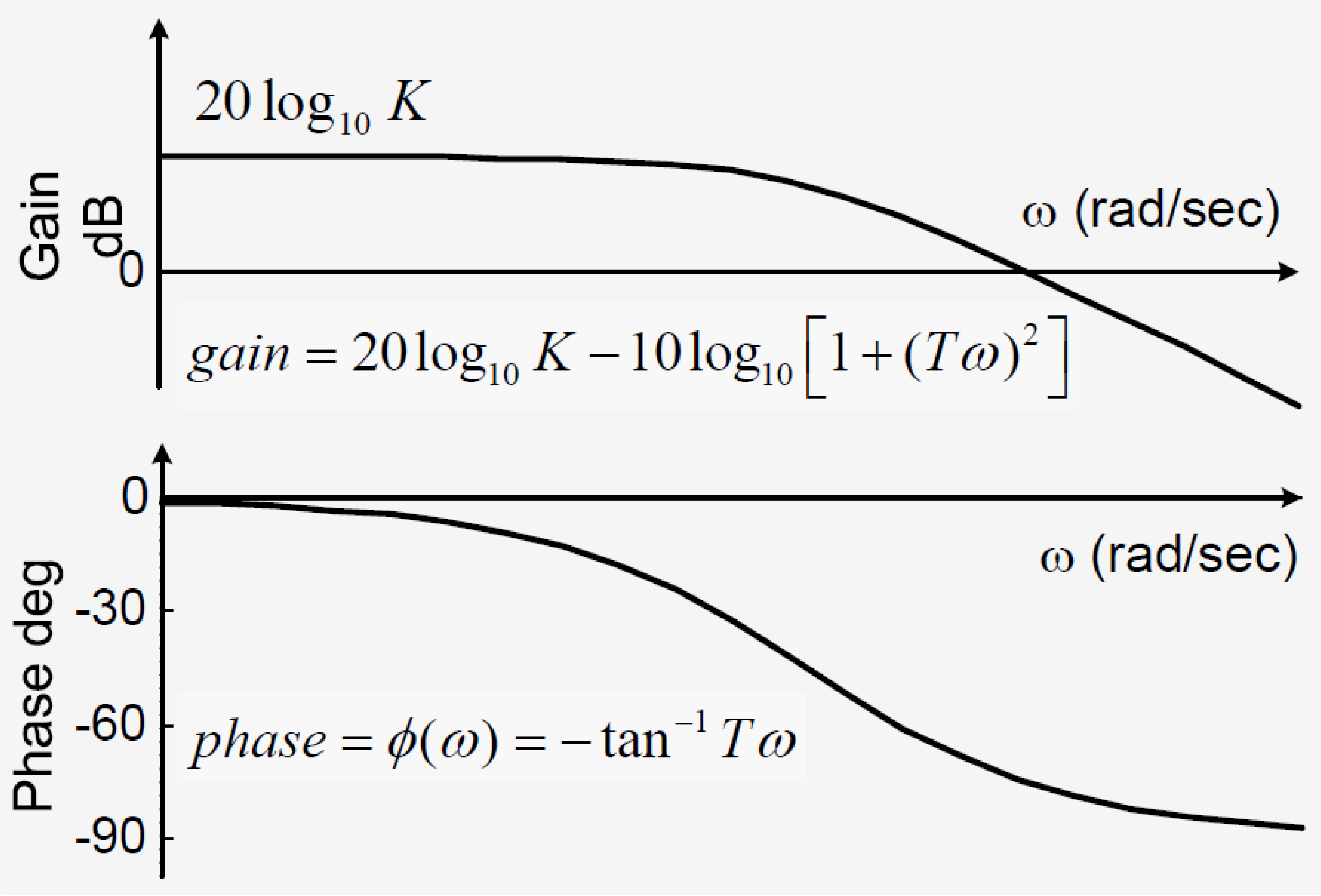
Phase $\phi(\omega) = -\tan^{-1} T\omega$

$$20\log_{10}\left|G(j\omega)\right| = 20\log_{10}\frac{K}{\sqrt{1 + (T\omega)^2}}$$

$$= 20\log_{10} K - 20\log_{10}\left[1 + (T\omega)^2\right]^{1/2}$$

$$= 20\log_{10} K - 10\log_{10}\left[1 + (T\omega)^2\right]$$

Bode Plot Example



Bode Plot Using Matlab

$$G(s) = \frac{1}{s(s+1)(0.5s+1)}$$

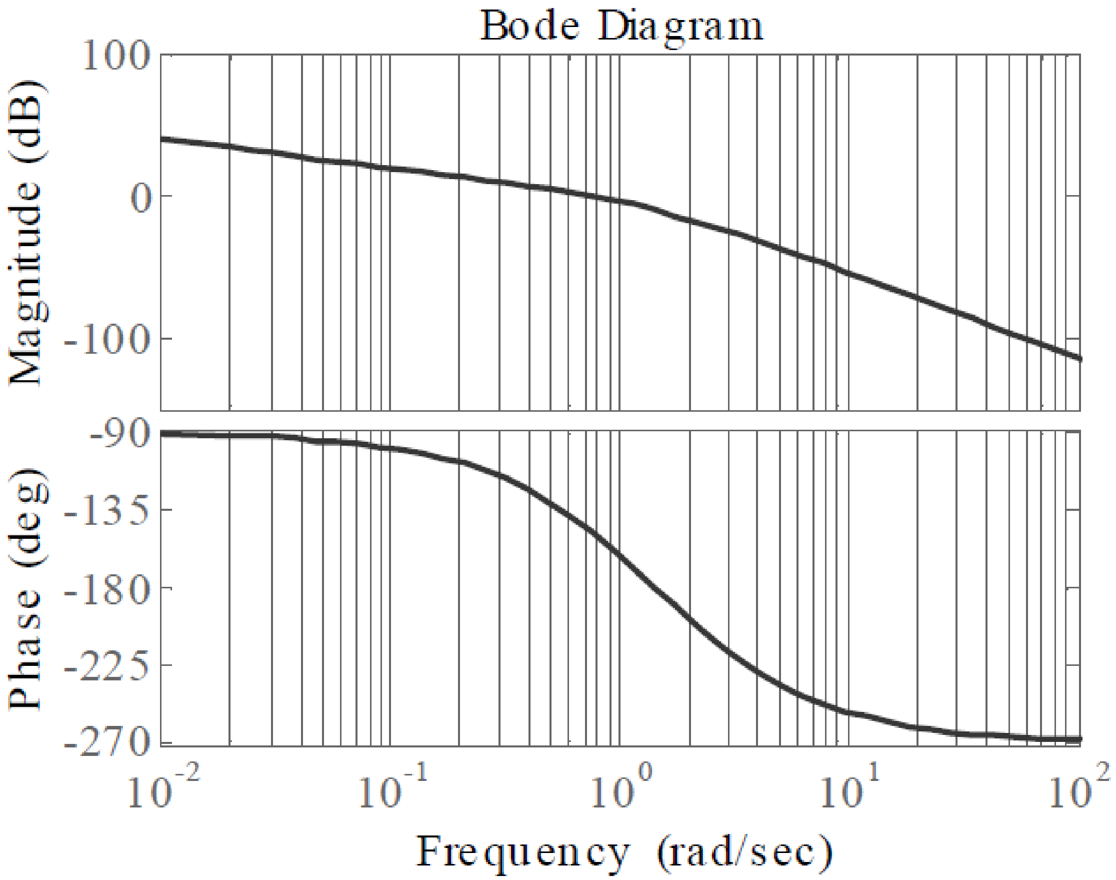
```
>>G=tf([1],[0.5 1.5 1 0])
```

Or

```
>>G=zpk([],[0 -1 -2],[1])
```

```
>>bode(G)
```

```
>>grid
```



First Order System Frequency Response

$$G(s) = \frac{K}{Ts+1} \xrightarrow{\text{Replacing } s \text{ with } j\omega} G(j\omega) = \frac{K}{jT\omega + 1}$$

$$M(\omega) = |G(j\omega)| = \left|\frac{K}{1 + jT\omega}\right| = \frac{|K|}{|1 + jT\omega|} = \frac{K}{\sqrt{1^2 + (T\omega)^2}}$$

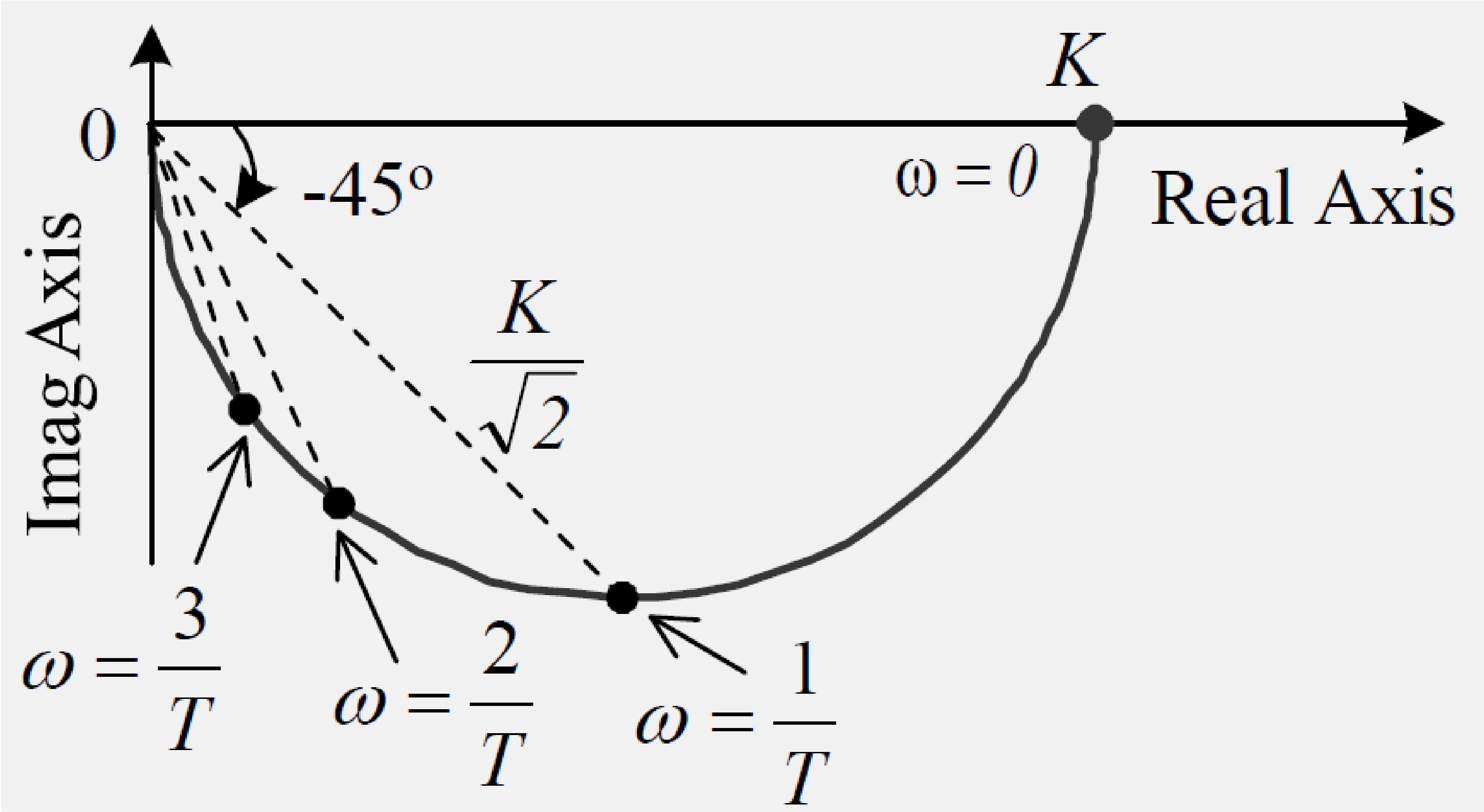
$$\phi(\omega) = \angle G(j\omega) = \angle\left(\frac{K}{1 + jT\omega}\right)$$

$$= \angle(K) - \angle\left(1 + jT\omega\right) = 0 - \tan^{-1}\left(\frac{T\omega}{1}\right)$$

$$= -\tan^{-1} T\omega$$

First Order System Frequency Response

$$M(\omega) = \frac{K}{\sqrt{1+(T\omega)^2}} \qquad \phi(\omega) = -\tan^{-1} T\omega$$



Frequency Response of A Special First Order System

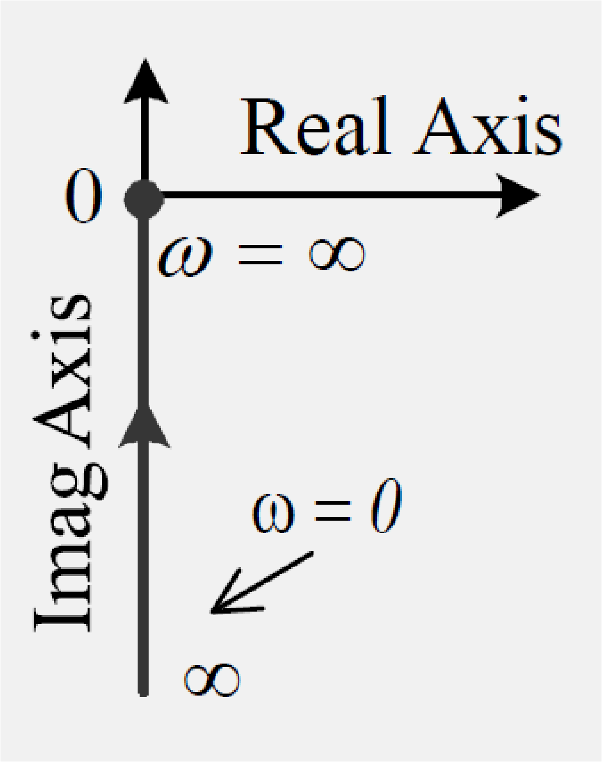
Pure Integration

$$G(s) = \frac{1}{s} \xrightarrow{s = j\omega} G(j\omega) = \frac{1}{j\omega}$$

$$|G(j\omega)| = \left|\frac{1}{j\omega}\right| = \frac{1}{\omega}$$

$$\angle G(j\omega) = \angle\left(\frac{1}{j\omega}\right)$$

$$= \angle(1) - \angle(j\omega) = -90^\circ$$



Frequency Response of A Special Second Order System

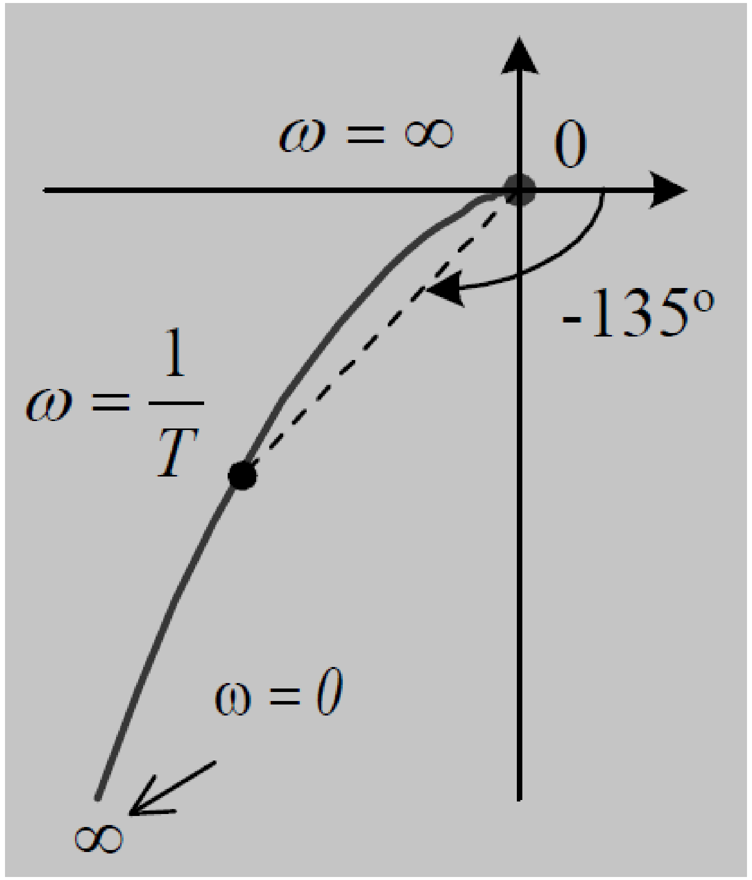
Pure Integration + First Order $$G(s) = \frac{K}{s(1+Ts)} = \frac{1}{s} \times \frac{K}{1+Ts}$$

$s = j\omega$

$$G(j\omega) = \frac{K}{j\omega(jT\omega+1)}$$

$$|G(j\omega)| = \frac{K}{\omega\sqrt{1+(T\omega)^2}}$$

$$\angle G(j\omega) = -90^o - \tan^{-1} T\omega$$



Second Order System Frequency Response

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Replacing s with $j\omega$

Frequency response

$$G(j\omega)=\frac{\omega_n^2}{-\omega^2+j2\zeta\omega_n\omega+\omega_n^2}$$

$$=\frac{\omega_n^2}{\omega_n^2-\omega^2+j2\zeta\omega_n\omega}$$

Gain:

$$M(\omega)=\left|G(j\omega)\right|=\left|\frac{\omega_n^2}{\omega_n^2-\omega^2+j2\zeta\omega_n\omega}\right|=\frac{\left|\omega_n^2\right|}{\left|\omega_n^2-\omega^2+j2\zeta\omega_n\omega\right|}$$

$$=\frac{\omega_n^2}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega\omega_n\right)^2}}$$

Second Order System Frequency Response

Phase:

$$\phi(\omega) = \angle G(j\omega) = \angle\left(\frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega}\right)$$

$$= \angle(\omega_n^2) - \angle\left(\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega\right)$$

$$= 0 - \tan^{-1}\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}$$

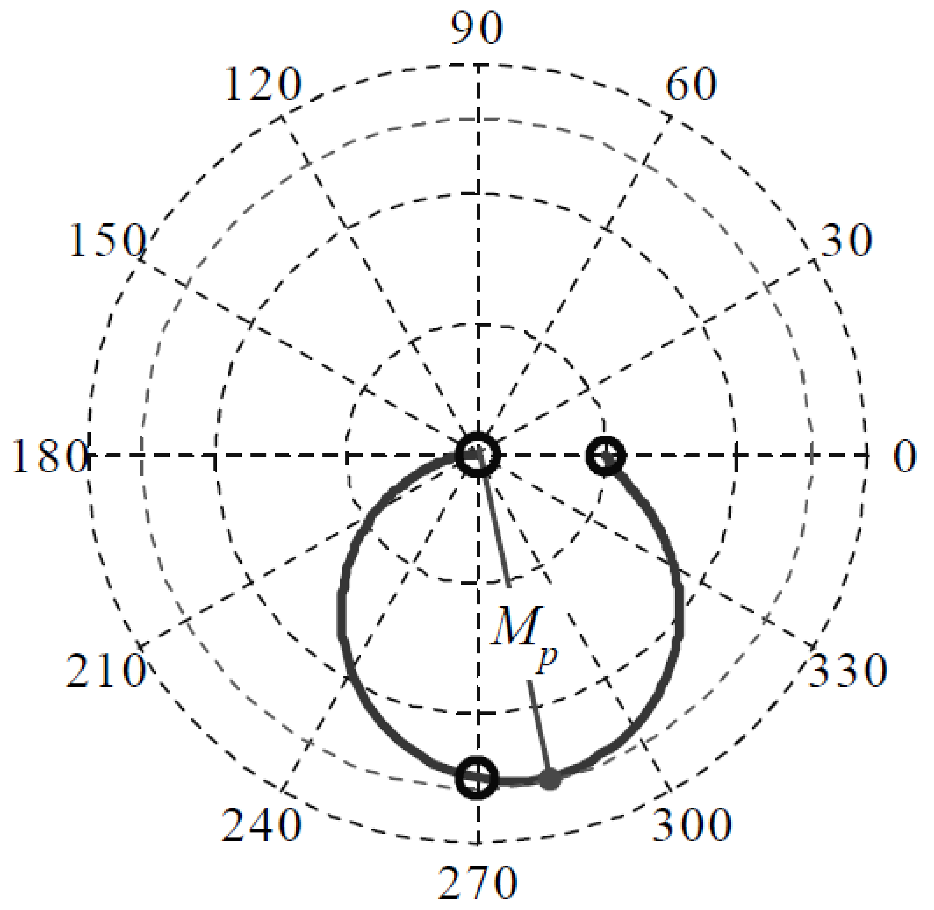
$$= -\tan^{-1}\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}$$

Second Order System Frequency Response

$$\omega = 0 \quad M(\omega) = 1 \quad \phi(\omega) = 0°$$

$$\omega = \omega_n \quad M(\omega) = \frac{1}{2\zeta} \quad \phi(\omega) = -90°$$

$$\omega = \infty \quad M(\omega) = 0 \quad \phi(\omega) = -180°$$



2nd Order System Frequency Response Example

The output amplitude increase to a *maximum value* then reduce when the frequency ω increase

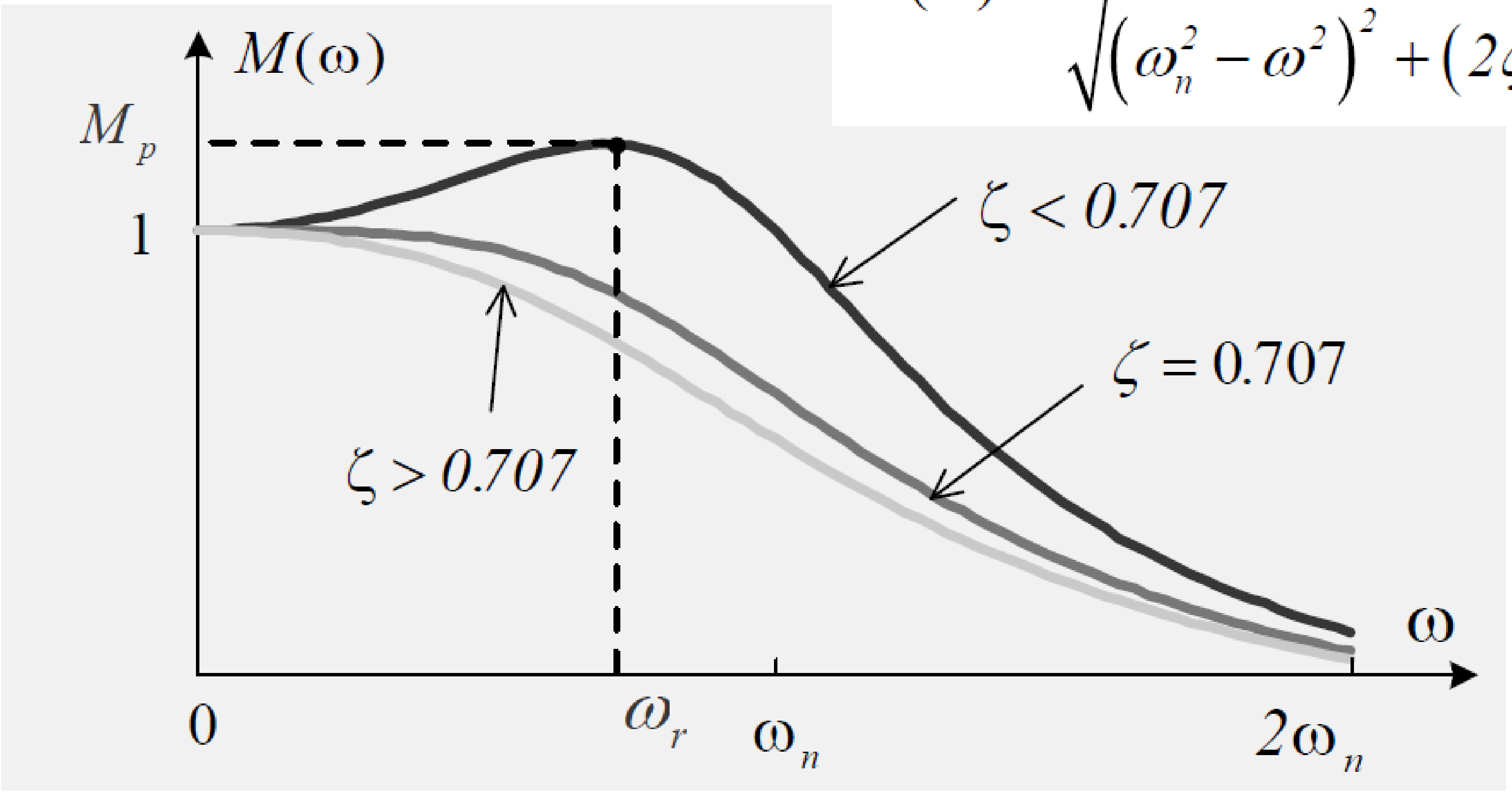
Resonance & Resonant Frequency

$u(t)$: $X\sin(\omega t)$ → $\dfrac{25}{s^2+2s+25}$ → $y(t)$: $Y\sin(\omega t+\phi)$

ω rad/s	Input Amplitude X	Output Amplitude Y	Gain $M = Y/X$	Phase ϕ
1	2	2.08	1.04	-4.76°
2	2	2.34	1.17	-10.8°
3	2	2.92	1.46	-20.6°
4.9	2	5.08	**2.54**	-84.2°
6	2	3.08	1.54	-132°
7	2	1.80	0.90	-150°
12	2	0.42	0.21	-168°

Gain vs Frequency

$$M(\omega) = \frac{\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega\omega_n\right)^2}}$$



- ***Resonant peak*** M_p: The resonant peak M_p is the maximum value of the gain $M(\omega)$.
- ***Resonant frequency*** ω_r: The resonant frequency ω_r is the frequency at which the peak resonance M_p occurs.

Phase vs Frequency

$$\phi(\omega) = -\tan^{-1}\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}$$