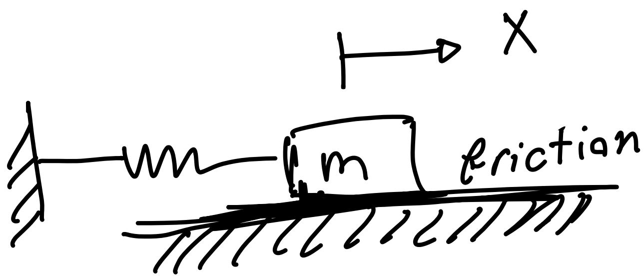
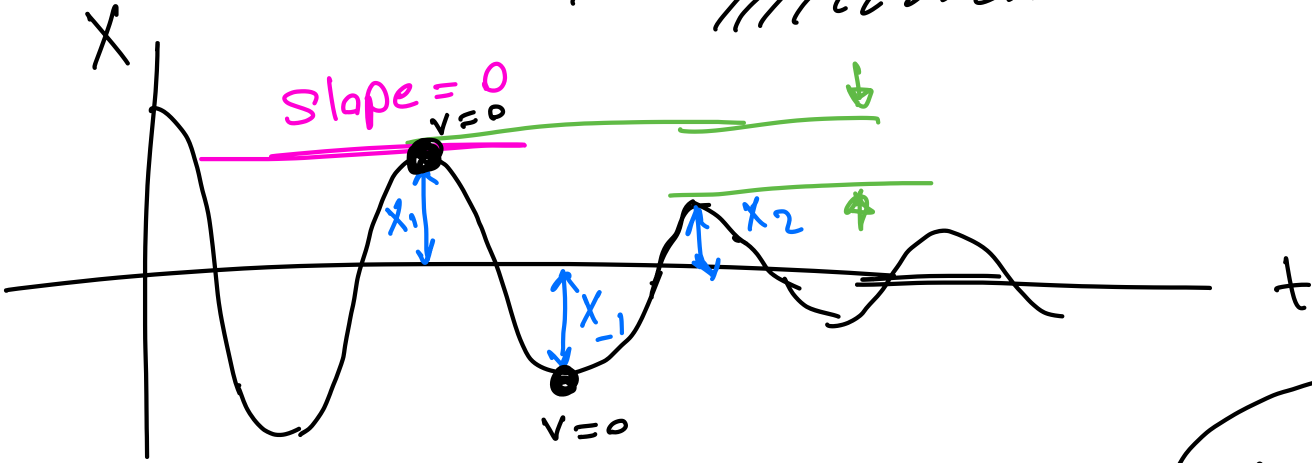
Topic left from chapter 2

2.8 coulomb damping





$$v = \frac{dx}{dt} = 0$$

Choose half-cycle starting at the extreme position with the velocity equal to zero and the amplitude equal to X_1.

The change in the kinetic energy is zero, Therefore

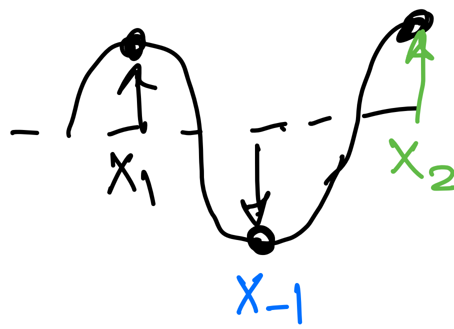
$$\frac{1}{2}k(x_1^2 - x_{-1}^2) - F_d(x_1 + x_{-1}) = 0$$

or

$$\frac{1}{2}k(x_1 - x_{-1}) = F_d$$

$$x_1 - x_{-1} = \frac{2F_d}{k}$$

half cycle



$$x_1 - x_2 = \frac{4F_d}{k}$$

full cycle from x_1 to x_2

Chapter 3

section 3.7 : Energy dissipated by damping

Dissipation can be due to heat or radiated away.

The energy lost per cycle due to a damping force F_d is computed from the general Equation:

$$W_d = \oint F_d \, dx$$

W_d depends on many factors, such as temperature, frequency, or amplitude.

we consider the simplest case here, which is mass-spring with viscous damping. Therefore, the damping force is:

$$F_d = c\dot{x}$$

Assume the motion as:

$$x = X \sin(\omega t - \phi)$$

$$\dot{x} = \omega X \cos(\omega t - \phi)$$

$$W_d = \oint c\dot{x}\, dx = \oint c \frac{dx}{dt} \frac{dx}{dt} dt$$

$$= \int c \dot{x}^2 dt = \int c \left[\omega X \cos(\omega t - \phi)\right]^2 dt$$

$$= c\omega^2 X^2 \int_0^{\frac{2\pi}{\omega}} \cos^2(\omega t - \phi)\, dt$$

$$= \pi c \omega X^2$$

$$\cos(\omega t - \phi) = u$$

$$-\sin(\omega t - \phi) dt = du$$

of particular interest is the

energy dissipated in forced vibration at resonance:

$$W_d = 2\zeta\pi K X^2$$

$$\boxed{\begin{array}{l} w_n = \sqrt{\frac{K}{m}} \\ c = 2\zeta\sqrt{Km} \end{array}}$$

The energy dissipated per cycle by the damping force can be presented by

$$\dot{x} = \omega X \cos(\omega t - \phi)$$

$$= \pm\omega X \sqrt{1 - \sin^2(\omega t - \phi)}$$

Initial assumption $x = X \sin(\omega t - \phi)$

$$\dot{x} = \pm\omega\sqrt{X^2 - X^2 \sin^2(\omega t + \phi)}$$

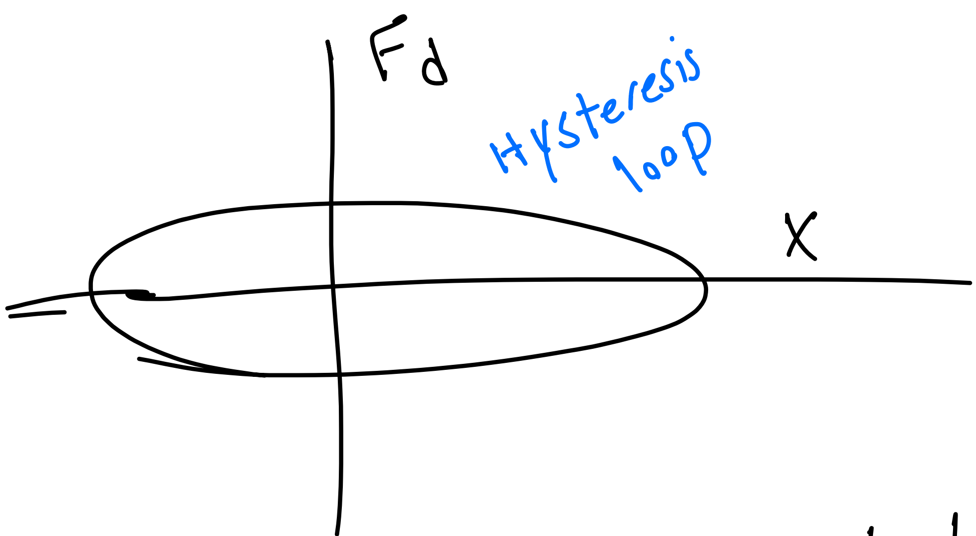
$$\dot{x} = \pm \omega \sqrt{X^2 - x^2}$$

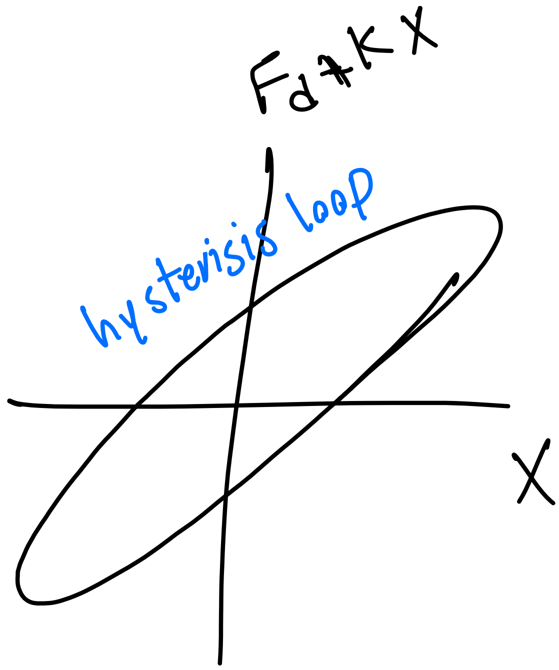
The damping force:

$$F_d = c\dot{x} = \pm c\omega\sqrt{X^2 - x^2}$$

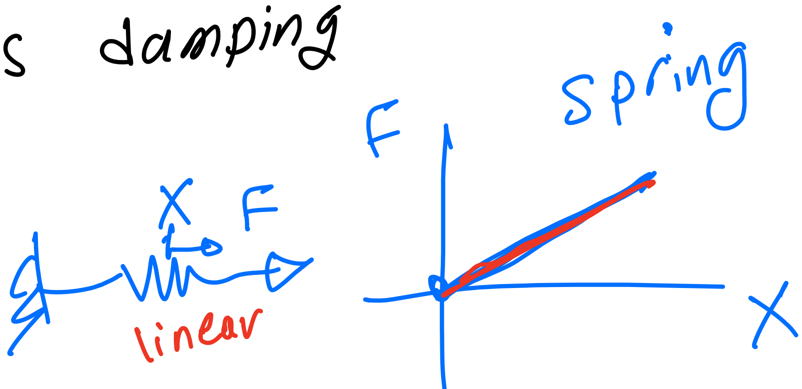
By rearranging:

$$\left(\frac{F_d}{c\omega X}\right)^2 + \left(\frac{x}{X}\right)^2 = 1$$





Energ dissipated by viscans damping



Specific damping capacity

$$\frac{W_d}{U}$$

W_d ← Energ loss per cycle

U ← peak potential energy

loss coefficient:

Defined as the ratio of damping energy loss per radian $\frac{W_d}{2\pi}$ divided by peak potential energy.

section 3.8

Equivalent viscons damping

The primary influence of damping on oscillatary systems is that of limiting the amplitude of