Example 5.9 in the 3rd edition of the book (in Chapter 5)

Example (3.9 Book)

(Chapter 3 in the 2nd edition of the book)

using Equation (3.23), find the elements of the second row of the Jacobian for the simple revolute joint.

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} \cdots \\ S_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ \cdots \\ 1 \end{bmatrix} \quad (3.23) \text{ Book}$$

$$J_{21} = \frac{\partial p_y}{\partial \theta_1} = C_1(C_{234}a_4 + C_{23}a_3 + C_2a_2)$$

$$J_{22} = \frac{\partial p_y}{\partial \theta_2} = S_1(-S_{234}a_4 - S_{23}a_3 - S_2a_2)$$

$$J_{23} = \frac{\partial p_y}{\partial \theta_3} = S_1(-S_{234}a_4 - S_{23}a_3)$$

$$J_{24} = \frac{\partial p_y}{\partial \theta_4} = S_1(-S_{234}a_4)$$

$$J_{25} = \frac{\partial p_y}{\partial \theta_5} = 0 \qquad J_{26} = \frac{\partial p_y}{\partial \theta_6} = 0$$

Example (3.10 Book)

Find the ${}^{T_6}J_{11}$ and ${}^{T_6}J_{41}$ elements of the Jacobian for the simple revolute robot of example 2.25.

To calculate two elements of the first column of the Jacobian, we need to use $A_1 A_2 \cdots A_6$ matrix.

$$ {}^{R}T_H = A_1 A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Equation (3.26) for revolute joints:

$$ {}^{T_6}J_{11} = (-n_x p_y + n_y p_x) $$

Column 1 : $A_1 A_2 A_3 A_4 A_5 A_6 =$ result

$$
= \begin{bmatrix}
c_1(c_{234}c_5c_6 - s_{234}s_6) - s_1s_5c_6 & \cdots & c_1(c_{234}a_4 + c_{23}a_3 + c_2a_2) \\
s_1(c_{234}c_5c_6 - s_{234}s_6) + c_1s_5c_6 & \cdots & s_1(c_{234}a_4 + c_{23}a_3 + c_2a_2) \\
s_{234}c_5c_6 + c_{234}c_6 & & \\
\vdots & &
\end{bmatrix}
$$

$$
{}^{T_6}J_{11} = \cdots\cdots = s_5c_6(c_{234}a_4 + c_{23}a_3 + c_2a_2)
$$

(Equation 3.29)

$$
{}^{T_6}J_{41} = n_z = s_{234}c_5c_6 + c_{234}s_6
$$

column 1 : $A_1A_2A_3A_4A_5A_6 \rightarrow$ find n_z in the result of the multiplication

How to relate the Jacobian to the differential operator (sec. 3.11)

<u>Example</u> (3.11 Book)

The hand frame of a 5-DOF robot, its numerical Jacobian for this instance, and a set of differential motions are given. The robot has 2RP2R $\rightarrow$ 2 revolute joints

2 Revolute Joints

Prismatic jont

Find the new location of the hand after the differential motion.

$$T_6 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 3 & 0 & 0 & 0 & \\ -2 & 0 & 1 & 0 & \\ 0 & 4 & 0 & 0 & \\ 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} d\theta_1 \\ d\theta_2 \\ dS_1 \\ d\theta_4 \\ d\theta_5 \end{bmatrix} = \begin{bmatrix} 0.1 \\ -0.1 \\ 0.05 \\ 0.1 \\ 0 \end{bmatrix}$$

It is assumed that the robot can can only rotate about the x- and y-axes, since it has only 5 DOF.

$$[D] = \begin{bmatrix} dx \\ dy \\ dz \\ \delta x \\ \delta y \end{bmatrix}$$

$$[D] = [J][D_\theta] = \begin{bmatrix} 3 & 0 & 0 & 0 & \\ -2 & 0 & 1 & 0 & \\ 0 & 4 & 0 & 0 & \\ 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 \\ -0.1 \\ 0.05 \\ 0.1 \\ 0 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 0.3 \\ -0.15 \\ -0.4 \\ 0 \\ -0.1 \end{bmatrix} \longleftarrow \begin{bmatrix} dx \\ dy \\ dz \\ \delta x \\ \delta y \end{bmatrix}$$

$$[T_6]_{new} = [T_6]_{original} + [dT_6]$$

$$[dT_6] = \Delta [T_6]$$

Equation (3.16)

$$\Delta = \begin{bmatrix} 0 & -\delta z & \delta y & dx \\ \delta z & 0 & -\delta x & dy \\ -\delta y & \delta x & 0 & dz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Equation (3.17)

$$\Delta = \begin{bmatrix} 0 & 0 & -0.1 & 0.3 \\ 0 & 0 & 0 & -0.15 \\ 0.1 & 0 & 0 & -0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[dT_6] = [\Delta][T_6] = \begin{bmatrix} 0 & -0.1 & 0 & 0.1 \\ 0 & 0 & 0 & -0.15 \\ 0.1 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The new location of the frame after the differential motion is:

$$T_6 = dT_6 + [T_6]_{original}$$

$$= \begin{bmatrix} 1 & -0.1 & 0 & 5.1 \\ 0 & 0 & -1 & 2.85 \\ 0.1 & 1 & 0 & 2.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example (3.12 Book)

The differential motions applied to a frame (T_1), described as $D = [dx, \delta y, \delta z]^T$ and resulting (T_2) positions and orientations of the end of a 3-DoF robot are given.

The corresponding Jacobian is also given.

$$D = \begin{bmatrix} 0.01 \\ 0.02 \\ 0.03 \end{bmatrix} \qquad T_2 = \begin{bmatrix} -0.03 & 1 & -0.02 & 4.97 \\ 1 & 0.03 & 0 & 8.15 \\ 0 & -0.02 & -1 & 9.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 5 & 10 & 0 \\ 3 & 10 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) Find the original frame T_1 before the differential motions were applied to it.

Using Equations (3.16) and (3.21)

$$\left.\begin{array}{l} dT = T_2 - T_1 \\ dT = \Delta T_1 \end{array}\right\} \Rightarrow \Delta T_1 = T_2 - T_1$$

$$\Rightarrow T_2 = \Delta T_1 + T_1$$

$$\Rightarrow T_2 = (\Delta + I) T_1$$

$$\Rightarrow T_1 = (\Delta + I)^{-1} T_2$$

$$\Delta = \begin{bmatrix} 0 & -\delta z & \delta y & dx \\ \delta z & 0 & -\delta x & dy \\ -\delta y & \delta x & 0 & dz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

substituting the values form the D matrix into Δ, adding I to it, and then inverting it, we get:

$$\Delta = \begin{bmatrix} 0 & -0.03 & 0.02 & 0.01 \\ 0.03 & 0 & 0 & 0 \\ -0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(\Delta + I)^{-1} = \begin{bmatrix} 0.999 & 0.03 & -0.02 & -0.01 \\ -0.03 & 0.999 & 0.001 & 0.0003 \\ 0.02 & 0.001 & 1 & -0.002 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Find ${}^{T}\Delta$.

$$^{T}\Delta = T_1^{-1}\,\Delta\,T_1$$

$$= \begin{bmatrix} 0 & 0.03 & 0 & 0.15 \\ -0.03 & 0 & -0.02 & -0.03 \\ 0 & 0.02 & 0 & 0.1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Inverse Jacabian (3.12)

Next week