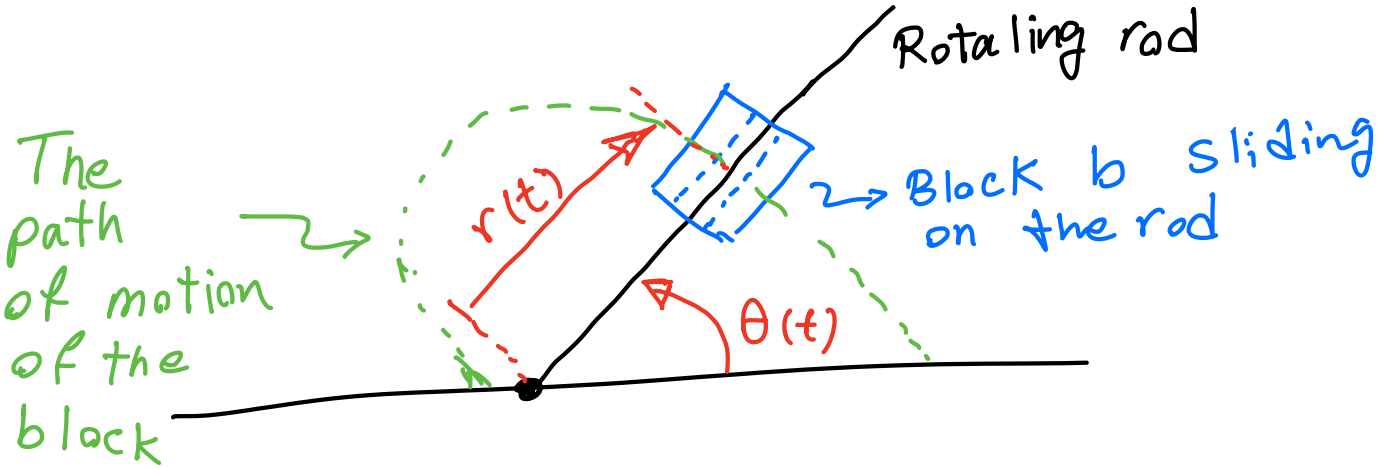
Example

Find the velocity and acceleration of block b as a function of time.



Given by $r(t) = r_0 (1 + \cos\theta)$

r_0: Constant

Velocity $$V = \frac{dr(t)}{dt} = r_0 \frac{d(1+\cos\theta)}{dt}$$

$$V = r_0 \frac{d\cos\theta}{dt} = r_0 \frac{d\cos\theta}{d\theta}\frac{d\theta}{dt}$$

$$V = r_0 (-\sin\theta)\,\dot{\theta}$$

Acceleration: $$a = \frac{dV}{dt}$$

$$a = -r_0 \frac{d(\sin\theta)\dot{\theta}}{dt}$$

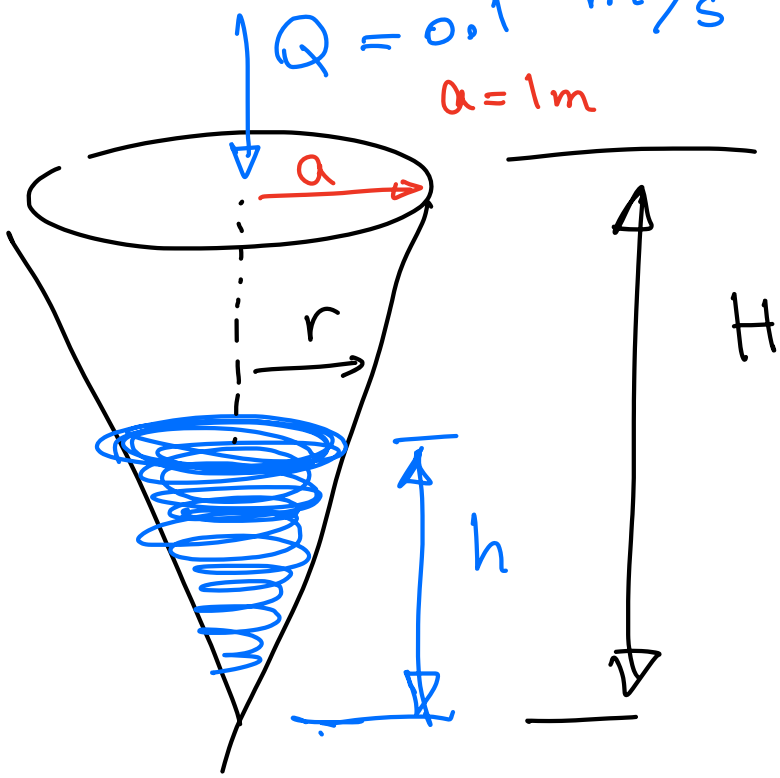
$$a = -r_0 \left[\frac{d(\sin\theta)}{dt}\dot{\theta} + \sin\theta \frac{d\dot{\theta}}{dt} \right]$$

$$a = -r_0 \left[\frac{d\sin\theta}{d\theta} \underbrace{\frac{d\theta}{dt}}_{\dot{\theta}} \dot{\theta} + (\sin\theta)\ddot{\theta} \right]$$

$$a = -r_0 \left[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta} \right]$$

Example

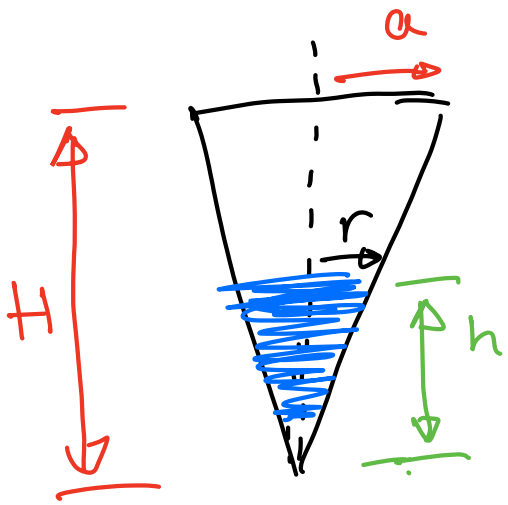
Find the rate at which h rises: $\left(\frac{dh}{dt} = ? \right)$



flow of the fluid (volumetic flow rate)

Volume

$$V = \frac{1}{3}\pi r^2 h$$



similar triangles:

$$\frac{H}{a} = \frac{h}{r}$$

substitute $r = \frac{ah}{H}$ into the Volume equation, we have

$$V = \frac{1}{3}\pi \left(\frac{ah}{H}\right)^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{a}{H}\right)^2 h^3$$

looking for $\frac{dh}{dt}$

Take the derivative of both sides with respect to time t

$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi \frac{a^2}{H^2} \frac{dh^3}{dt}$$

$$\frac{dh^3}{dt} = \frac{dh^3}{dh}\frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi a^2}{3H^2} \, 3h^2 \frac{dh}{dt}$$

$\frac{dV}{dt}$

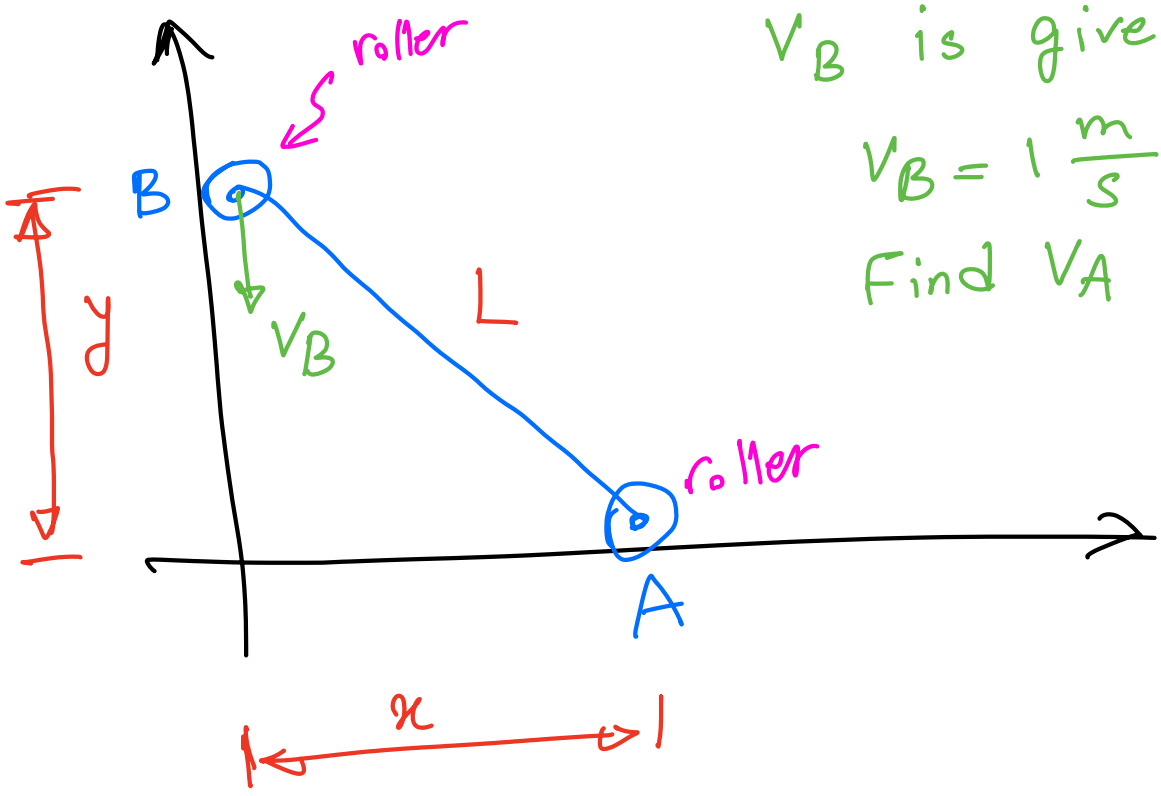
$$Q = 0.1 \frac{m^3}{s}$$

$$\Rightarrow \frac{dh}{dt} = \frac{(0.1)H^2}{\pi a^2 h^2}$$

$$\Rightarrow \int_0^h h^2 \, dh = \int_0^t \frac{0.1 H^2}{\pi a^2} dt$$

$$\frac{h^3}{3} = \frac{0.1 H^2}{\pi a^2} t$$

Example



V_B is given

$V_B = 1 \frac{m}{s}$

Find V_A.

$$x^2 + y^2 = L^2$$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt} L^2$$

$$\frac{d}{dt} x^2 + \frac{d}{dt} y^2 = 0$$

$$\frac{d}{dx} x^2 \underbrace{\frac{dx}{dt}} + \frac{d}{dy} y^2 \frac{dy}{dt} = 0$$

Velocity in x ↓

Velocity in y ↓

$$V_B = V_y = 1 \frac{m}{s}$$

$$2x V_x + 2y V_y = 0$$

$$V_x = -\frac{y V_y}{x} = -\frac{y}{x}$$

$$\frac{y}{x} = \tan\theta$$