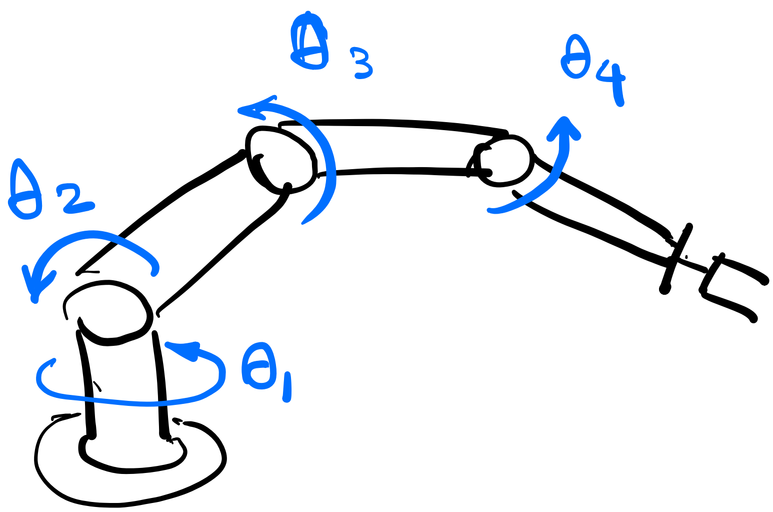
Chapter 2

Kinematics of Robots:
Position Analysis

Kinematics of Robots $\begin{cases} \text{Forward Kinematics} \\ \text{Inverse Kinematics} \end{cases}$

Forward Kinematics:
we determine where the robot's end (hand) will be if all joint variables are Known.

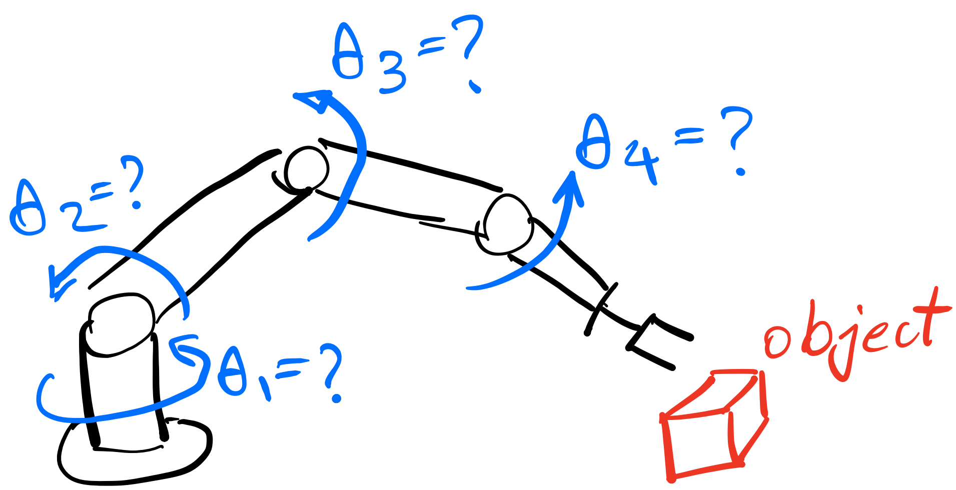


θ_1, θ_2, θ_3, θ_4 are given

$\longrightarrow$ Robot hand position

Inverse Kinematics:

The position of the hand is given, it is required to calculate the joint variables



IF the position of the hand is given, find $\theta_1, \theta_2, \theta_3, \theta_4 ..$

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we will study the forward and inverse kinematics of different robot configurations such as cartesian, cylindrical, and spherical coordinates.

Finally, we will use the Denavit-Hartenberg representaion to derive forward and inverse kinematic equations of all possible configurations of robots, regardless of number of joints.

Conventions

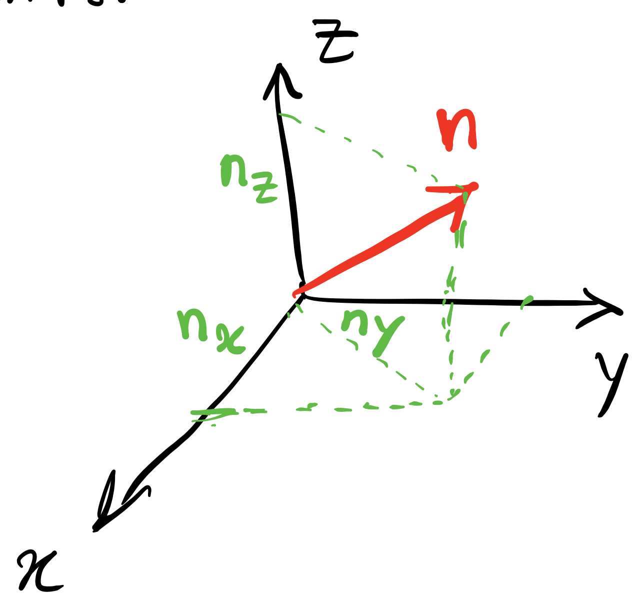
we will use the following conventions for describing vectors, frames, transformations, and so on.

Vectors $\mathbf{i}$ (in Bold) → in Book

$\vec{i}, \vec{j}, \vec{K}, \vec{x}, \vec{y}, \vec{z}, \vec{n}, \vec{o}, \vec{a}, \vec{P}$

Vector components:

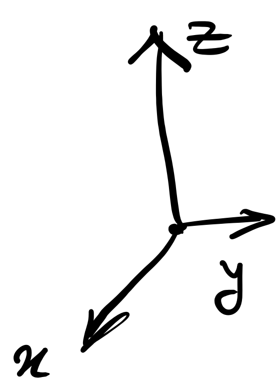
n_x, n_y, n_z



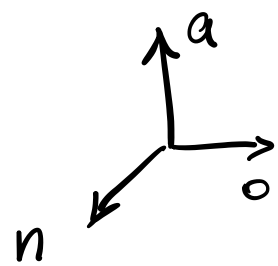
or for a "a" vector → a_x, a_y, a_z

Frames :

F_{xyz} , xyz



F_{noa} , noa



F_{camera} , ...

Transformations:

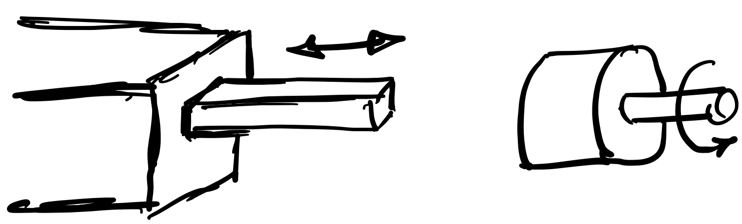
T_1 , T_2 , ${}^{u}T$, ${}^{B}P$

Transformation T relative to frame u

${}^{U}T_{R}$ (Transformation of the robot relative to universe, where universe is a fixed frame.

Matrix Representation

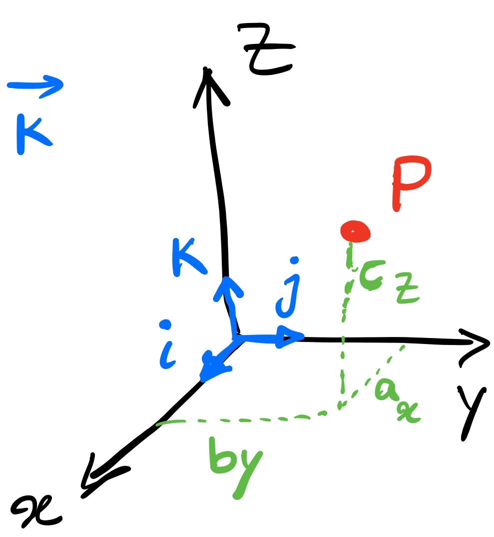
Matrices can be used to represent points, vectors, frames, translations, rotations, transformations.



Representation of a point in space.

A point P in space can be represented by its three coordinates relative to a reference frame as:

$$P = a_x \vec{i} + b_y \vec{j} + c_z \vec{k}$$



where a_x, b_y, and c_z are the

three coordinates of the point represented in the reference frame.

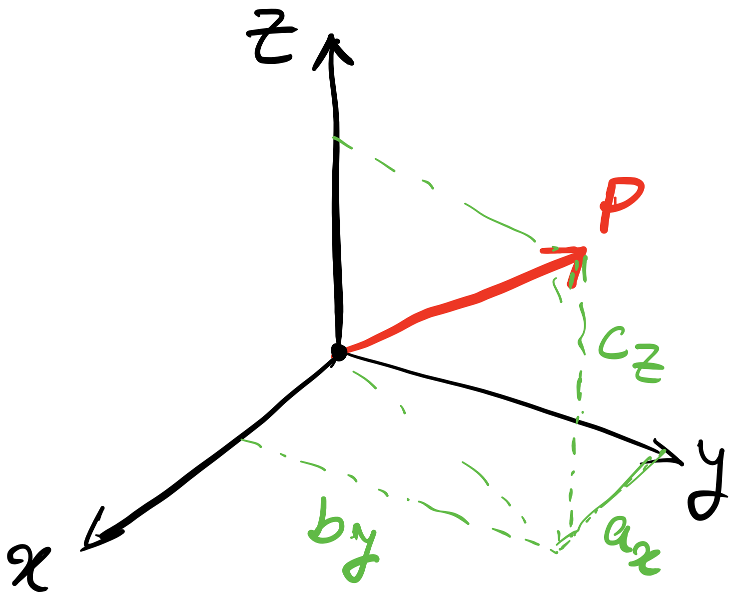
Representation of a vector in space

A vector can be represented by three coordinates of its tail and its head. If the vector starts at point A and ends at point B, then it can be represented by

$$\vec{P}_{AB} = (B_x - A_x)\vec{i} + (B_y - A_y)\vec{j}$$

$$+ (B_z - A_z)\vec{K}$$

If the vector starts at the origin



$$\vec{P} = a_x \vec{i} + b_y \vec{j} + c_z \vec{K}$$

where a_x, b_y, c_z are the three components of the vector in the reference frame.

The three components of the vector can also be written in matrix form,

$$\vec{P} = \begin{bmatrix} a_x \\ b_y \\ c_z \end{bmatrix}$$

This representation can be slightly modified to also include a scale factor W such that if P_x, P_y, and P_z are divided by W, they will yield a_x, b_y, and c_z.

Therefore the vector is:

$$\vec{P} = \begin{bmatrix} P_x \\ P_y \\ P_z \\ W \end{bmatrix} \qquad \begin{aligned} P_x &= W\, a_x \\ P_y &= W\, b_y \\ P_z &= W\, c_z \end{aligned}$$

Example

A vector is described as

$$\vec{P} = 3\vec{i} + 5\vec{j} + 2\vec{k}.$$

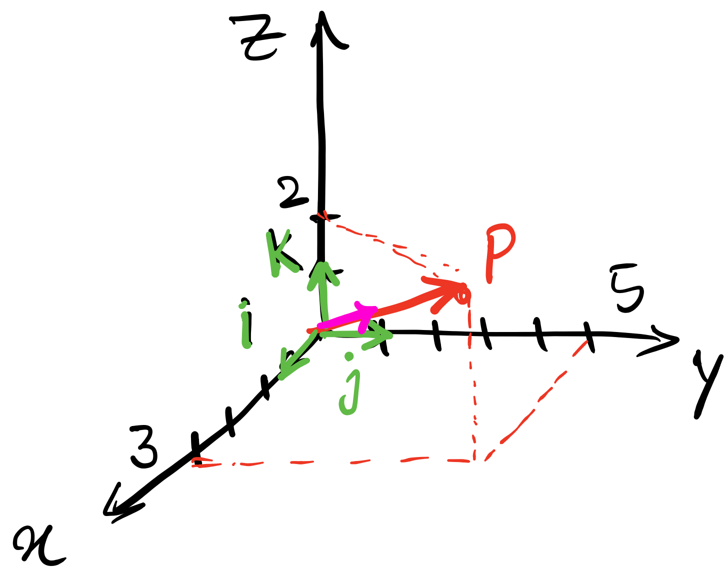
Express the vector in matrix form.

$$\vec{p} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

Express the vector in matrix form with a scale factor of 2.

$$\vec{p} = \begin{bmatrix} 6 \\ 10 \\ 4 \\ 2 \end{bmatrix}$$

write the vector as a unit vector.



In order to make the vector into a unit vector, we normalize the length to be equal to 1. To do this, each component of the vector is divided by the square root of the sum of the squares of three components

$$\lambda = \sqrt{P_x^2 + P_y^2 + P_z^2}$$ ← length of the vector

$$\lambda = \sqrt{3^2 + 5^2 + 2^2} = 6.16$$

$$\vec{P} = 3\vec{i} + 5\vec{j} + 2\vec{k}$$

$$\vec{P}_{unit} = \frac{3}{6.16}\vec{i} + \frac{5}{6.16}\vec{j} + \frac{2}{6.16}\vec{k}$$

$$\vec{P}_{unit} = \begin{pmatrix} 3/6.16 \\ 5/6.16 \\ 2/6.16 \end{pmatrix} = \begin{bmatrix} 0.487 \\ 0.811 \\ 0.324 \end{bmatrix}$$

The length of the unit vector:

$$\sqrt{0.487^2 + 0.811^2 + 0.324^2} = 1$$

Example

A vector $\vec{p}$ is 5 units long and is in the direction of a unit vector $\vec{q}$ described below. Express the vector in matrix form.

$$\vec{q}_{unit} = \begin{bmatrix} 0.371 \\ 0.557 \\ q_z \end{bmatrix}$$

$q_z = ?$

The unit vector's length must be 1.

$$\lambda = \sqrt{q_x^2 + q_y^2 + q_z^2}$$

$$= \sqrt{0.138 + 0.310 + q_z^2} = 1$$

$$q_z = 0.743$$

$$\vec{q}_{unit} = \begin{bmatrix} 0.371 \\ 0.557 \\ 0.743 \end{bmatrix}$$

$$\vec{P} = \vec{q}_{unit} \times 5 = \begin{bmatrix} 1.855 \\ 2.785 \\ 3.715 \end{bmatrix}$$