ME018B - Introduction to Computational Modeling in Mechanical Engineering

Practice Quiz - 2024

Last name: ……………………………First name:…………………………Student ID:………………

Be neat and clear in your work. Show all work. Don't skip steps.

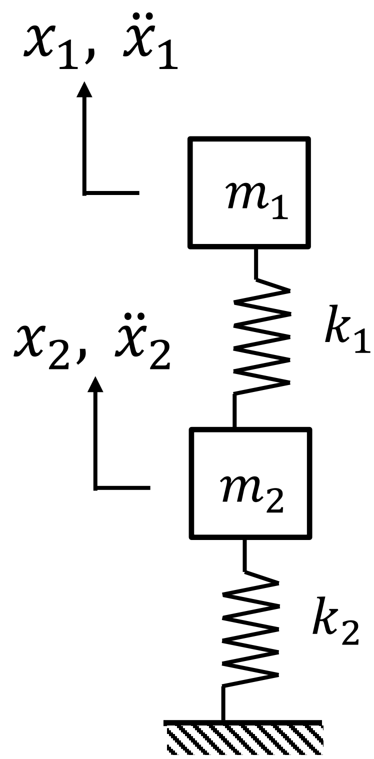
A two degrees of freedom mass-spring system is shown in the figure below.

$m_1 = 2\ kg$

$m_2 = 5\ kg$

$k_1 = 12\ \frac{N}{m}$

$k_2 = 14\ \frac{N}{m}$



a) Draw the free-body-diagram (FBD) for both masses. Assume $x_1 > x_2$.
b) Write the equations of motion (using the FBD in part (a) and applying the Newton's second law of motion)
c) Assume the solution for x_1 and x_2 as:

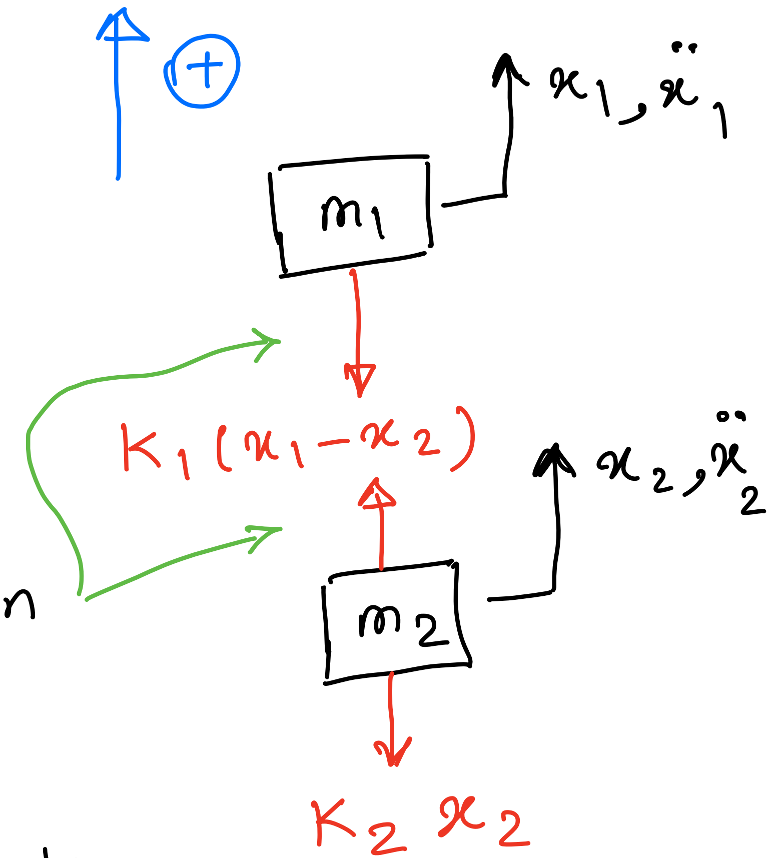
$$x_1 = A_1 e^{i\omega t}$$
$$x_2 = A_2 e^{i\omega t}$$

$\omega^2 = \lambda$

Assume ~~$\omega_1{}^2 = \lambda_1$ and $\omega_2{}^2 =$~~ λ_2, and substitute in the equations of motion.
d) Write the equations of motion in matrix form.
e) Solve the eigenvalue problem to find λ_1 and λ_2, as well as ω_1 and ω_2.
f) Find the eigenvectors (also known as mode shapes in vibrations) corresponding to the frequencies ω_1 and ω_2. use $A_2 = 1$ unit and calculate A_1
g) Illustrate the mode shapes (the relative amplitude of motion of the masses, A_1 and A_2 for both ω_1 and ω_2).

A blank page to show the work:

a) F.B.D.



$x_1 > x_2$

Therefore, the spring is in tension

b) Equations of Motion ($\Sigma F = m\ddot{x}$)

$$\begin{cases} -K_1(x_1 - x_2) = m_1 \ddot{x}_1 \\ K_1(x_1 - x_2) - K_2 x_2 = m_2 \ddot{x}_2 \end{cases}$$

c) $x_1 = A_1 e^{i\omega t}$ $\qquad$ $x_2 = A_2 e^{i\omega t}$

substitute in the equations of motion:

$\ddot{x}_1 = -A_1 \omega^2 e^{i\omega t}$ $\qquad$ $\ddot{x}_2 = -A_2 \omega^2 e^{i\omega t}$

$$\begin{cases} -K_1\left(A_1e^{iwt}-A_2e^{iwt}\right)=m_1\left(-A_1w^2e^{iwt}\right) \\ K_1\left(A_1e^{iwt}-A_2e^{iwt}\right)-K_2\left(A_2e^{iwt}\right) \\ \quad =m_2\left(-A_2w^2e^{iwt}\right) \end{cases}$$

$$\begin{cases} -K_1(A_1-A_2)\cancel{e^{iwt}}=-m_1A_1w^2\cancel{e^{iwt}} \\ K_1(A_1-A_2)\cancel{e^{iwt}}-K_2A_2\cancel{e^{iwt}} \\ \quad =-m_2A_2w^2\cancel{e^{iwt}} \end{cases}$$

$$\begin{cases} -K_1(A_1-A_2)=-m_1A_1w^2 \\ K_1(A_1-A_2)-K_2A_2=-m_2A_2w^2 \end{cases}$$

Assume $w^2=\lambda$
and substitute:

$$\begin{cases} -K_1(A_1-A_2)=-m_1A_1\lambda \\ K_1(A_1-A_2)-K_2A_2=-m_2A_2\lambda \end{cases}$$

d)

$$\begin{cases} (m_1\lambda - K_1) A_1 + K_1 A_2 = 0 \\ K_1 A_1 + (-K_1 - K_2 + m_2\lambda) A_2 = 0 \end{cases}$$

$$\begin{pmatrix} m_1\lambda - K_1 & K_1 \\ K_1 & -K_1 - K_2 + m_2\lambda \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

e)

$$\Rightarrow \begin{vmatrix} 2\lambda - 12 & 12 \\ 12 & -12 - 14 + 5\lambda \end{vmatrix} = 0 \qquad \begin{cases} m_1 = 2\ kg \\ m_2 = 5\ kg \\ k_1 = 12\ \frac{N}{m} \\ k_2 = 14\ \frac{N}{m} \end{cases}$$

$$\begin{vmatrix} 2\lambda - 12 & 12 \\ 12 & -26 + 5\lambda \end{vmatrix} = 0$$

$$(2\lambda - 12)(-26 + 5\lambda) - 12 \times 12 = 0$$

$$-52\lambda + 10\lambda^2 + 312 - 60\lambda - 144 = 0$$

$$10\lambda^2 - 112\lambda + 168 = 0$$

$$\begin{cases} \lambda_1 = \dfrac{28 - 2\sqrt{91}}{5} \cong 1.78 \\ \lambda_2 = \dfrac{28 + \sqrt{91}}{5} \cong 9.4 \end{cases}$$

Eigenvalues

$$\begin{cases} w_1^2 = \lambda_1 \Rightarrow w_1 = 1.33 \\ w_2^2 = \lambda_2 \Rightarrow w_2 = 3.06 \end{cases}$$

natural frequencies

f)

$$\begin{cases} m_1 = 2\,kg \\ m_2 = 5\,kg \\ k_1 = 12\,\dfrac{N}{m} \\ k_2 = 14\,\dfrac{N}{m} \end{cases}$$

$$\begin{bmatrix} 2\lambda - 12 & 12 \\ 12 & -26 + 5\lambda \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} (2\lambda - 12) A_1 + 12 A_2 = 0 \\ 12 A_1 + (-26 + 5\lambda) A_2 = 0 \end{cases}$$

$\lambda = \lambda_1 = 1.78$

$-8.44 A_1 + 12 A_2 = 0$

$\Longrightarrow A_1 = 1.42 A_2$

only one equation is enough because the second equation will be the same

$$\begin{cases} (2\lambda - 12) A_1 + 12 A_2 = 0 \\ 12 A_1 + (-26 + 5\lambda) A_2 = 0 \end{cases}$$

$\lambda = \lambda_2 = 9.4$

$$6.8 A_1 + 12 A_2 = 0$$

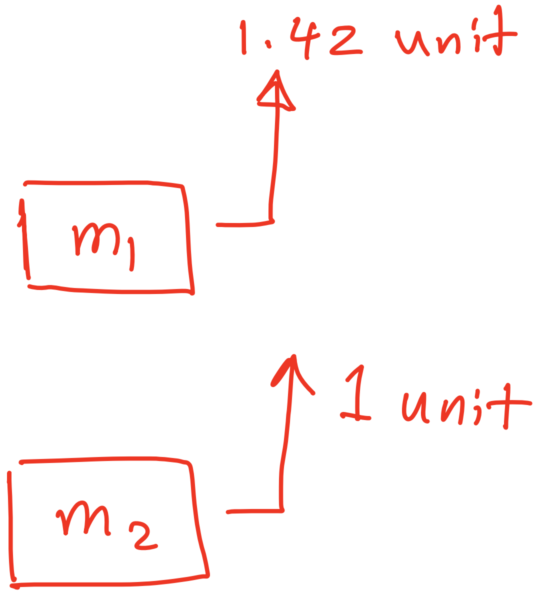
$$\Rightarrow A_1 = -1.76 A_2$$

Eigenvectors are:

$$V_1 = \begin{bmatrix} 1.42 \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} -1.76 \\ 1 \end{bmatrix}$$

g) Mode shapes:

$$V_1 = \begin{bmatrix} 1.42 \\ 1 \end{bmatrix}$$



$$V_2 = \begin{bmatrix} -1.76 \\ 1 \end{bmatrix}$$