we have this determinant from the previous lecture.

$$\begin{vmatrix} K_1 - m_1 w^2 & -K_1 \\ -K_1 & K_1 + K_2 - m_2 w^2 \end{vmatrix} = 0$$

$$(K_1 - m_1 w^2)(K_1 + K_2 - m_2 w^2) - (-K_1)(-K_1) = 0$$

If $K_1 = K_2 = 1$ & $m_1 = m_2 = 2$

solve for w.

$$w^2 \longrightarrow \lambda$$

$$\begin{vmatrix} 1 - 2\lambda & -1 \\ -1 & 2 - 2\lambda \end{vmatrix} = 0$$

$$(1 - 2\lambda)(2 - 2\lambda) - 1 = 0$$

$$\Longrightarrow \quad 4\lambda^2 - 6\lambda + 1 = 0$$

$$\lambda = \frac{6 \pm \sqrt{36-16}}{8} = \frac{6 \pm \sqrt{20}}{8} = \frac{6 \pm 4.5}{8}$$

$$\Rightarrow \begin{cases} \lambda_1 = 1.3 \\ \lambda_2 = 0.19 \end{cases}$$

These λ values are the eigenvalues of this problem

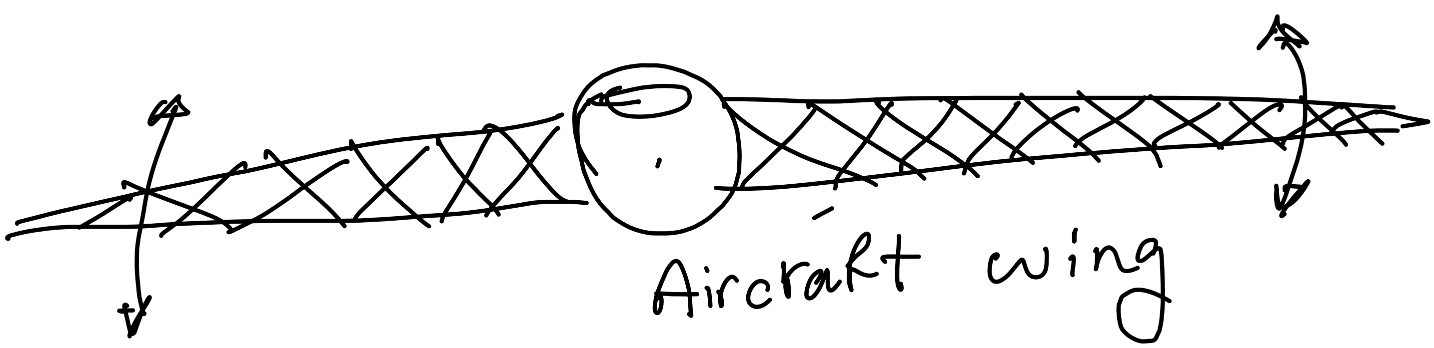
$$\omega^2 = \lambda \Rightarrow \begin{cases} \omega_1 = 1.14 \ \text{rad/s} \\ \omega_2 = 0.43 \ \text{rad/s} \end{cases}$$

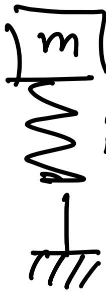
ω_1 and ω_2 are (in vibrations) the natural frequencies of the 2 DoF mass-spring system.

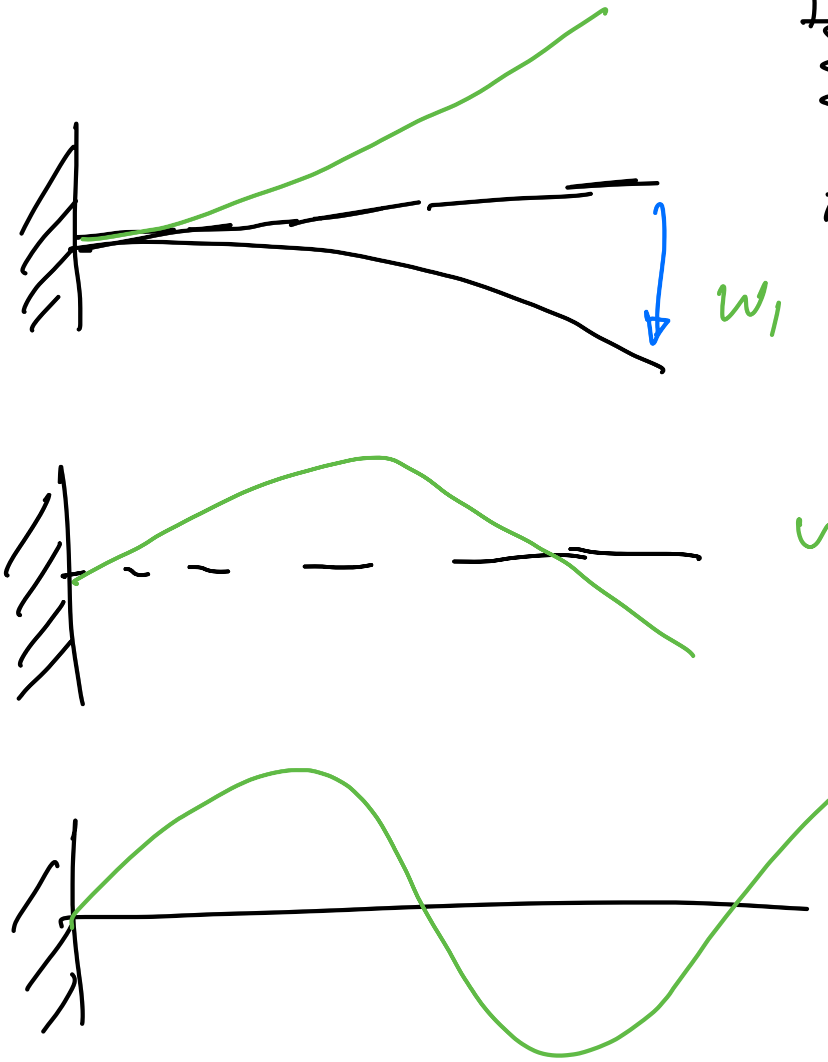
when the system is excited with natural frequencies, the amplitude of vibration is maximum.

The natural frequency is a characteristic of a vibratory system. It is an important parameter.

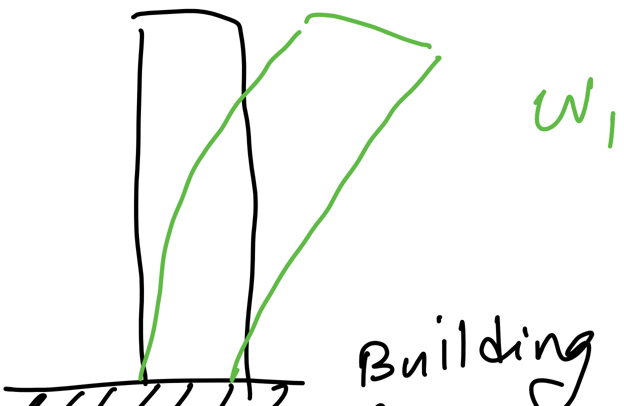
In engineering, we try to avoid the natural frequency or design a system that is not excited with its natural frequency. The excitation amplitude of the vibration at natural frequency can lead to failure of the system.



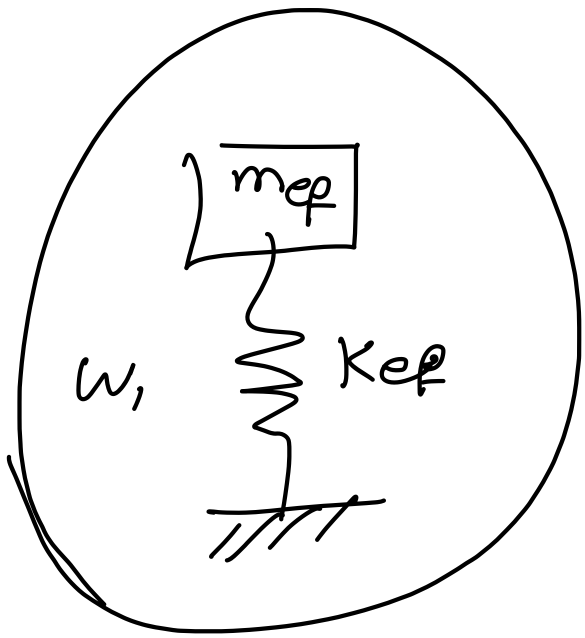




The most deflection happens in the first natural frequency.



Building first natural frequency.



The equations of motion from the previous lecture:

$$\begin{bmatrix} K_1 - m_1 w^2 & -K_1 \\ -K_1 & K_1 + K_2 - m_2 w^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 - 2\lambda & -1 \\ -1 & 2 - 2\lambda \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} (1-2\lambda) A_1 - A_2 = 0 \\ -A_1 + (2 - 2\lambda) A_2 = 0 \end{cases}$$

$\lambda_1 = 1.3$

subtitute the eigenvalue into the equations of motion.

$$\begin{cases} (1-2.6)A_1 - A_2 = 0 \\ -A_1 + (2-2.6)A_2 = 0 \end{cases}$$

and solve for A_1 and A_2

$$\begin{cases} -1.6A_1 - A_2 = 0 \Longrightarrow A_2 = -1.6A_1 \quad * \\ -A_1 - 0.6A_2 = 0 \Longrightarrow A_2 = \dfrac{-A_1}{0.6} \end{cases}$$

or $A_2 = -1.6A_1$

A_1 and A_2 can not be solved because both equations are the same. Therefore, we have 1 equation and 2 unknowns.

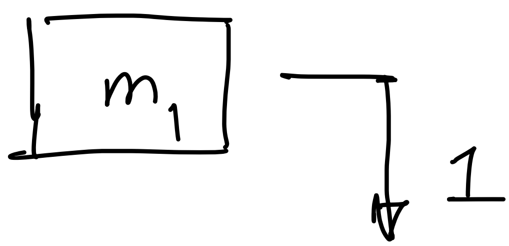
However, we can find the relative motion of A_1 and A_2 amplitudes. This approach leads to the eigenvector solution.

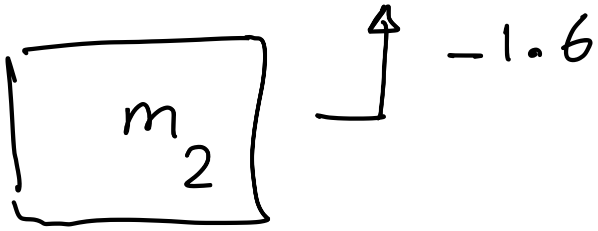
To do that, we can assume that A_1 has moved 1 unit, and find the relative motion of A_2 relative to A_1.

$$\text{If } A_1 = 1 \xRightarrow{*} A_2 = -1.6$$

Therefore, for $\omega_1 = 1.14$ rad/s (or $\lambda_1 = 1.3$) the eigenvector solution is as below.

$$\omega_1 = 1.14 \Longrightarrow \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1.6 \end{Bmatrix}$$





This eigenvector solution is reffered to as mode shapes in vibrations.

For $\begin{cases} \lambda_2 = 0.19 \quad (\sim 0.2) \\ w_2 = 0.43 \end{cases}$

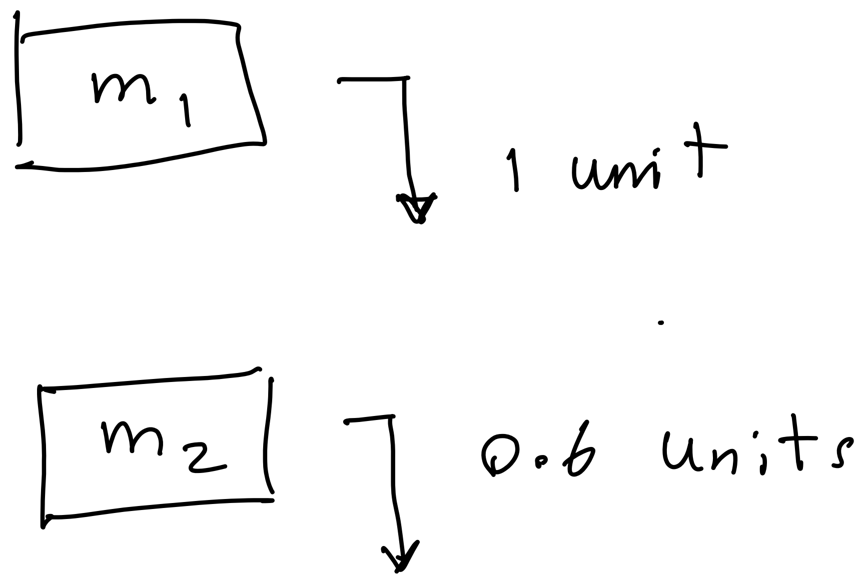
$$\begin{cases} (1-2\lambda)A_1 - A_2 = 0 \\ -A_1 + (2-2\lambda)A_2 = 0 \end{cases}$$

substitute λ_2

$$\begin{cases} (1-0.4)A_1 - A_2 - A_2 = 0 \\ -A_1 + 1.6 A_2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} A_2 = 0.6\,A_1 \\ A_2 = 0.6\,A_1 \end{cases}$$

IF $A_1 = 1$ unit $\Rightarrow A_2 = 0.6$



Both masses move in the same direction. This is the other mode shape (or Eigenvector).