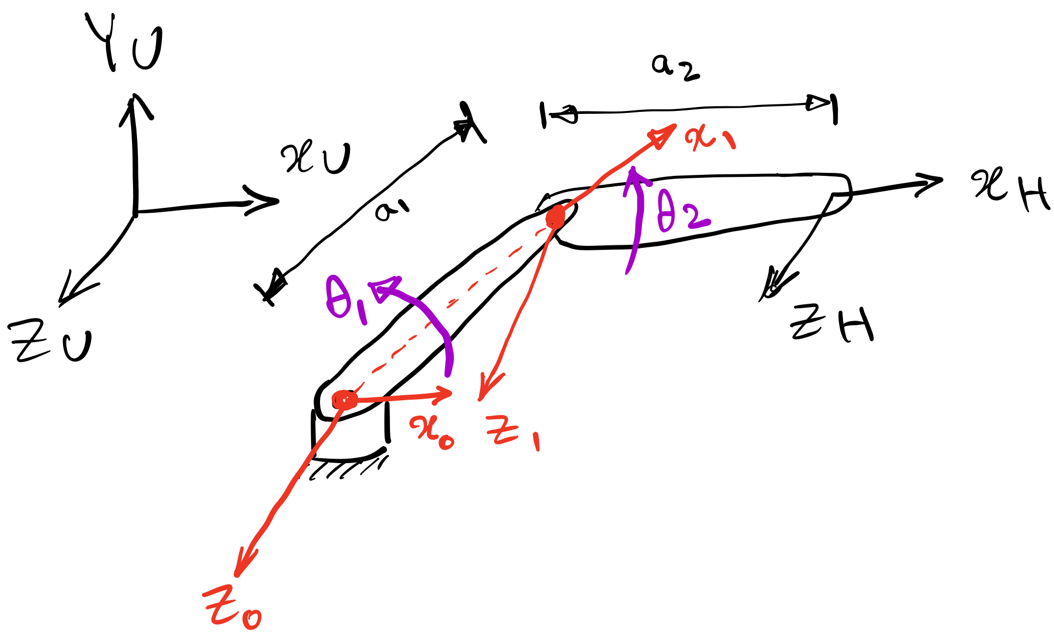
Example

For the simple 2-axis planar robot in the figure, assign the necessary coordinate systems based on the D-H representation, fill out the parameters table, and derive the forward kinematic equations for the robot.



D-H parameters Table

#	θ	d	a	α
	Rotate x_n to x_{n+1} about z_n by θ_{n+1}	Translate x_n to x_{n+1} along z_n by d_{n+1}	Translate z_n to z_{n+1} along x_{n+1} by a_{n+1}	Rotate z_n to z_{n+1} about x_{n+1} by α_{n+1}
0-1	Rotate x_0 to x_1 about z_0 θ_1	Translate x_0 to x_1 along z_0 0	Translate z_0 to z_1 along x_1 a_1	Rotate z_0 to z_1 about x_1 by 0
1-H	Rotate x_1 to x_H about z_1 θ_2	Translate x_1 to x_H along z_1 0	Translate z_1 to z_H along x_H a_2	Rotate z_1 to z_H about x_H by 0

substituting these parameters into the corresponding A matrices as follows:
(A matrix is equation (2.53) in the book)

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}^0T_H = A_1 \times A_2 =$ See the book for this result

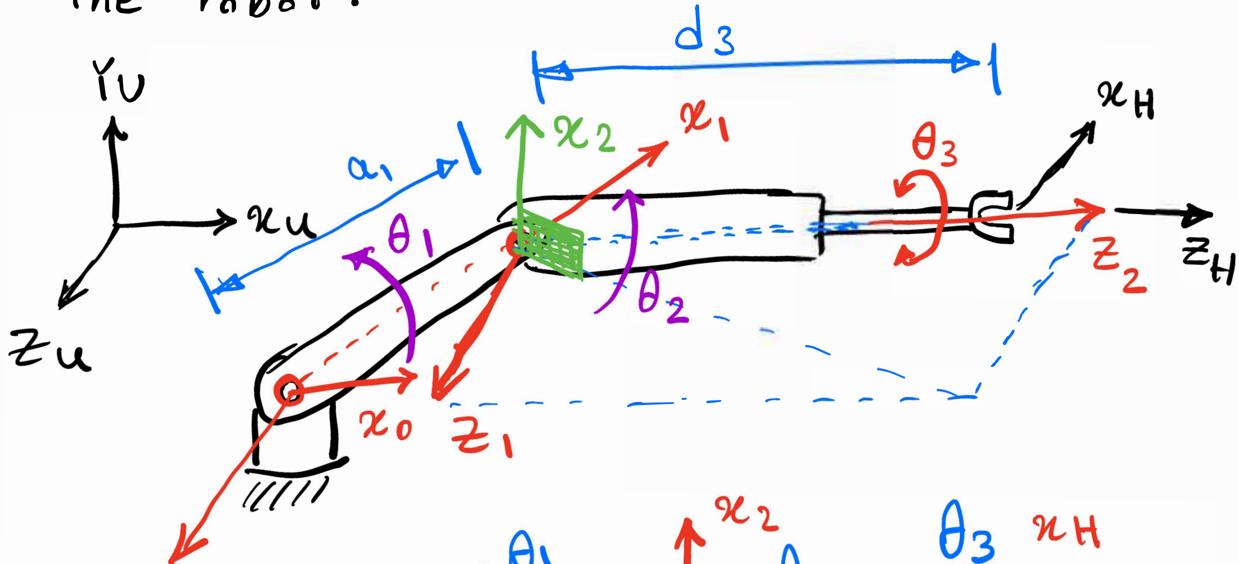
$${}^0T_H = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_2 c_{12} + a_1 c_1 \\ s_{12} & c_{12} & 0 & a_2 s_{12} + a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

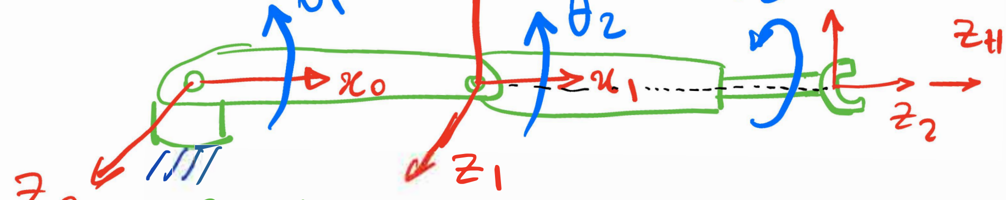
$$C_{12} = C(\theta_1 + \theta_2) = C_1 C_2 - S_1 S_2$$

$$S_{12} = S(\theta_1 + \theta_2) = S_1 C_2 + C_1 S_2$$

Example

Assign the necessary frames to the robot in the figure, and derive the forward kinematic equation of the robot.





Reset position (Home position)

D-H parameters Table

#	θ	d	a	α
	Rotate x_n to x_{n+1} about z_n by θ_{n+1}	Translate x_n to x_{n+1} along z_n by d_{n+1}	Translate z_n to z_{n+1} along x_{n+1} by a_{n+1}	Rotate z_n to z_{n+1} about x_{n+1} by α_{n+1}
0-1	Rotate x_0 to x_1 about z_0 θ_1	Translate x_0 to x_1 along z_0 0	Translate z_0 to z_1 along x_1 a_1	Rotate z_0 to z_1 about x_1 by 0
1-2	Rotate x_1 to x_2 about z_1 $90+\theta_2$	Translate x_1 to x_2 along z_1 0	Translate z_1 to z_2 along x_2 0	Rotate z_1 to z_2 about x_2 90
2-H	Rotate x_2 to x_H about z_2 θ_3	Translate x_2 to x_H along z_2 d_3	Translate z_2 to z_H along x_H 0	Rotate z_2 to z_H about x_H 0

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_H = A_1 A_2 A_3$$

= See page 80 for this result

Simplifying the matrix with

$c_1 c_2 - s_1 s_2 = c_{12}$

$s_1 c_2 + c_1 s_2 = s_{12}$

$$
{}^{0}T_H = \begin{pmatrix} -s_{12} c_3 & s_{12} s_3 & c_{12} & c_{12} d_3 + a_1 c_1 \\ c_{12} c_3 & -c_{12} s_3 & s_{12} & s_{12} d_3 + a_1 s_1 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$
