Instrumentation and Controls

ETM 3301

Lecture 14

Instructor

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Routh Table

$$CE: \quad a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	0
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	0
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	0

Example: $CE: \quad s^4 + 5s^3 + 7s^2 + 3s + 5 = 0$

s^4	1	7	5		$+$
s^3	5	3	0		$+$
s^2	$\dfrac{5\times 7 - 1\times 3}{5} = 6.4$	$\dfrac{5\times 5 - 1\times 0}{5} = 5$	0		$+$
s^1	$\dfrac{6.4\times 3 - 5\times 5}{6.4} = -0.9$	0	0		-
s^0	$\dfrac{-0.9\times 5 - 6.4\times 0}{-0.9} = 5$	0	0		$+$

Example: $CE: \quad s^4 + 5s^3 + 7s^2 + 3s + 5 = 0$

There are two changes in sign (+ to - and - to +) in the first column, hence the system is unstable and there are two poles with positive real parts.

Verification:

Poles are: $-2.55 \pm 0.546j,$

$0.055 \pm 0.854j$

Unstable system

Routh Table: Special Case I

- If the 1st element in a row is zero, it is replaced by a very small positive number ε, and the sign changes when $\varepsilon \rightarrow 0$ are counted after completing the array.

Example: $CE: \quad s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 = 0$

s^5	1	3	5
s^4	2	6	3
s^3	$0 \Longrightarrow \varepsilon$	3.5	0
s^2	$\dfrac{6\varepsilon - 7}{\varepsilon}$	3	0
s^1	$\dfrac{42\varepsilon - 49 - 6\varepsilon^2}{12\varepsilon - 14}$	0	0
s^0	3	0	0

Example: $CE: \quad s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 = 0$

	$\varepsilon \to 0$	sign
s^5	1	+
s^4	2	+
s^3	ε	+
s^2	$-\infty$	-
s^1	49/14	+
s^0	3	+

Two sign changes.

Two poles with positive real parts.

$0.343 \pm 1.508j$

-1.67

$-0.51 \pm 0.7j$

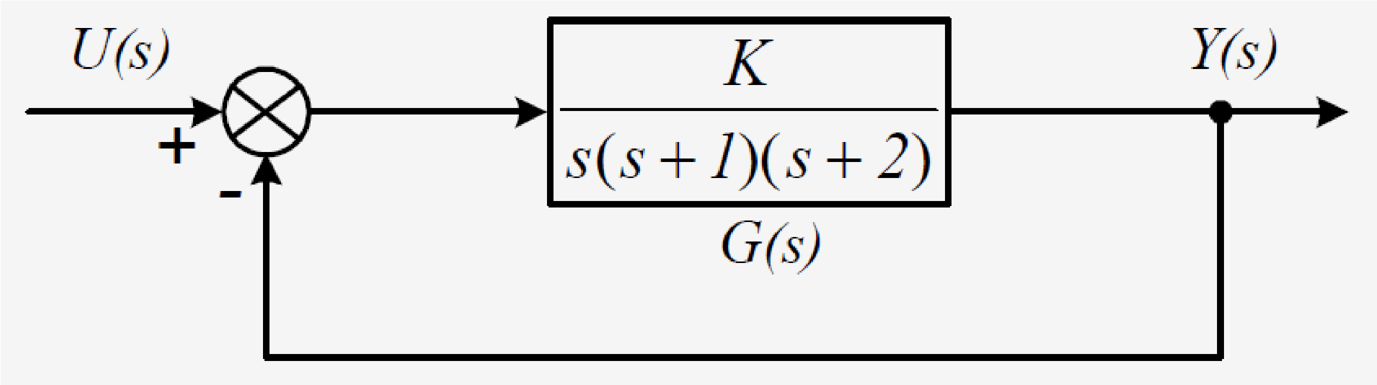
Unstable system

Routh Table: Special Case II

- If all the elements in a row are zero, the system has poles with positive or zero real parts.
  - The system is either unstable or critically stable.
  - Need other methods to check stability.

Routh-Hurwitz Stability Criterion

- Check the system stability.
- Find the number of poles with positive real part.
- An important application of Routh-Hurwitz stability criterion is to find critical values of certain parameters in a control system for ensuring stability.

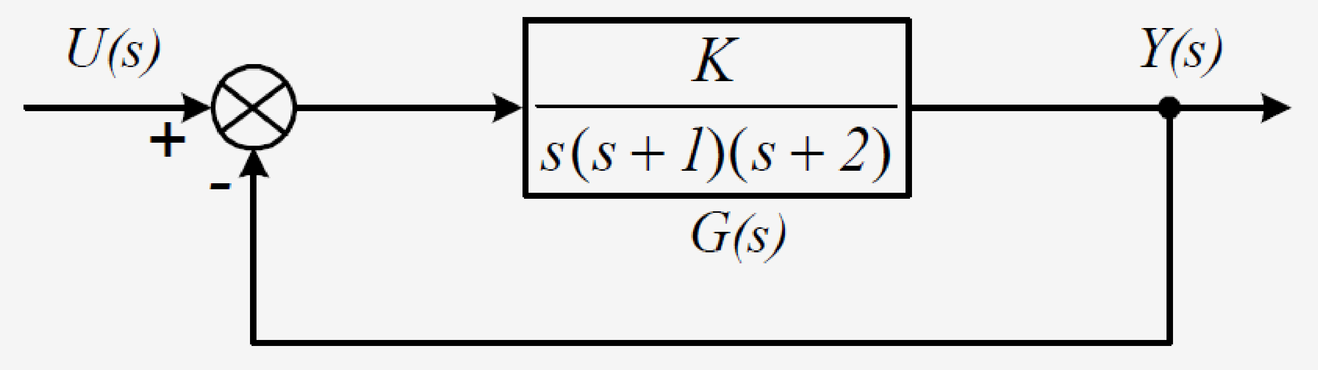


$K=4$, poles: $-0.102 \pm 1.192j, -2.796$, stable

$K=10$, poles: $0.155 \pm 1.732j, -3.309$, unstable

Example

- Find the range of K where the following system is stable.



Closed loop transfer function:

$$T(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)} = \frac{K}{s(s+1)(s+2) + K}$$

CE:

$$s(s+1)(s+2) + K = 0$$

$$or \quad s^3 + 3s^2 + 2s + K = 0$$

Example: Routh Table

$$CE: \quad s^3 + 3s^2 + 2s + K = 0$$

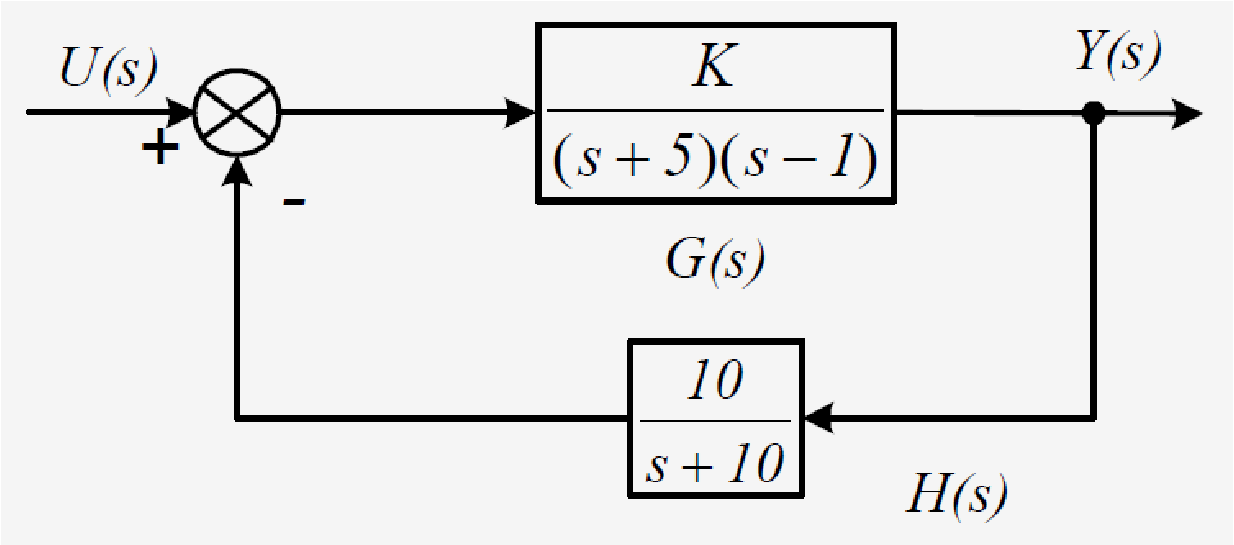
s^3	1	2	0
s^2	3	K	0
s^1	$\dfrac{-\begin{vmatrix} 1 & 2 \\ 3 & K \end{vmatrix}}{3} = \dfrac{6-K}{3} = a$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix}}{3} = 0$	0
s^0	$\dfrac{-\begin{vmatrix} 3 & K \\ a & 0 \end{vmatrix}}{a} = K$	0	0

Stability Condition:

$$\left.\begin{aligned} \frac{6-K}{3} &> 0 \\ K &> 0 \end{aligned}\right\} \Rightarrow 0 < K < 6$$

Example

- For what values of gain K, if any, is the system show below stable?



Closed loop transfer function:

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K(s+10)}{s^3+14s^2+35s+(10K-50)}$$

CE: $s^3+14s^2+35s+(10K-50)=0$

Routh Table

$$CE: \quad s^3 + 14s^2 + 35s + (10K - 50) = 0$$

s^3	1	35	0
s^2	14	$10K-50$	0
s^1	$\dfrac{14 \times 35 - (10K - 50)}{14}$	0	0
s^0	$10K-50$	0	0

The system is stable if and only if:

$$\left.\begin{aligned} \frac{14 \times 35 - (10K - 50)}{14} > 0 \\ 10K - 50 > 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 490 > 10K - 50 \\ 10K > 50 \end{aligned}\right\} \Rightarrow 5 < K < 54$$