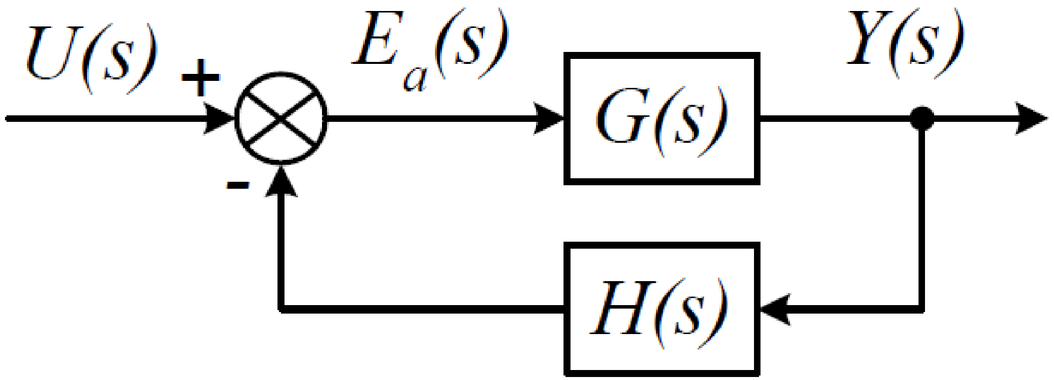
# Instrumentation and Controls
## ETM 3301

# Lecture 17

Instructor
Dr. Farbod Khoshnoud

# Steady state errors for nonunity feedback systems (1)




- Note: $E_a(s)$ is NOT error signal!

Error: $E(s) = U(s) - Y(s)$

- **Method**: Directly use the error definition and the final value theorem to find the steady state error.

# Steady state errors for nonunity feedback systems (2)

Closed-loop TF $$T(s)=\frac{G(s)}{1+G(s)H(s)}$$

$$E(s)=U(s)-Y(s)=\left[1-T(s)\right]U(s)$$

$$=\frac{1+G(s)H(s)-G(s)}{1+G(s)H(s)}U(s)$$

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s\left[1-T(s)\right]U(s)$$

$$=\lim_{s\to 0}s\frac{1+G(s)H(s)-G(s)}{1+G(s)H(s)}U(s)$$

# Example (1)

- A feedback system with forward path and feedback path transfer functions

$$G(s) = \frac{100}{s(s+10)} \qquad H(s) = \frac{1}{s+5}$$

- Find the steady state error for a unit step input.

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{100}{s(s+10)}}{1+\frac{100}{s(s+10)}\times\frac{1}{s+5}}$$

$$= \frac{100(s+5)}{s(s+10)(s+5)+100} = \frac{100(s+5)}{s^3+15s^2+50s+100}$$

# Example (2)

Step input $u(t) = 1 \qquad U(s) = \frac{1}{s} \qquad Y(s) = T(s)U(s)$

$$E(s) = U(s) - Y(s) = \left[1 - T(s)\right]U(s)$$

$$= \frac{s^3 + 15s^2 - 50s - 400}{s(s^3 + 15s^2 + 50s + 100)}$$

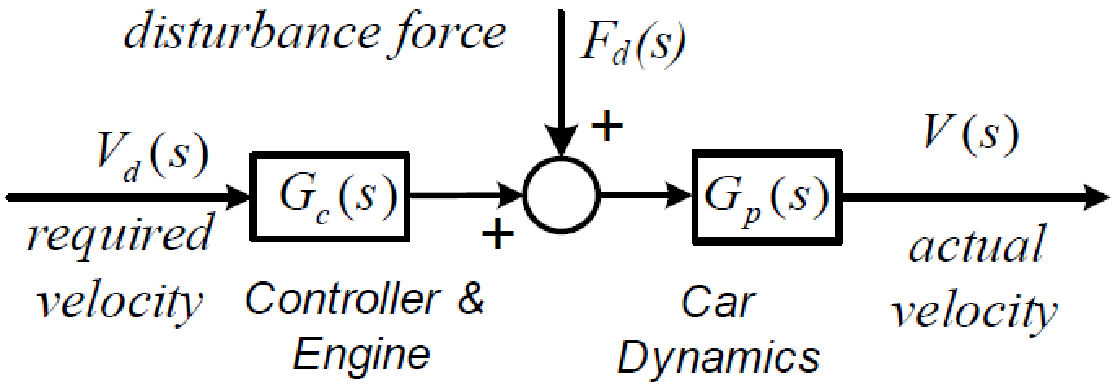
$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{s^3 + 15s^2 - 50s - 400}{s(s^3 + 15s^2 + 50s + 100)}$$

$$= \lim_{s \to 0} \frac{s^3 + 15s^2 - 50s - 400}{s^3 + 15s^2 + 50s + 100}$$

$$= -4$$

# Steady state errors for systems with external disturbances

Car speed control example (open-loop control):




Car $$G_p(s) = \frac{K_p}{Ts+1}$$ Controller $$G_c(s) = K_c$$

$$V(s) = G_c(s)G_p(s)V_d(s) + G_p(s)F_d(s)$$

# Steady state errors for systems with external disturbances

$$V(s) = G_c(s)G_p(s)V_d(s) + G_p(s)F_d(s)$$

$$E(s) = V_d(s) - V(s) = \left(1 - G_c(s)G_p(s)\right)V_d(s) - G_p(s)F_d(s)$$

$$E(s) = V_d(s) - V(s) = \frac{Ts + 1 - K_p K_c}{Ts + 1} V_d(s) - \frac{K_p}{Ts + 1} F_d(s)$$

Assume: $v_d(t) = A \longleftarrow$ known constant value

$f_d(t) = W \longleftarrow$ unknown constant value

$$V_d(s) = \frac{A}{s} \qquad F_d(s) = \frac{W}{s}$$

# Steady state errors for systems with external disturbances

$$E(s) = \frac{Ts + 1 - K_p K_c}{Ts + 1} V_d(s) - \frac{K_p}{Ts + 1} F_d(s)$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \left( \frac{Ts + 1 - K_p K_c}{Ts + 1} \frac{A}{s} - \frac{K_p}{Ts + 1} \frac{W}{s} \right)$$

$$= (1 - K_p K_c) A - K_p W$$

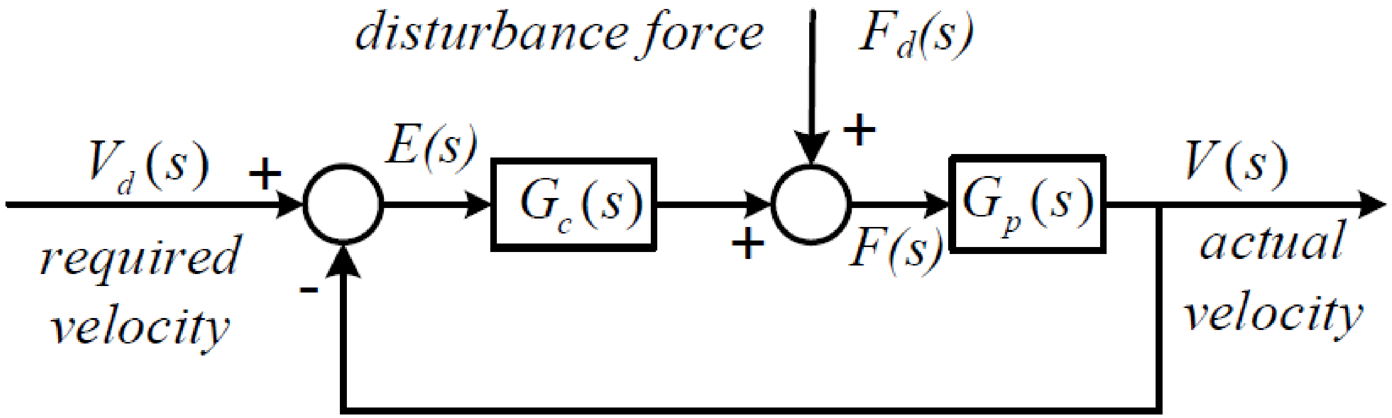
Can be made zero by setting controller

$$K_c = \frac{1}{K_p}$$

Controller has no control over this term

# Steady state errors for systems with external disturbances

Car speed control example (closed loop control):




Car $G_p(s) = \dfrac{K_p}{Ts+1}$ Controller $G_c(s) = K_c$

# Steady state errors for systems with external disturbances

$$V(s) = G_p(s)F(s) = G_p(s)[G_c(s)E(s) - F_d(s)]$$
$$= G_p(s)[G_c(s)(V_d(s) - V(s)) - F_d(s)]$$

Output:

$$V(s) = G_p(s)G_c(s)V_d(s) - G_p(s)G_c(s)V(s) - G_p(s)F_d(s)$$

$$V(s) = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}V_d(s) - \frac{G_p(s)}{1 + G_p(s)G_c(s)}F_d(s)$$

input

disturbance

# Steady state errors for systems with external disturbances

Error:

$$E(s) = V_d(s) - V(s)$$

$$= \frac{1}{1+G_p(s)G_c(s)} V_d(s) + \frac{G_p(s)}{1+G_p(s)G_c(s)} F_d(s)$$

input → $V_d(s)$     disturbance → $F_d(s)$

$$G_p(s) = \frac{K_p}{Ts+1} \qquad G_c(s) = K_c$$

$$E(s) = \frac{Ts+1}{Ts+1+K_pK_c} V_d(s) + \frac{K_p}{Ts+1+K_pK_c} F_d(s)$$

# Steady state errors for systems with external disturbances

Assume: $v_d(t) = A \longleftarrow$ known constant value

$f_d(t) = W \longleftarrow$ unknown constant value

$$E(s) = \frac{Ts+1}{Ts+1+K_pK_c}\frac{A}{s} + \frac{K_p}{Ts+1+K_pK_c}\frac{W}{s}$$

Steady state speed control (tracking) error:

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\left(\frac{Ts+1}{Ts+1+K_pK_c}A + \frac{K_p}{Ts+1+K_pK_c}W\right)$$

$$= \frac{1}{1+K_pK_c}A + \frac{K_p}{1+K_pK_c}W \neq 0$$

# Steady state errors for systems with external disturbances

Error:
$$E(s) = \frac{1}{1 + G_p(s)G_c(s)} V_d(s) + \frac{G_p(s)}{1 + G_p(s)G_c(s)} F_d(s)$$

$$G_p(s) = \frac{K_p}{Ts + 1} \qquad G_c(s) = \frac{K_1 s + K_2}{s}$$

$$E(s) = \frac{s(Ts + 1)}{s(Ts + 1) + K_p(K_1 s + K_2)} V_d(s) + \frac{sK_p}{s(Ts + 1) + K_p(K_1 s + K_2)} F_d(s)$$

# Steady state errors for systems with external disturbances

Assume: $v_d(t) = A$ ⟵ known constant value

$f_d(t) = W$ ⟵ unknown constant value

$$E(s) = E(s) = \frac{s(Ts+1)}{s(Ts+1) + K_p(K_1 s + K_2)} \frac{A}{s}$$

$$+ \frac{sK_p}{s(Ts+1) + K_p(K_1 s + K_2)} \frac{W}{s}$$

# Steady state errors for systems with external disturbances

Steady state speed control (tracking) error:

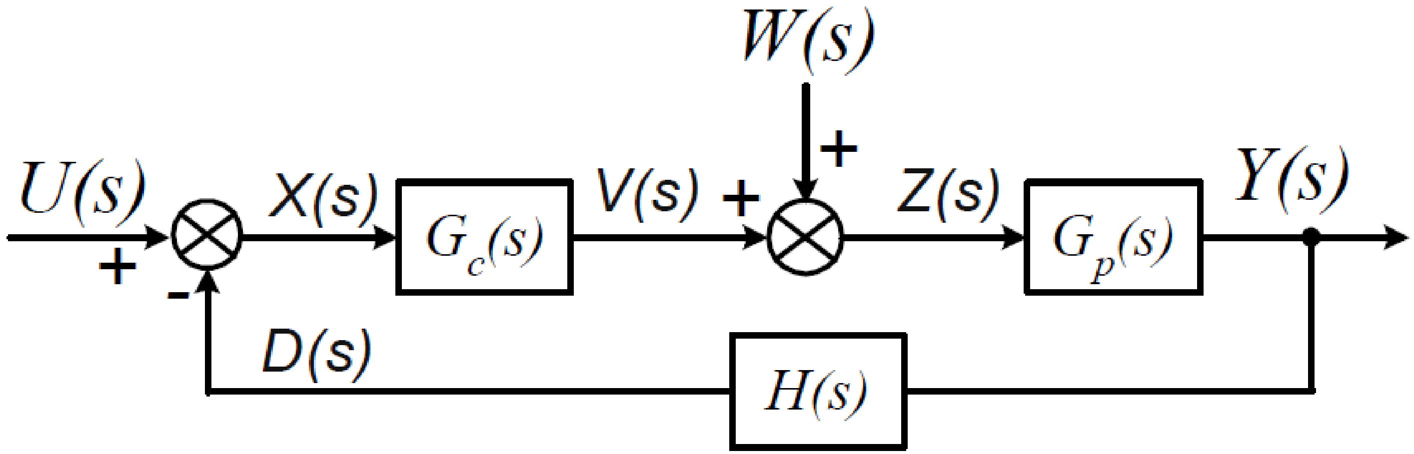
$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} \left( \frac{s(Ts+1)}{s(Ts+1) + K_p(K_1 s + K_2)} A + \frac{sK_p}{s(Ts+1) + K_p(K_1 s + K_2)} W \right)$$

$$= 0$$

Always zero!!!!

# Steady state errors for systems with external disturbances (1)




$U(s)$: input $\quad$ $Y(s)$: output $\quad$ $W(s)$: external disturbance

$$Y(s) = Y_u(s) + Y_w(s) = G_{yu}(s)U(s) + G_{yw}(s)W(s)$$

response due to input

response due to disturbance

# Steady state errors for systems with external disturbances (2)

$$Y(s) = G_p Z(s) = G_p (V(s) + W(s))$$
$$= G_p (G_c X(s) + W(s))$$
$$= G_p G_c (U(s) - D(s)) + G_p W(s)$$
$$= G_p G_c U(s) - G_p G_c H Y(s) + G_p(s) W(s)$$

$$(1 + G_p G_c H) Y(s) = G_p G_c U(s) + G_p W(s)$$

$$Y(s) = \frac{G_p G_c}{1 + G_p G_c H} U(s) + \frac{G_p}{1 + G_p G_c H} W(s)$$

# Steady state errors for systems with external disturbances (2)

Error:

$$E(s) = U(s) - Y(s)$$

$$= \left( 1 - \frac{G_p G_c}{1 + G_p G_c H} \right) U(s) - \frac{G_p}{1 + G_p G_c H} W(s)$$

$$= E_u(s) + E_w(s)$$

Steady State Error:

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} sE_u(s) + \lim_{s \to 0} sE_w(s)$$

$$= e_u + e_w$$

# Steady state errors for systems with external disturbances (3)

Steady State Error: $e_{ss} = e_u + e_w$

$$e_u = \lim_{s \to 0} sE_u(s) = \lim_{s \to 0} \left[ s \left( 1 - \frac{G_p G_c}{1 + G_p G_c H} \right) U(s) \right]$$

- state-state error due to reference input $u$

$$e_y = \lim_{s \to 0} sE_w(s) = -\lim_{s \to 0} \left[ s \left( \frac{G_p}{1 + G_p G_c H} \right) W(s) \right]$$

- state-state error due to external disturbance $w$