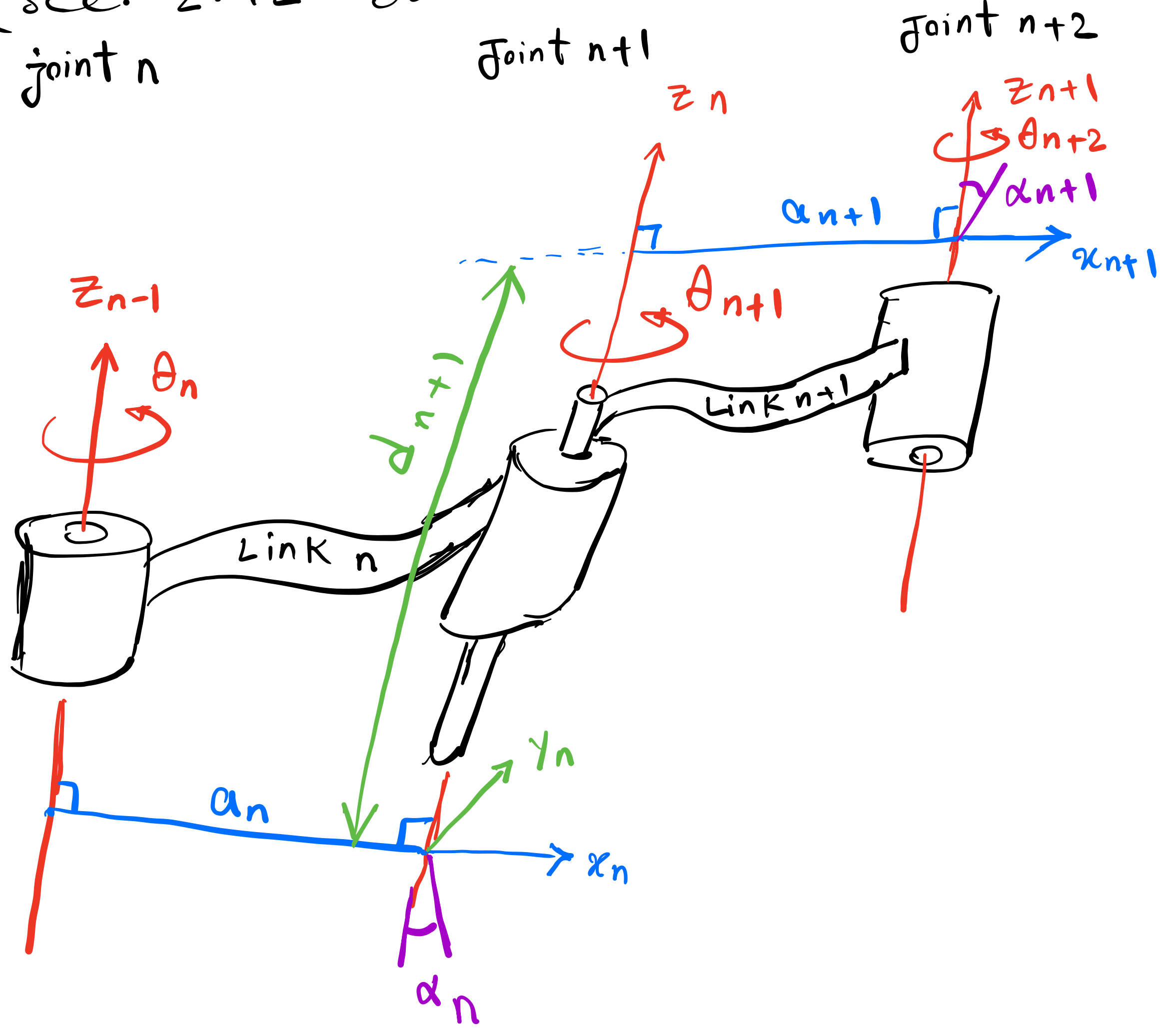
Denavit-Hartenberg (D-H) Representation of Forward Kinematic Equations of Robots

(see. 2.12 Book : Important)



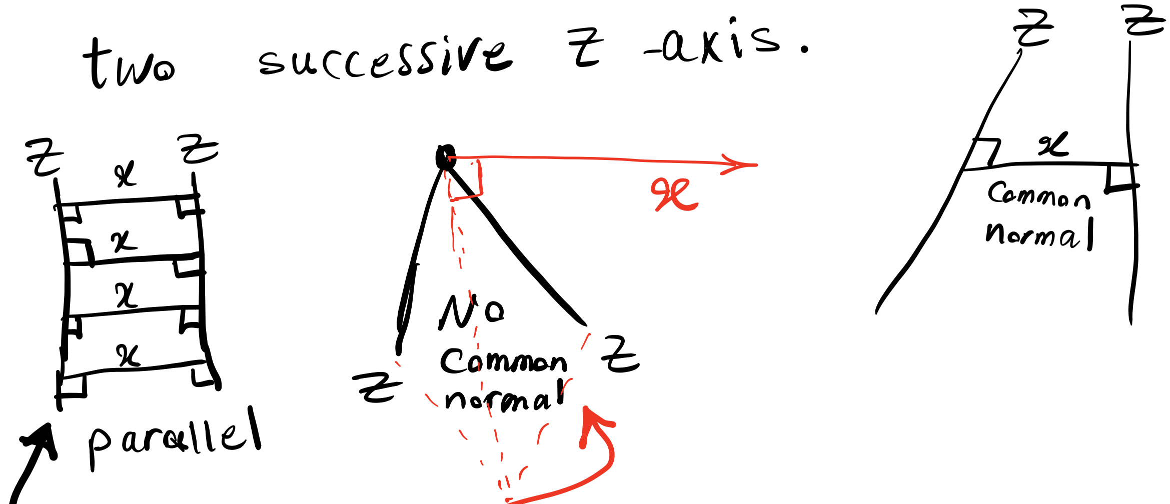
How to assign the z-axis:

If the joint is revolute, the z-axis is in the direction of rotation as followed by the right-hand rule for rotations.

If the joint is prismatic, the z-axis for the joint is along the direction of the linear movement.

How to assign the x-axis:

The x-axis is the common normal of two successive z-axis.



Exceptions:

- If two z-axes are parallel, there are an infinite number of common normals between them. We will pick the common normal that is colinear with the common normal of the previous joint.
- If the z-axes of two successive joints are intersecting, there is no

common normal between them. we will assign the x-axis along a line perpendicular to the plane formed by the two z-axes. (page 75 book)

How to find θ, d, α, a parameters:
(page 75 and 76)

1. Rotate about the z_n-axis an angle of θ_{n+1}. This will mak x_n and x_{n+1} parallel to each other.

2. Translate along the z_n-axis a distance of d_{n+1} to make x_n and x_{n+1} colinear.

3. Translate along x_n-axis a distance of a_{n+1} to bring the origins of x_n and x_{n+1} together.

4. Rotate z_n about x_{n+1} axis an angle of α_{n+1} to align z_n-axis with z_{n+1}-axis.

The transformation ${}^{n}T_{n+1}$ (called A_{n+1}) between two successive frames representing the preceding four movements is the product of the four matrices representing them.

$$
{}^{n}T_{n+1} = A_{n+1}
$$

$$
= Rot(z, \theta_{n+1}) \times Trans(0, 0, d_{n+1}) \times Trans(a_{n+1}, 0, 0) \times Rot(x, \alpha_{n+1})
$$

$$
= \begin{bmatrix} c\theta_{n+1} & -s\theta_{n+1} & 0 & 0 \\ s\theta_{n+1} & c\theta_{n+1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$\times \begin{bmatrix} 1 & 0 & 0 & a_{n+1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{n+1} & -s\alpha_{n+1} & 0 \\ 0 & s\alpha_{n+1} & c\alpha_{n+1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{n+1} = \begin{bmatrix} c\theta_{n+1} & -s\theta_{n+1}c\alpha_{n+1} & s\theta_{n+1}s\alpha_{n+1} & a_{n+1}c\theta_{n+1} \\ s\theta_{n+1} & c\theta_{n+1}c\alpha_{n+1} & -c\theta_{n+1}s\alpha_{n+1} & a_{n+1}s\theta_{n+1} \\ 0 & s\alpha_{n+1} & c\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation (2.53) Book

Let's assume (as an example) we have 6 joints $\longrightarrow$ 6 frames

Next step $\longrightarrow$ Build the D-H parameter Table.

D-H parametes Table (Example for 6 frames)

#	θ	d	a	α
0-1				
1-2				
2-3	θ_3	d_3	a_3	α_3
3-4				
4-5				
5-6				

As an example, the transformation between joints 2 and 3 (2-3 in the table) of a generic robot will simply be:

$$
{}^{2}T_3 = A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 c\alpha_3 & s\theta_3 s\alpha_3 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 c\alpha_3 & -c\theta_3 s\alpha_3 & a_3 s\theta_3 \\ 0 & s\alpha_3 & c\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The total transformation between the base of the robot and the hand will be:

$$^{R}T_{H} = {}^{R}T_{1}\,{}^{1}T_{2}\,{}^{2}T_{3}\cdots{}^{n-1}T_{n} = A_1 A_2 A_3 \cdots A_n$$

For a 6-DOF robot, there will be six A matrices.

Study the examples in the D-H section for the next Lecture.