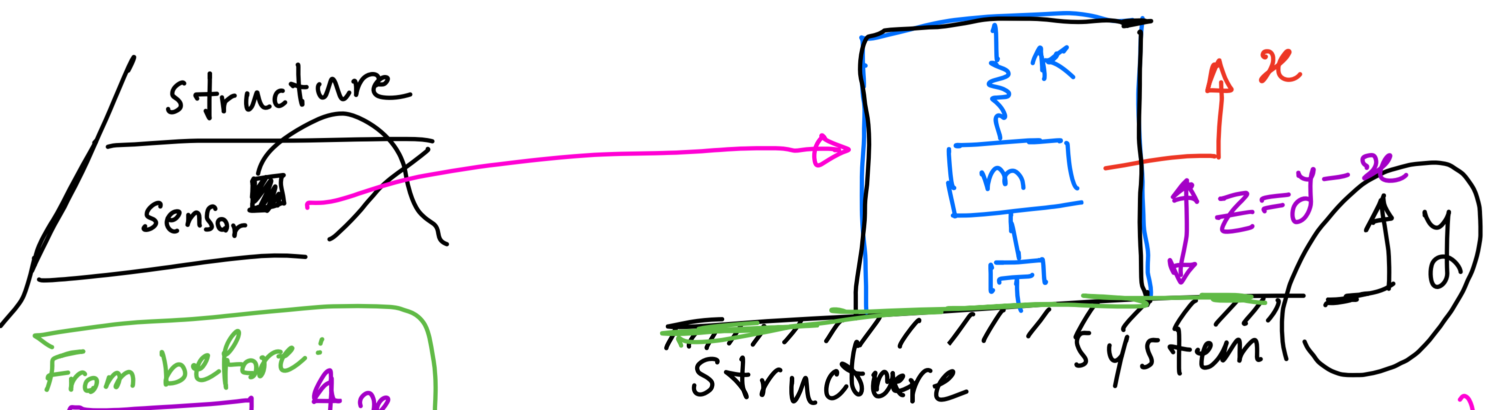
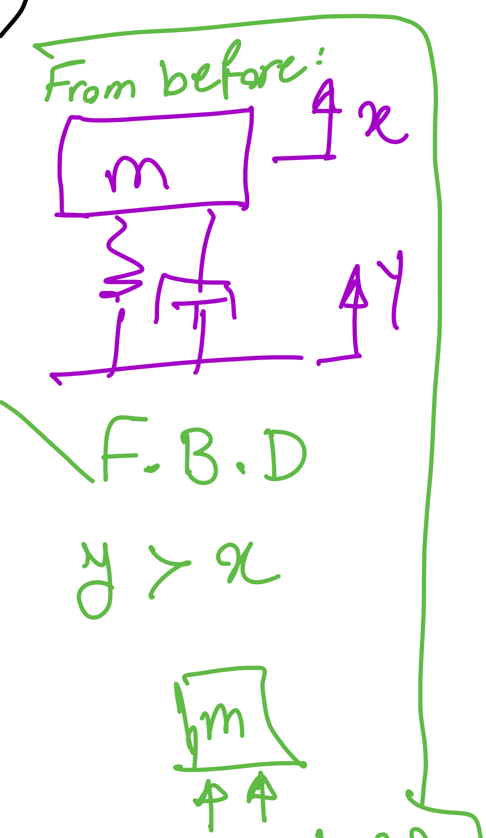
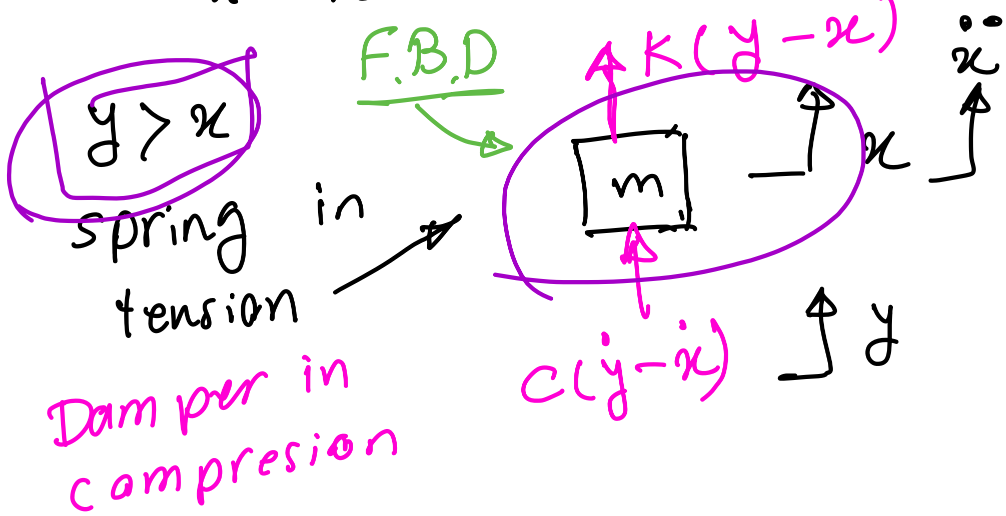
Vibration-measuring instruments

Section 3.11 Book

The basic element of many vibration measuring instruments is the seismic unit.







$K(y-x)$ $c(\dot{y}-\dot{x})$

The determine the behaviour of such instruments, The equation of motion of m is :

$z = y - x \rightarrow x = y - z$

$$\sum F = m\ddot{x}$$

$$m\ddot{x} = K(y-x) + c(\dot{y}-\dot{x})$$

$$m(\ddot{y}-\ddot{z}) = Kz + c\dot{z}$$

$$\boxed{m\ddot{z} + c\dot{z} + Kz = m\ddot{y}}$$

If assuming sinusoidal motion

$y = Y\sin\omega t \rightarrow Ye^{i\omega t} \rightarrow \ddot{y} = -\omega^2 Ye^{i\omega t}$

$$z = Ze^{i(\omega t-\phi)}$$

$$-mZ\omega^2 e^{i(\omega t-\phi)} + ciZ\omega e^{i(\omega t-\phi)} + KZe^{i(\omega t-\phi)} = mYe^{i\omega t}$$

$\bar{Z} = Ze^{-i\phi}$

$$\underbrace{-m\bar{z}\omega^2 + K\bar{z}}_{\text{Real}} + i\,\underbrace{c\bar{z}\omega}_{\text{Imag.}} = -mY\omega^2$$

$$\bar{z} = \frac{-m\omega^2 Y}{(-m\omega^2 + K) + ic\omega}$$

$$\omega_n^2 = \frac{K}{m}$$

$$|\bar{z}| = \frac{m\omega^2 Y}{\sqrt{(-m\omega^2+K)^2 + (c\omega)^2}}$$

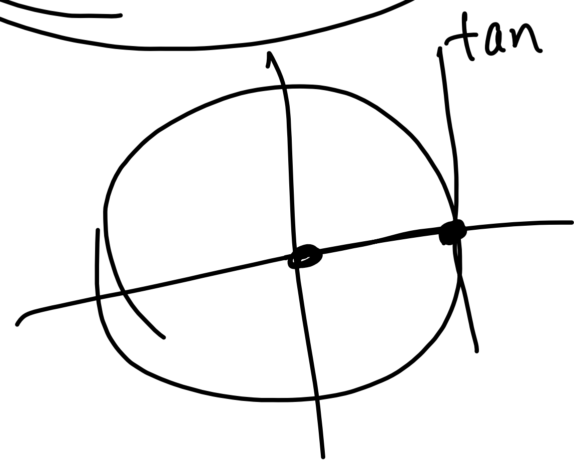
$$= \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\phi = \angle\underbrace{\left[Y\left(\frac{\omega}{\omega_n}\right)^2\right]}_{\text{Real}} - \angle\left[\underbrace{(-m\omega^2+K)}_{\text{Real}} + i\,\underbrace{c\omega}_{\text{Imag.}}\right]$$

$$\text{time delay} = \frac{T \text{ (period)}}{2\pi}\,\Phi$$

$$\text{Real} \rightarrow a + ib \leftarrow \text{Imag}$$

$$\Phi = \tan^{-1}\frac{b}{a}$$



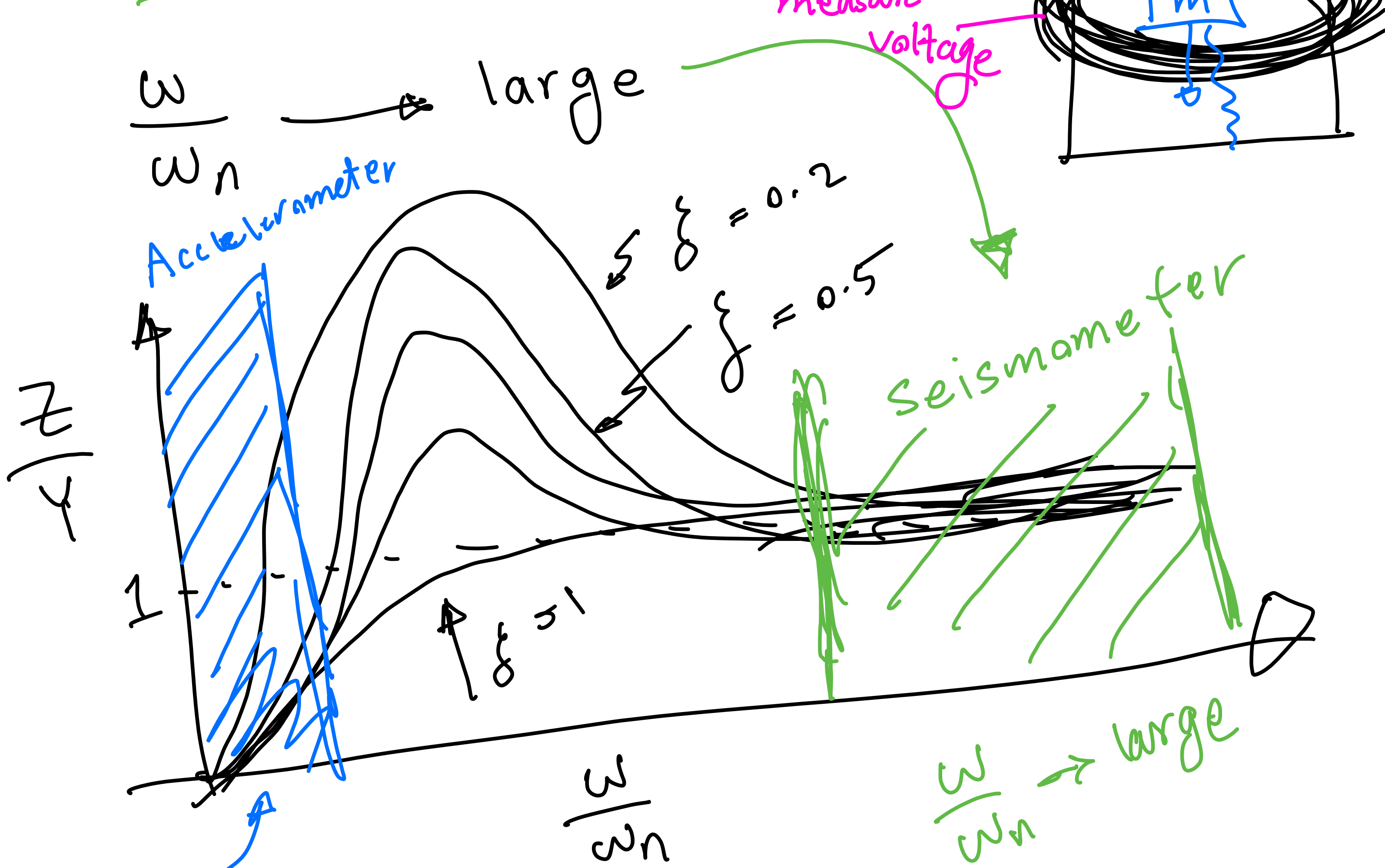
$$\Phi = \tan^{-1}\frac{0}{Y\left(\frac{w}{w_n}\right)^2}$$

$$-\tan^{-1}\frac{cw}{-mw^2+K}$$

$$\Phi = -\tan^{-1}\frac{cw}{K-mw^2} = \tan^{-1}\frac{2\xi\frac{w}{w_n}}{1-\left(\frac{w}{w_n}\right)^2}$$

Seismometer (Instrument with low natural frequency)

when the w_n of the instrument is low compare with the input (excitation) frequency



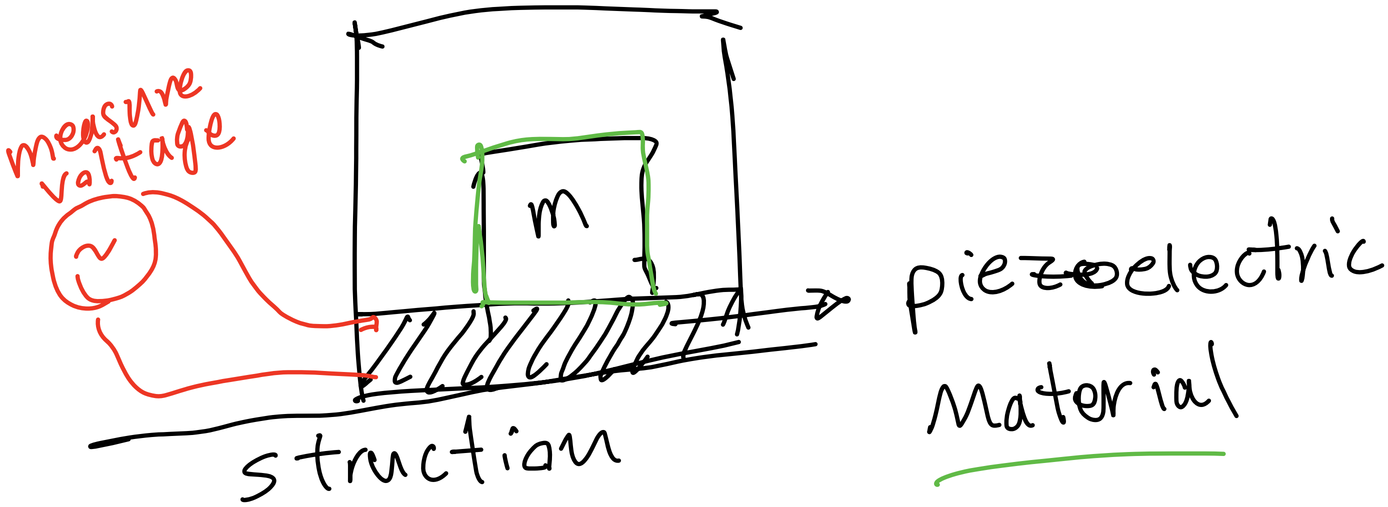
Accelerometer: instrument with high natural frequency:

$$\omega_n \gg \omega$$

$\frac{\omega}{\omega_n} \to$ small

$\omega_n = \sqrt{\frac{K}{m}}$ if K is large

ω_n is large



chapter 4 Transient Vibration

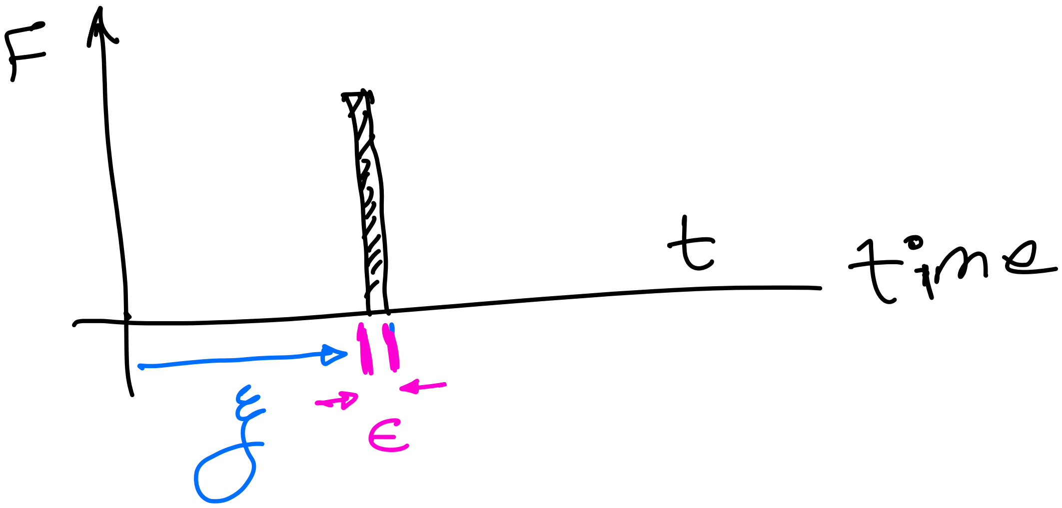
Impulse Excitation

section 4.1

Impulse is the time interal of the force, designate it by notation $\hat{F}$

$$\hat{F} = \int F(t)\,dt$$

very large magnitude of force acting for a very short time.



A delta function at $t = \xi$ is identified by the symbol $\delta(t-\xi)$ and has the following properties:

$\delta(t-\xi) = 0$ for all $t \neq \xi$

for $t = \xi$

$$\int_0^\infty \delta(t-\xi)\, dt = 1$$

$$\hat{F} = \int f(t)\, dt$$

$$f(\xi) = \int_0^\infty f(t)\, \delta(t-\xi)\, dt$$

$$F\, dt = m\, dv$$

$$F = m \frac{dv}{dt}$$

$$\underbrace{F\, dt}_{\hat{F}} = m\, dv$$

the impulse $\hat{F}$ acting on the mass will result in a sudden change in its velocity equal to $\dfrac{\hat{F}}{m}$, without any initial change in its displacement under free vibration, we found

$$x = \frac{\dot{x}(0)}{\omega_n} \sin \omega_n t + x(0) \cos \omega_n t$$

solution of free vibration in chapter 2

The response of a spring-mass system initially at rest and excited by an impulse $\hat{F}$

$$x = \frac{\hat{F}}{m\omega_n} \sin \omega_n t = \hat{F}\, h(t)$$

$$h(t) = \frac{1}{m\omega_n} \sin \omega_n t \rightarrow$$ response to a unit impulse $(\hat{F} = 1)$

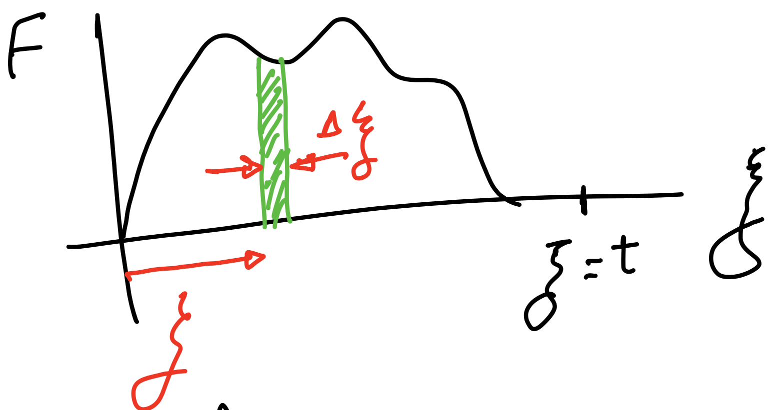
when damping is present (Eq. 2.6-16) with $x(0) = 0$

$$x = \frac{\dot{x}(0)\, e^{\xi \omega_n t}}{\omega_n \sqrt{1-\xi^2}} \sin \sqrt{1-\xi^2}\, \omega_n t$$

substituting for the initial condition $\dot{x}(0) = \frac{\hat{F}}{m}$

$$x = \frac{\hat{F}}{m \omega_n \sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \sqrt{1-\xi^2}\, \omega_n t$$

Arbitrary excitation (4.2 Book)



$$\hat{F} = f(\xi)\, \Delta \xi$$

$$f(\xi)\, \Delta \xi\, h(t-\xi)$$

$$x(t) = \int_0^t f(\xi)\, h(t-\xi)\, d\xi$$

Example 4.2-1

Next week