# Differential Motions of a Robot and its hand Frame (section 5.9)

The Jacobian of the robot will create the link between the joint movements and the hand movement:

$$\begin{bmatrix} dx \\ dy \\ dz \\ \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} \text{Robot} \\ \text{Jacobian} \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \\ d\theta_5 \\ d\theta_6 \end{bmatrix}$$

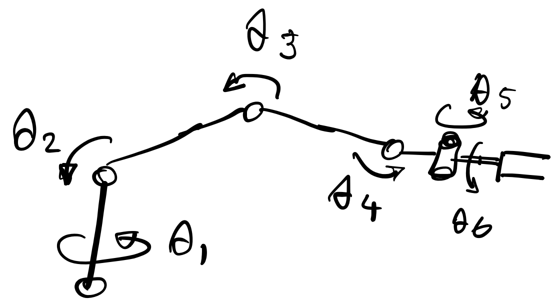
Hand movement

Joint movements

$$[D] = [J][D_\theta]$$

# Calculation of the Jacobian (section 5.10 Book)

As an example, consider the simple revolute arm of Example 2.25.




The last column of the forward kinematic equation of the robot is:

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} C_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ S_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ S_{234}a_4 + S_{23}a_3 + S_2a_2 \\ 1 \end{bmatrix}$$

$C_1 \longrightarrow \cos\theta_1$

$C_{234} \longrightarrow \cos(\theta_2 + \theta_3 + \theta_4)$

## Jacobian (Equation 5.a)

$$\begin{bmatrix} \delta y_1 \\ \delta y_2 \\ \vdots \\ \delta y_i \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_j} \\ \frac{\partial f_2}{\partial x_1} & \cdots & & \\ \vdots & & & \\ \frac{\partial f_i}{\partial x_1} & \cdots & & \frac{\partial f_i}{\partial x_j} \end{bmatrix}}_{\text{Jacobian}} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_j \end{bmatrix}$$

---

$$\text{Jacobian} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & \cdots \\ J_{21} & J_{22} & J_{23} & \cdots \\ J_{31} & J_{32} & J_{33} & \cdots \\ \vdots & & & \end{bmatrix}$$

$$J_{11} = \frac{\partial P_x}{\partial \theta_1} = -S_1 \left[ C_{234} a_4 + C_{23} a_3 + C_2 a_2 \right]$$

$$\frac{\partial \cos(\theta_2 + \theta_3 + \theta_4)}{\partial \theta_2}$$

$$J_{12} = \frac{\partial p_x}{\partial \theta_2} = C_1\left[-S_{234}a_4 - S_{23}a_3 - S_2 a_2\right]$$

$$J_{13} = \frac{\partial p_x}{\partial \theta_3} = C_1\left[-S_{234}a_4 - S_{23}a_3\right]$$

$$J_{14} = \frac{\partial p_x}{\partial \theta_4} = C_1\left[-S_{234}a_4\right]$$

$$J_{15} = \frac{\partial p_x}{\partial \theta_5} = 0$$

$$J_{16} = \frac{\partial p_x}{\partial \theta_6} = 0$$

The same can be done for next two rows. These are for translations.

There is no single equation for rotation

$$\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotations are obtained differently.

In reality, it is actually alot simpler to calculate the Jacobian relative to $T_6$ (frame 6), the last frame. Therefore we will instead use the following approach. We can write the velocity equation relative to the last frameas:

$$[{}^{T_6}D] = [{}^{T_6}J][D\theta]$$

The differential motion relationship of Equation (3.25) can be written as:

$$\begin{bmatrix} {}^{T_6}d_x \\ {}^{T_6}d_y \\ {}^{T_6}d_z \\ {}^{T_6}\delta_x \\ {}^{T_6}\delta_y \\ {}^{T_6}\delta_z \end{bmatrix} = \begin{bmatrix} {}^{T_6}J_{11} & {}^{T_6}J_{12} & \cdots & {}^{T_6}J_{16} \\ {}^{T_6}J_{21} & {}^{T_6}J_{22} & \cdots & {}^{T_6}J_{26} \\ {}^{T_6}J_{31} & \vdots & & {}^{T_6}J_{36} \\ {}^{T_6}J_{41} & \vdots & & {}^{T_6}J_{46} \\ {}^{T_6}J_{51} & & & {}^{T_6}J_{56} \\ {}^{T_6}J_{61} & & & {}^{T_6}J_{66} \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ \vdots \\ d\theta_6 \end{bmatrix}$$

The calculations of $J$ element has been done for you as below.

If joint $i$ under consideration is a revolute joint, then:

$$^{T_6}J_{1i} = (-n_x p_y + n_y p_x)$$

$$^{T_6}J_{2i} = (-o_x p_y + o_y p_x)$$

$$^{T_6}J_{3i} = (-a_x p_y + a_y p_x)$$

$$^{T_6}J_{4i} = n_z$$

$$^{T_6}J_{5i} = o_z$$

$$^{T_6}J_{6i} = a_z$$

Equations (5.26) in the Book

If joint $i$ under consideration is a prismatic joint, then:

$^{T_6}J_{1i} = n_z$ $\quad$ $^{T_6}J_{2i} = o_z$ $\quad$ $^{T_6}J_{3i} = a_z$

$^{T_6}J_{4i} = 0$ $\quad$ $^{T_6}J_{5i} = 0$ $\quad$ $^{T_6}J_{6i} = 0$

(5.27)

For equations (5.26) and (5.27),

For column i use ${}^{i-1}T_6$, meaning:

For column 1, use ${}^{0}T_6 = A_1 A_2 A_3 A_4 A_5 A_6$

For column 2, use ${}^{1}T_6 = A_2 A_3 A_4 A_5 A_6$

For column 3, use ${}^{2}T_6 = A_3 A_4 A_5 A_6$

For column 4, use ${}^{3}T_6 = A_4 A_5 A_6$

For column 5, use ${}^{4}T_6 = A_5 A_6$

For column 6, use ${}^{5}T_6 = A_6$

---

Next time study this section and examples 5.9 and 5.10