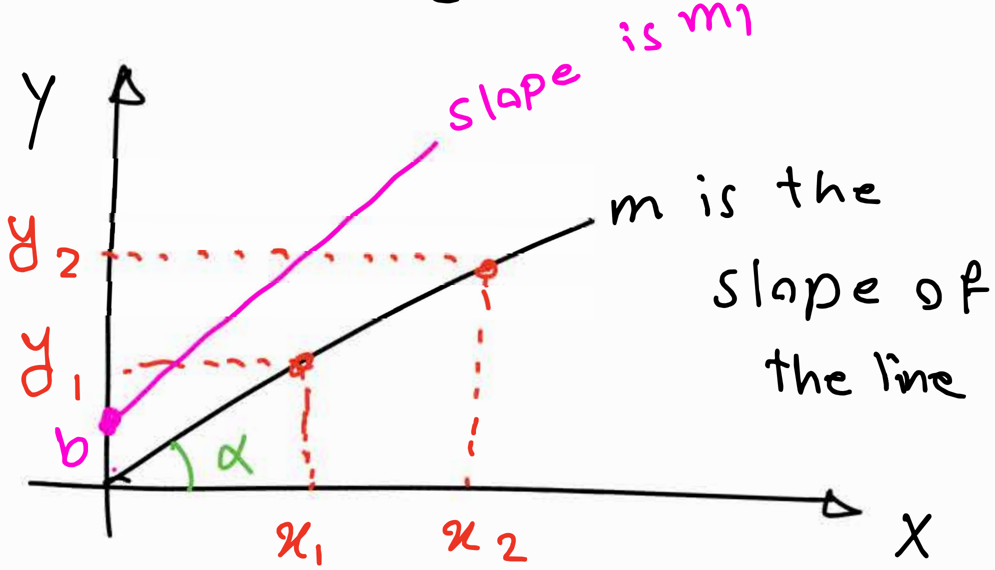
Equation of a line



$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\alpha$$

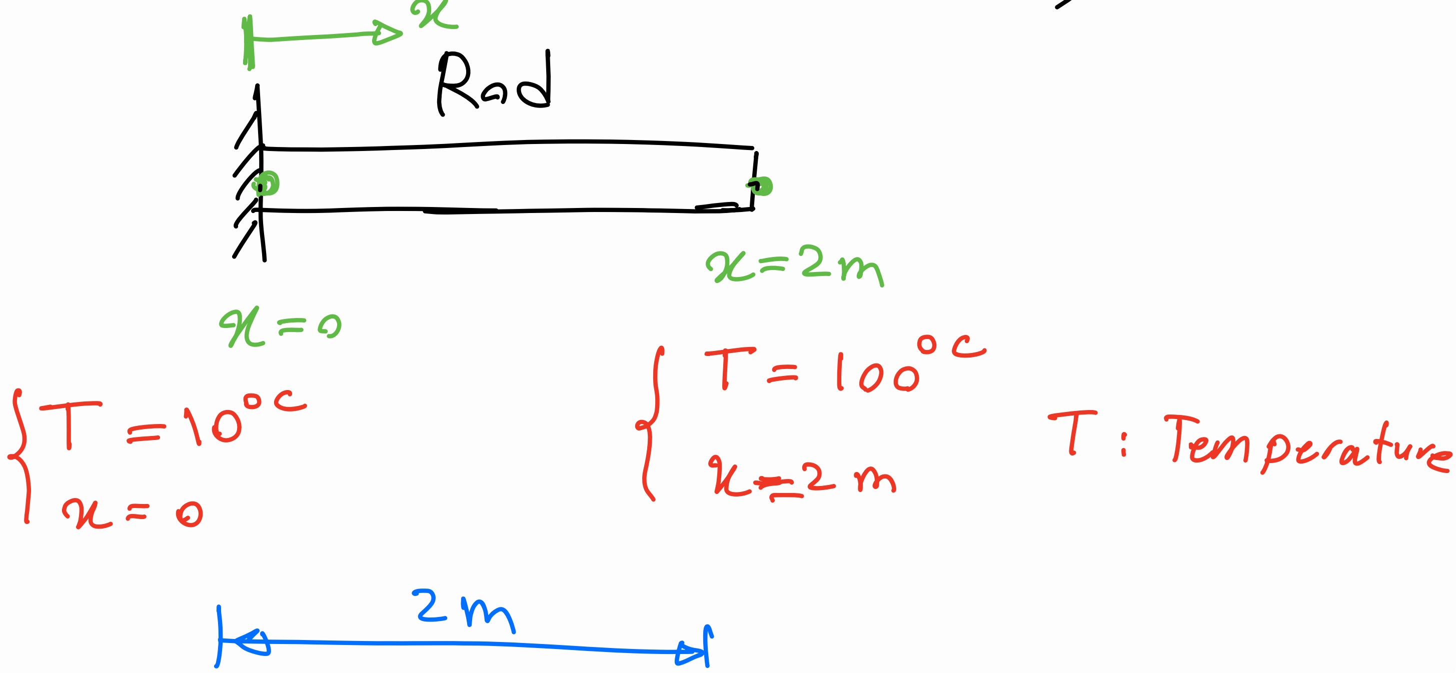
$$y = m_1 x + b$$

$y = ?$ when $x = 0$

$y = b$

Example

Heat distribution in a rod is given by a linear equation (because temperature varies linearly along the rod)



Find the equation for temperature as a function of x.

$$T(x) = ?$$

$$T(x) = Ax + B$$

A and B are constants to be found.

$$\begin{cases} T = 10^\circ c \\ x = 0 \end{cases} \xRightarrow{\text{substitute}} 10 = A(0) + B$$

$$B = 10$$

$$\begin{cases} T = 100^\circ c \\ x = 2\,m \end{cases} \xRightarrow{\text{substitute}} 100 = A(2) + 10$$

$$A = 45$$

$$\Rightarrow \quad T(x) = 45x + 10$$

Find the temperature at 2/3 of the rod.

$$T\left(\frac{2}{3}\,2\right) = 45\left(\frac{4}{3}\right) + 10 = 70^\circ c$$

Example

Equation of a line for displacement of a spring.

x_0 is the position of the spring before any force is applied or the unstretched length of the spring

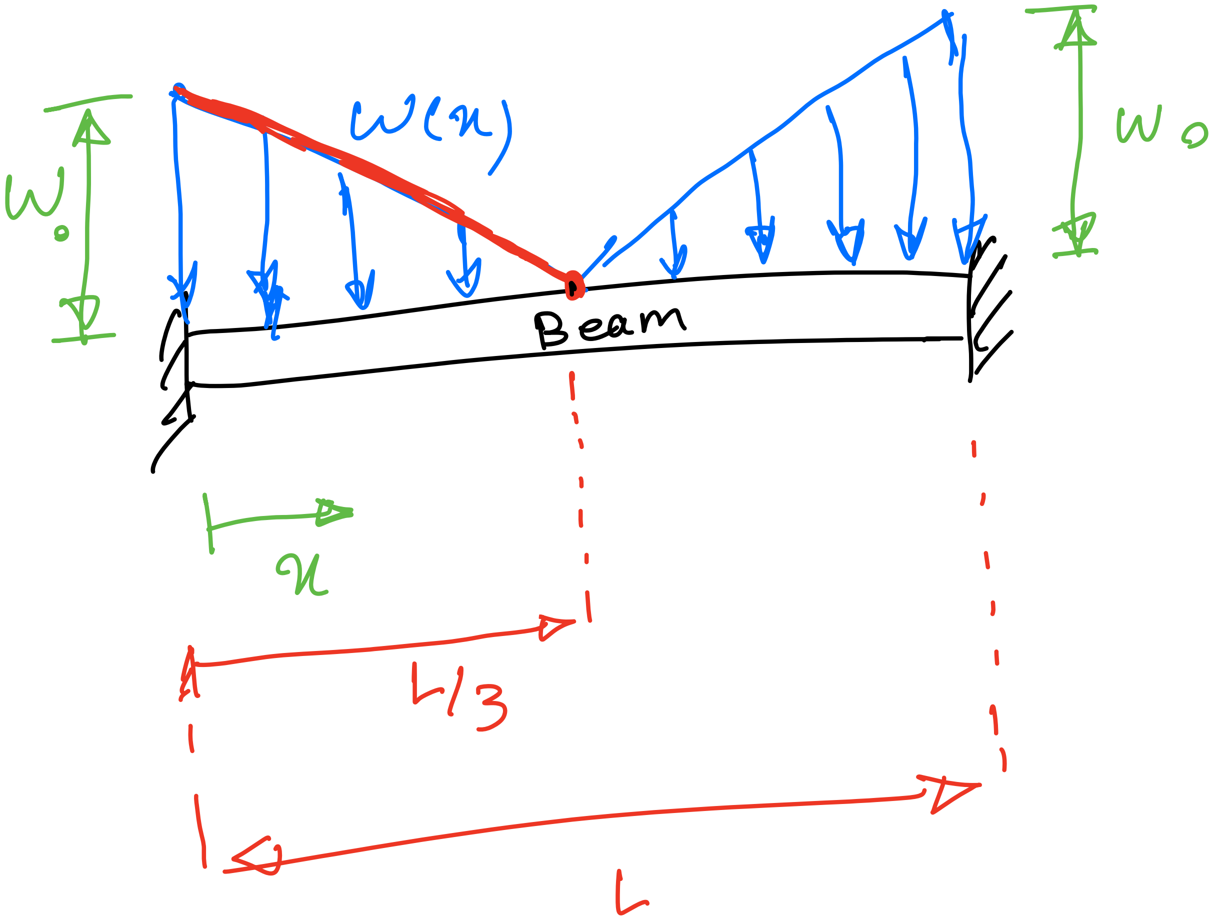
$$f = K(x - x_0)$$

$(x - x_0)$ → Elongation of the spring

K → stiffness of the spring.

Example:

Equation of the distributed load (linearly) on a beam.



$W(x) = ?$

$$\begin{cases} x = 0 \\ W = W_0 \end{cases} \qquad \begin{cases} x = L/3 \\ W = 0 \end{cases}$$

$$W(x) = Ax + B$$

$A \to ?$ $B \to ?$

$$\begin{cases} x = 0 \\ W = W_0 \end{cases} \Rightarrow W_0 = A(0) + B$$

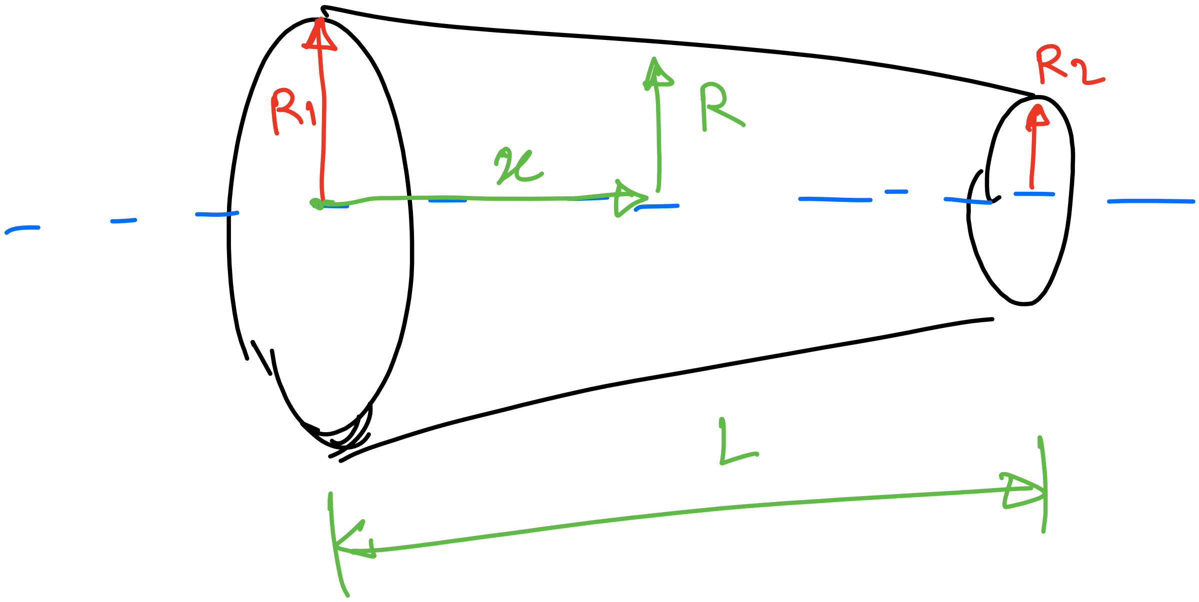
$$B = W_0$$

$$\begin{cases} x = L/3 \\ W = 0 \end{cases} \Rightarrow 0 = A(L/3) + W_0$$

$$\Rightarrow A = \frac{-3W_0}{L}$$

Example

Find the equation for the radius of the shape below as a function of x, if R_1 and R_2 are given.



$R(x) = ?$

$$R(x) = Ax + B$$

$$\begin{cases} x = 0 \\ R = R_1 \end{cases} \Longrightarrow R_1 = A(0) + B$$

$$R_1 = B$$

$$\begin{cases} x = L \\ R = R_2 \end{cases} \Longrightarrow R_2 = A(L) + R_1$$

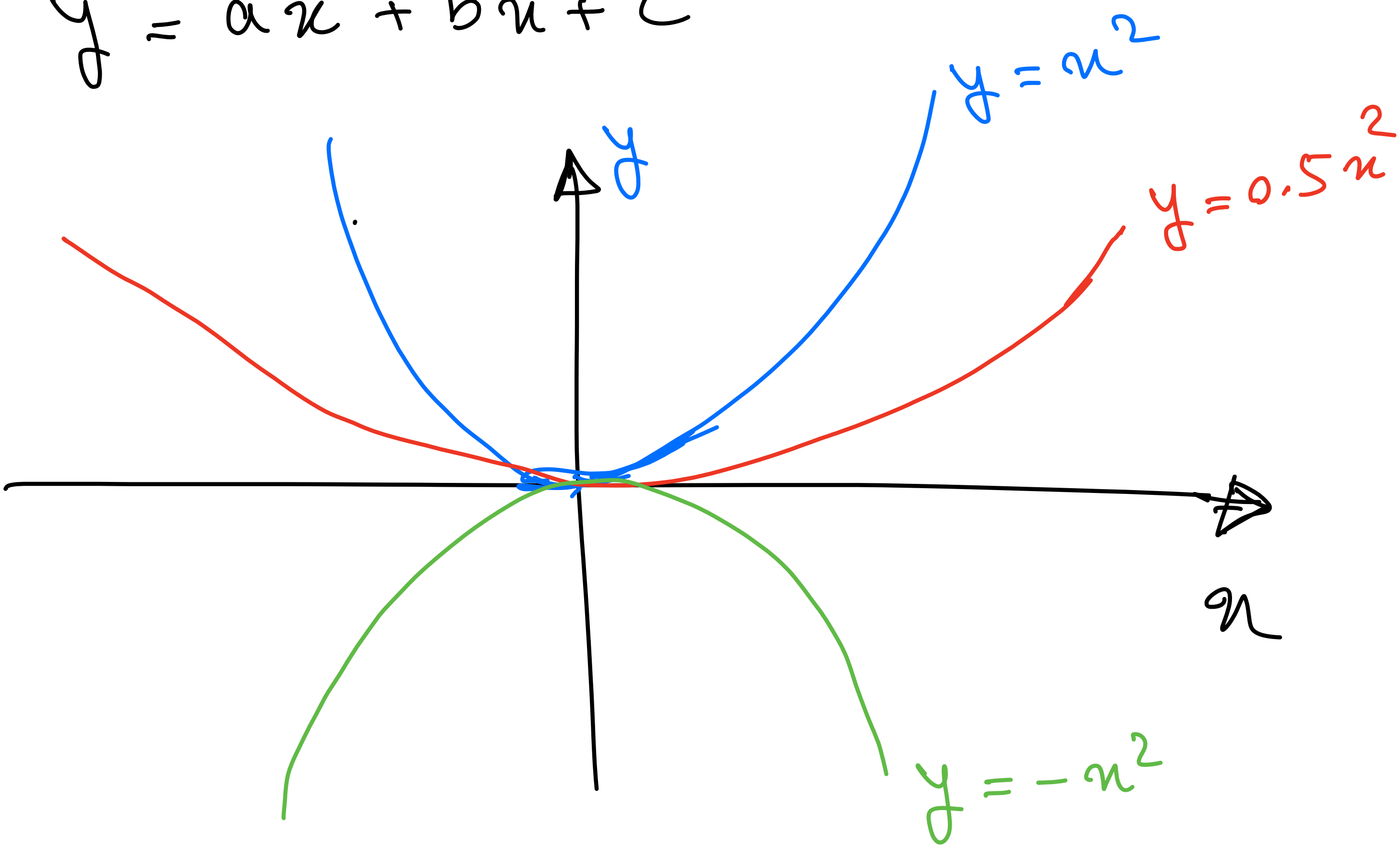
$$A = \frac{R_2 - R_1}{L}$$

$$R(x) = \frac{R_2 - R_1}{L} x + R_1$$

Quadratic functions

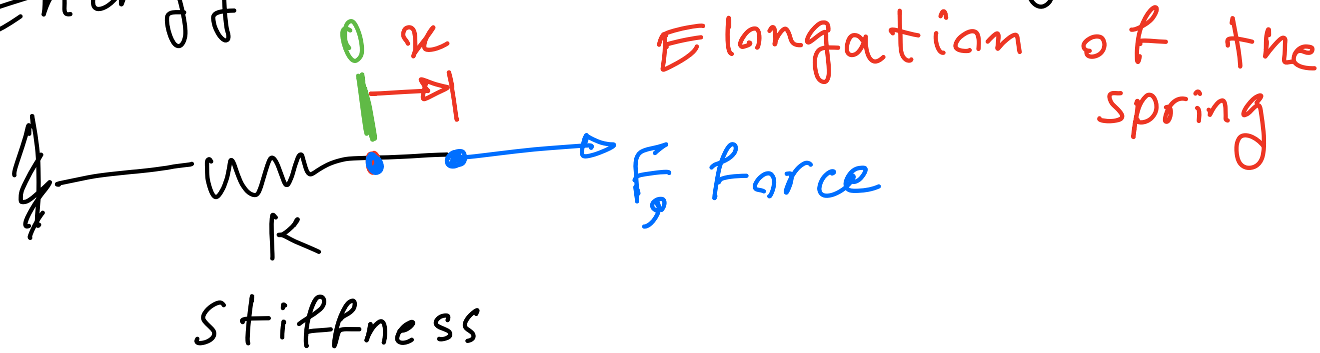
In the form of

$y = ax^2 + bx + c$



Example:

Energy stored in a spring.



$F = Kx$

Energy or work done by the force, F can be given by:

$$E = \int F \cdot dx = \int Kx\, dx = K \int x\, dx$$

$$E = K \frac{1}{2} x^2 \text{ or } \frac{1}{2} K x^2$$
