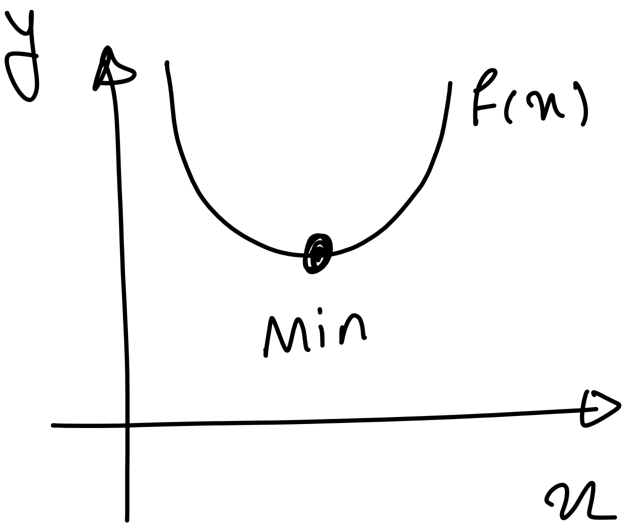
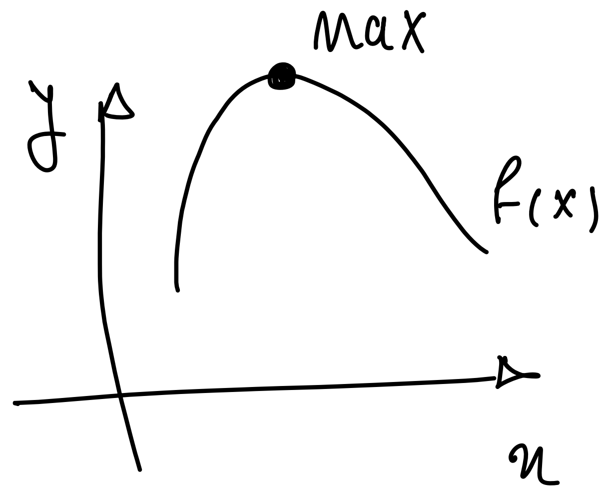
An application for derivative

Finding maximum and minimum.





Finding Max/min

$$\frac{df(x)}{dx} = 0$$

If Min $\frac{d^2f}{dx^2} > 0$

If max $\frac{d^2f}{dx^2} < 0$

Example

projectile problem with initial velocity of V_0

$$\begin{cases} x = V_0 (\cos\alpha) t \\ y = V_0 (\sin\alpha) t - \frac{1}{2} g t^2 \end{cases}$$

Determin. the maximum height reached at fixed α and the corresponding time.

Max. height $\longrightarrow \dfrac{dy}{dt} = 0$

$$\frac{dy}{dt} = V_0 \sin\alpha - g t = 0$$

$$\Longrightarrow \quad t_{max} = \frac{V_0 \sin\alpha}{g}$$

$$t_{max} \longrightarrow y$$

$$y_{max} = V_0 (\sin\alpha)\, t_{max} - \tfrac{1}{2} g (t_{max})^2$$

$$y_{max} = V_0 (\sin\alpha) \left(\frac{V_0 \sin\alpha}{g}\right) - \tfrac{1}{2} g \frac{V_0^2 \sin\alpha^2}{g^2}$$

$$y_{max} = \frac{V_0^2 \sin^2\alpha}{g} - \tfrac{1}{2} \frac{V_0^2 \sin^2\alpha}{g}$$

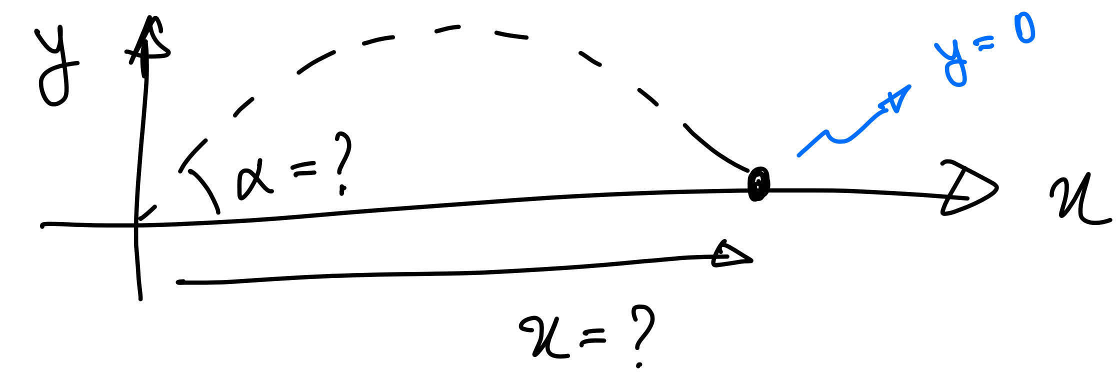
$$y_{max} = \tfrac{1}{2} \frac{V_0^2 \sin^2\alpha}{g}$$

Example

At what α does the ball travel the furthest (in x direction) before hits the ground?

and find the max travel distance.

$$\begin{cases} x = V_0 (\cos\alpha) t & (1) \\ y = V_0 \sin\alpha \, t - \frac{1}{2} g t^2 & (2) \end{cases}$$



$$y = 0 \Longrightarrow V_0 \sin\alpha \, t - \frac{1}{2} g t^2 = 0$$

$$t = \frac{2 V_0 \sin\alpha}{g} \quad (3)$$

substitute (3) into (1):

$$x = V_0 \cos\alpha \, t = V_0 \cos\alpha \, \frac{2 V_0 \sin\alpha}{g}$$

$$x = \frac{2 V_0^2 \sin\alpha \cos\alpha}{g} \quad (4)$$

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \sin\beta \cos\alpha$$

$$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \sin\beta \cos\alpha$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

$$\sin(\alpha+\alpha) + \underbrace{\sin(\alpha-\alpha)}_{0} = 2\sin\alpha\cos\alpha$$

$$\Longrightarrow \sin 2\alpha = 2\sin\alpha\cos\alpha \quad (5)$$

substitute (5) into (4)

$$x = \frac{v_0^2 \sin 2\alpha}{g}$$

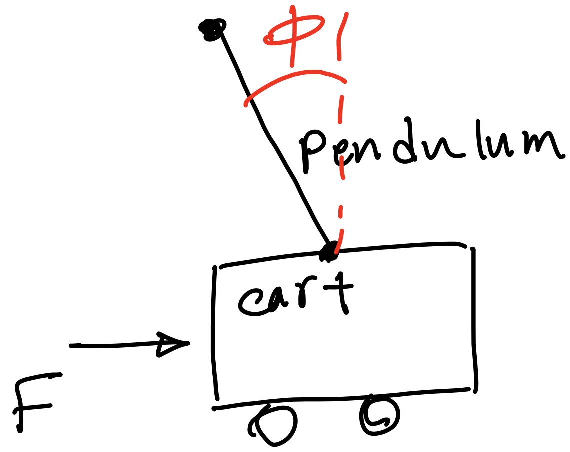
Find the max.

$$\frac{dx}{d\alpha} = \frac{v_0^2 (2\cos 2\alpha)}{g} = 0$$

$$\cos 2\alpha = 0$$

$$2\alpha = 90^\circ \rightarrow \alpha = 45^\circ$$

$$\max(x) = \frac{v_0^2 \sin(2\times 45^\circ)}{g} = \frac{v_0^2}{g}$$



Only application

Equations of motion were given in the previous lecture

- Rotation Equation of the pendulum
- Translation Equation of the cart

The equations then will be written in a matrix form called state-space model.

constants $\quad$ F

$$[\dot{x}] = [A][x] + [B][u]$$

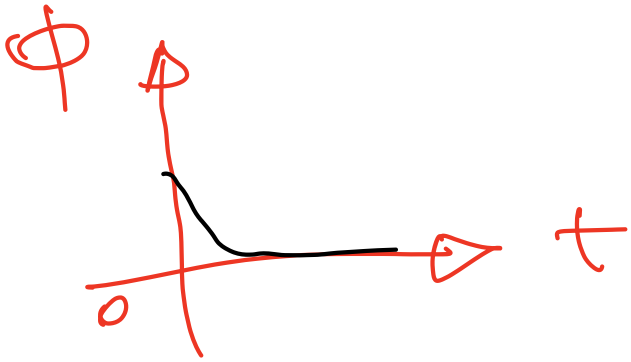
Input

Variables: displacement and velocity such as Φ

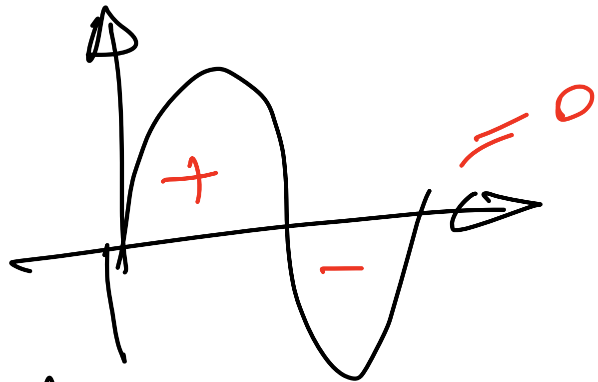
create a cost function:

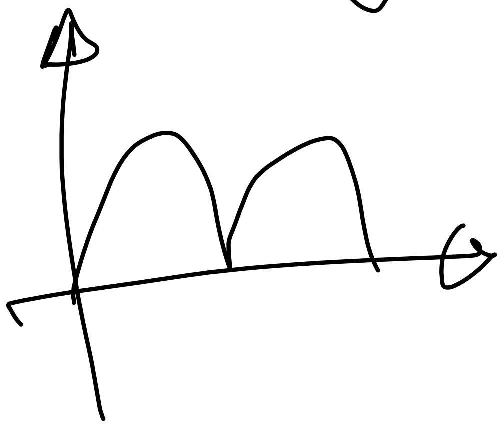
$$J = \int \left([x]^T [Q] [x] + \underbrace{[u]^T [R] [u]}_{\text{Force minimze}} \right) dt$$

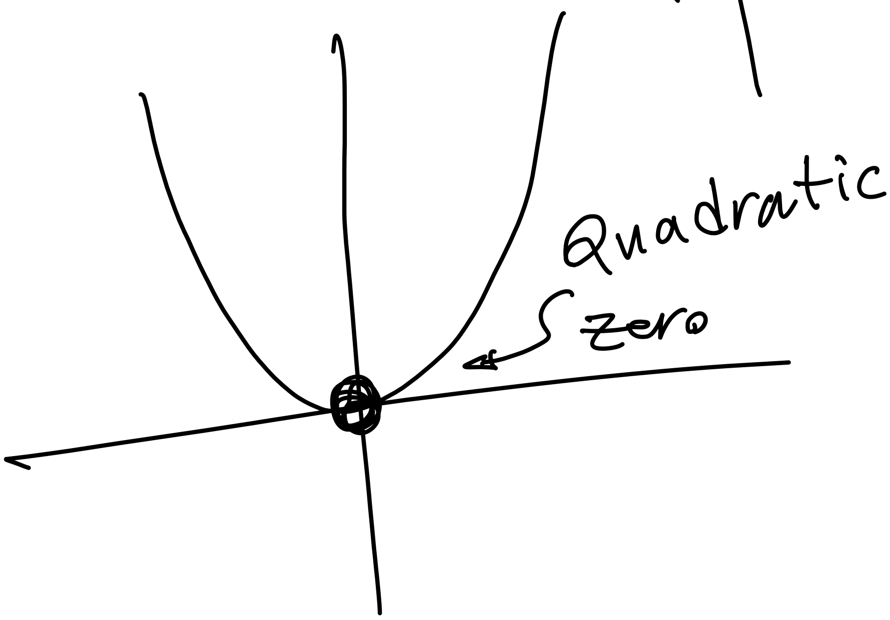
minimize Φ



$$x \cdot x = x^2$$







Linear Qudratic Regulator (LQR)

Most efficient techinque to optimize a control problem.

$$J = \int \Big([x]^T [Q] [x] + \underbrace{[u]^T [R] [u]}_{\text{Force minimize}} \Big)\, dt$$

minimize ϕ

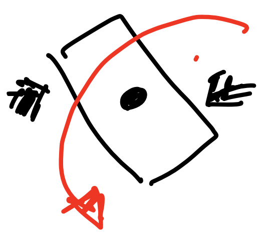
$$\underbrace{\begin{bmatrix} \dot{x} & \dot{\phi} & x & \phi \end{bmatrix}}_{1\times4} \underbrace{\begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & \\ 0 & & & Q_4 \end{bmatrix}}_{4\times4} \underbrace{\begin{bmatrix} \dot{x} \\ \dot{\phi} \\ x \\ \phi \end{bmatrix}}_{4\times1}$$

0.01

0.01

$$[F]^T \begin{bmatrix} \overset{1000}{R} \end{bmatrix} [F]$$

give R a big value



minimiz fuel in propulsion in space

$\Longrightarrow$ minimize F

only application