Section 3.8 Book

The primary influence of damping on oscillatory systems is that of limiting the amplitude of response at resonance.

In the case of viscous damping The amplitude at resonance is

$$X = \frac{F_0}{c\omega_n}$$

For other types of damping, no such simple expression exists.

Instead we can use an equivalent form of viscous damping (called c_{eq})

Using the dissipated energ in the system and equating it with the dissipated energy by viscous damping we have:

$$W_d = \pi c_{eq} \omega X^2$$

Then W_d must be evaluated for the system.

<u>Example</u> (3.8-1)

Find the equivalent damping for resistive force due to drag acting on an oscillatory system. Assume the damping force:

$$F_d = \pm a \dot{x}^2 \quad (\text{speed } 3 \text{ to } 20 \tfrac{m}{s})$$

Assume harmonic motion:

$$x = -X \cos \omega t$$

$$\dot{x} = X\omega \sin \omega t \rightarrow \frac{dx}{dt} = X\omega \sin\omega t$$

$$dx = (X\omega \sin \omega t)\, dt$$

The dissipated energy in each cycle:

$$W_d = 2\int_{-X}^{X} a\dot{x}^2\, dx$$

$$= 2a\omega^2 X^3 \int_0^{\pi} \sin^3 \omega t \; d(\omega t)$$

$$= \frac{8}{3} a\omega^2 X^3$$

$$W_d = \pi\, c_{eq}\, \omega X^2$$

$$\frac{8}{3} a\omega^2 X^3 = \pi\, c_{eq}\, \omega X^2$$

$$c_{eq} = \frac{8}{3\pi} a\omega X$$

The amplitude at resonance is found by substituting $c = c_{eq}$ in Eq (3.8-1) Book with $\omega = \omega_n$

$$X = \sqrt{\frac{3\pi F_0}{8 a \omega_n^2}}$$

Example (3.8-2)

Find the equivalent damping for coulomb damping.

Assume the motion as $x = X \sin \omega t$

work done per cycle by the coulomb force F_d is equal to

$$W_d = F_d \, (4X)$$

Substitute:

$$W_d = \pi C_{eq} \omega X^2$$

$$4 F_d X = \pi C_{eq} \omega X^2$$

$$C_{eq} = \frac{4 F_d}{\pi \omega X}$$

The amplitude of forced vibration can be found by substituting C_{eq} into Eq. (3.1-3) Book

$$X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + \left(\frac{4 F_d \omega}{\pi \omega X}\right)}}$$

solving for X:

$$|x| = \frac{\sqrt{F_o^2 - \left(\frac{4F_d}{\pi}\right)^2}}{K - m\omega^2}$$

$$= \frac{F_o}{K} \frac{\sqrt{1 - \left(\frac{4F_d}{\pi F_o}\right)^2}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$\omega = \omega_n \rightarrow |x| \rightarrow \infty$ Not realistic

$4F_d / \pi F_o < 1$

(3.9) structural Damping

In structures, for example metals such as steel or aluminum. the energy is dissipated proportional to the amplitude of vibration.

Energ dissipated by structural damping can be written as:

$$W_d = \alpha X^2$$

where α is a constant.

using the equivalent damping:

$$\pi C_{eq} \omega X^2 = \alpha X^2$$

$$C_{eq} = \frac{\alpha}{\pi \omega}$$

viscous damping, the equation of motion is $m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$

For structural damping, the equation

is:

$$m\ddot{x} + \left(\frac{\alpha}{\pi w}\right)\dot{x} + Kx = F_o \sin wt$$

Complex stiffness.

In case of flutter, speeds of airplane wings and tail surfaces, the concept of complex stiffness is used.

For harmonic oscillations; the equation of motion can be written by:

$$m\ddot{x} + \left(K - i\frac{\alpha}{\pi}\right)x = F_o e^{iwt}$$

stiffness

If we use $\gamma = \alpha/\pi K$

$$m\ddot{x} + \underbrace{K(1 + i\gamma)}_{\text{Complex stiffness}} x = F_0 e^{i\omega t}$$

γ is the structural damping factor.

Assume the solution as

$$x = \bar{X} e^{i\omega t}$$

the steady-state amplitude from Eq. (3.9-4) becomes

$$\bar{X} = \frac{F_0}{(K - m\omega^2) + i\gamma K}$$

The amplitud at resonance is $(\omega = \omega_n)$

$$|X| = F_0 / \gamma K$$

Remember for viscous damping

Case: $|X| = \frac{F_0}{2\zeta K}$

Aero elasticity very theoritical

too many coefficient

similar to γ.

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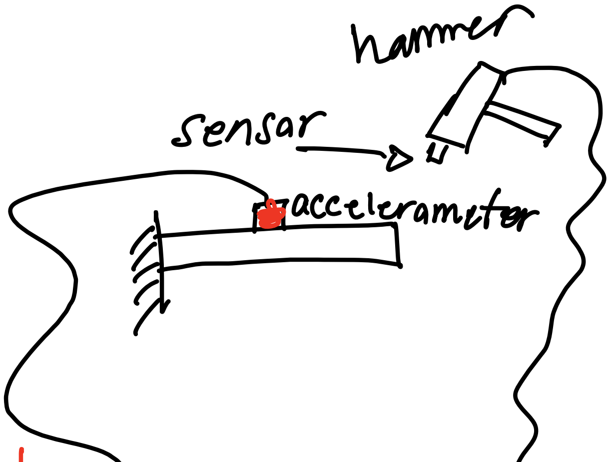
sharpness of resonance

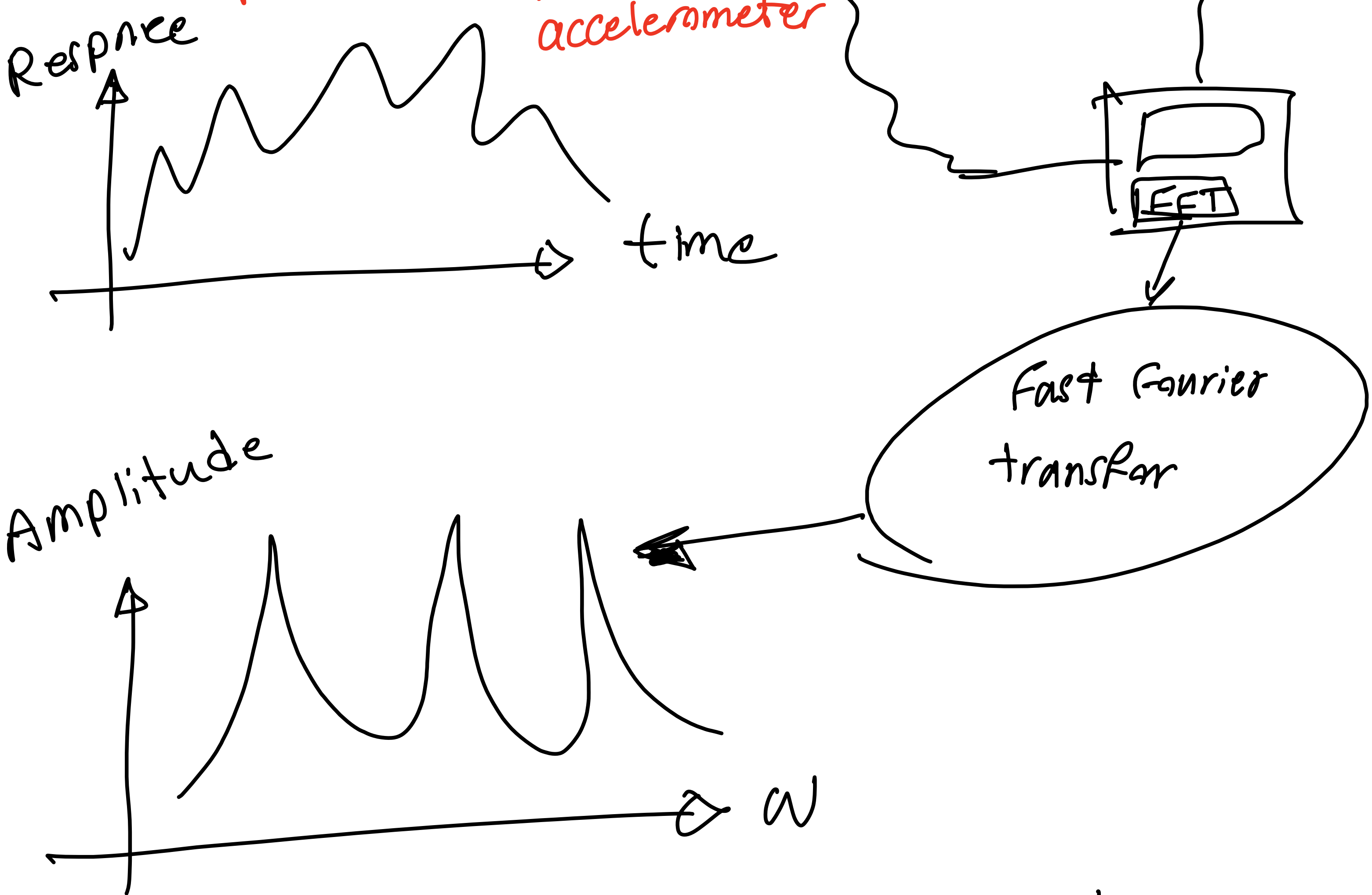
In force vibration :

using an impact using an instrumented hammer

or a shaker

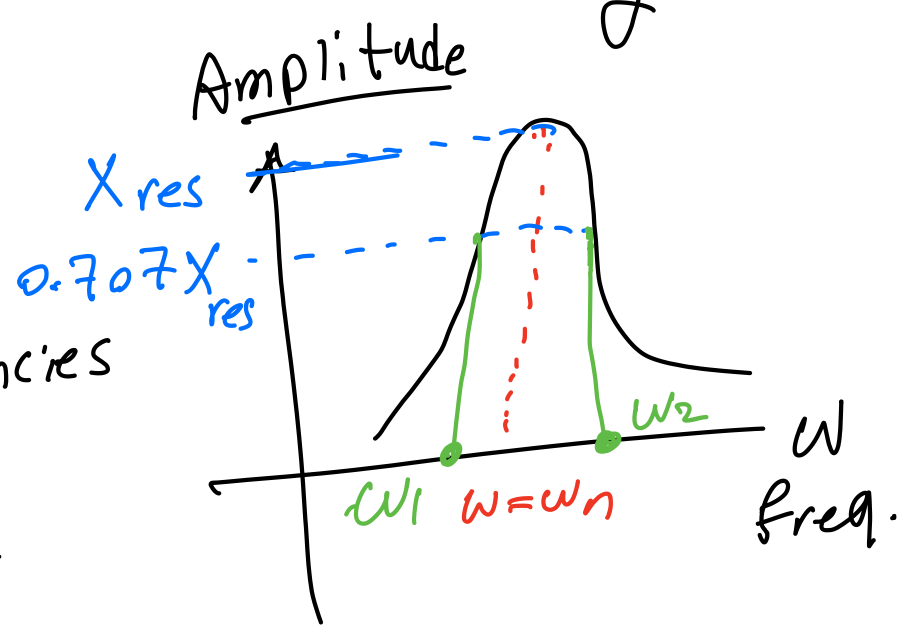


The response of the



when $\omega = \omega_n$ the resonant amplitude is $x_{res} = \dfrac{(F_0/k)}{2\zeta}$

we now seek the two frequencies on either side of resonance.



often called side bands

where $X = 0.707\, X_{res}$

substitute in equation 3.1-7

and saure both sides

$$\frac{1}{2}\left(\frac{1}{2\zeta}\right)^2 = \frac{1}{\left[1-\left(\frac{w}{w_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{w}{w_n}\right)\right]^2}$$

$$\left(\frac{w}{w_n}\right)^2 - 2(1-\zeta^2)\left(\frac{w}{w_n}\right)^2 - (1-8\zeta^2) = 0$$

solve for $\left(\frac{w}{w_n}\right)^2$

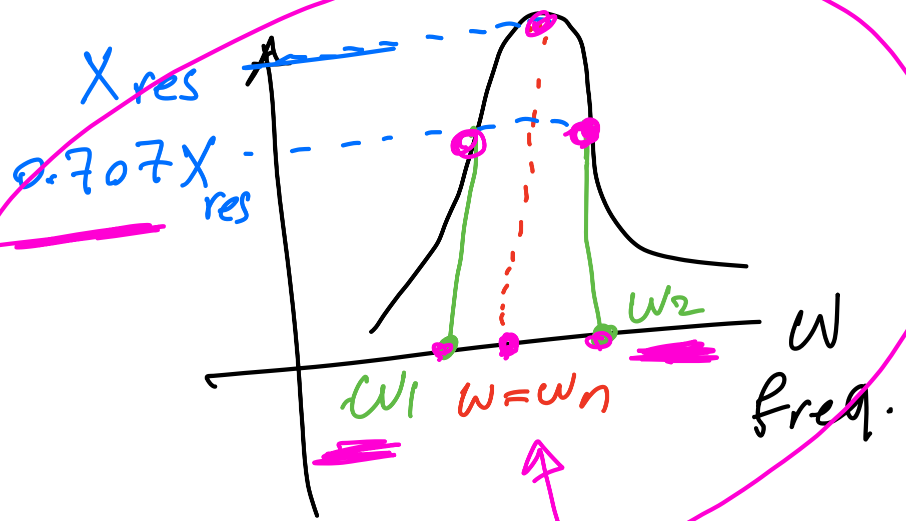
$$\left(\frac{w}{w_n}\right)^2 = (1-2\zeta^2) \pm 2\zeta\sqrt{1-\zeta^2}$$

Assuming $\zeta \ll 1$ and therfore

structural

$$1 \gg (0.05)^2$$ ← 5% Damping

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1 \pm 2\zeta$$ ← (3.10-3) Car damper 30%



Letting the two frequencies corresponding to the roots of Eq. (3.10.3) be ω_1 and ω_2

$$\omega_2 + \omega_1 = 2\omega_n$$

$$\omega_n = \frac{\omega_1 + \omega_2}{2}$$

$$4\zeta = \frac{\omega_2^2 - \omega_1^2}{\omega_n^2} = \frac{(\omega_2 - \omega_1)(\omega_2 + \omega_1)}{\omega_n^2}$$

$$4\zeta = 2\left(\frac{\omega_2 - \omega_1}{\omega_n}\right) \Rightarrow \zeta = \frac{1}{2}\left(\frac{\omega_2 - \omega_1}{\omega_n}\right)$$