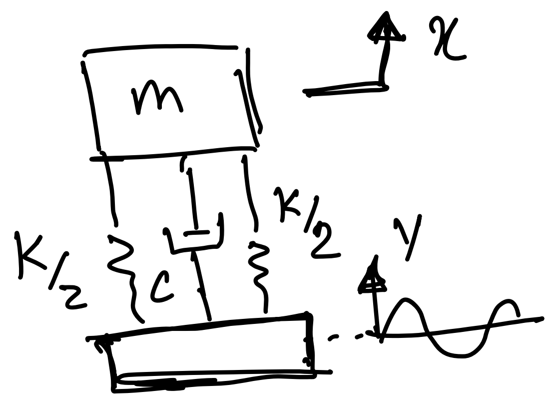
Vibration Isolation (section 3.6) Book

support Motion (section 3.5) Book



constant

$$m\ddot{z} + c\dot{z} + Kz = -m\ddot{y}$$

$$y = Ye^{i\omega t}$$

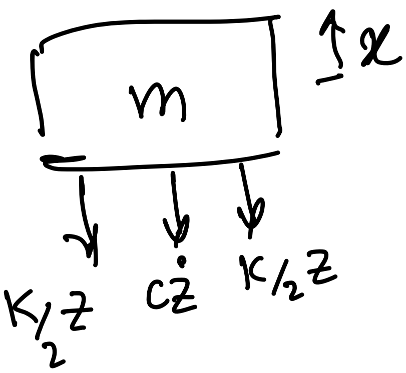
$$z = Ze^{i(\omega t - \phi)} = (Ze^{-i\phi})e^{i\omega t}$$

$$x = Xe^{i(\omega t - \psi)} = (Xe^{-i\psi})e^{i\omega t}$$

$x > y$

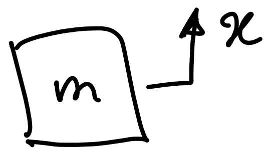
$z = x - y$

F.B.D



$$Ze^{i\phi} = \frac{m\omega^2 Y}{K - m\omega^2 + i\omega c}$$

$$x = z + y$$



$$x = \left(Ze^{-i\phi} + Y\right)e^{i\omega t}$$

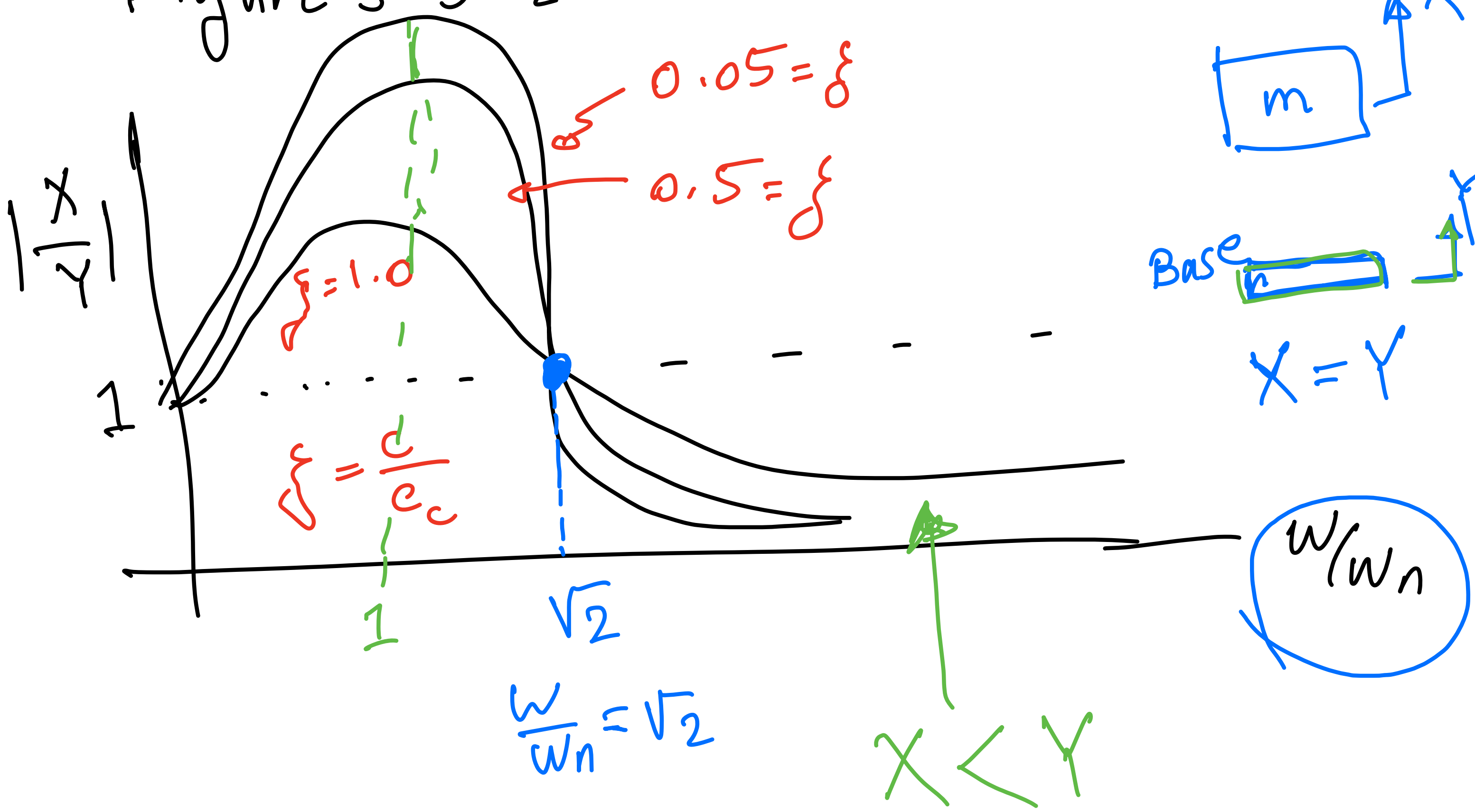
$$= \left(\frac{\overset{\text{real}}{K} + \overset{\text{Imag.}}{i\omega c}}{\underbrace{K - m\omega^2}_{\text{Real}} + \underbrace{i\omega c}_{\text{Imag.}}}\right) Y e^{i\omega t}$$

$$\underbrace{a}_{\text{Real}} + \underbrace{ib}_{\text{Imag.}} \rightarrow \sqrt{a^2 + b^2}$$

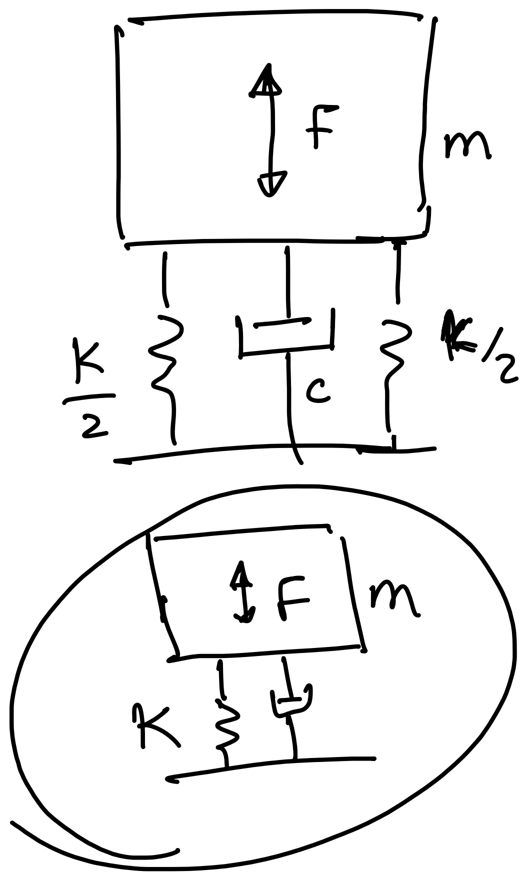
The steady-state amplitue is :

$$\left|\frac{X}{Y}\right| = \sqrt{\frac{K^2 + (\omega c)^2}{(K - m\omega^2)^2 + (c\omega)^2}}$$

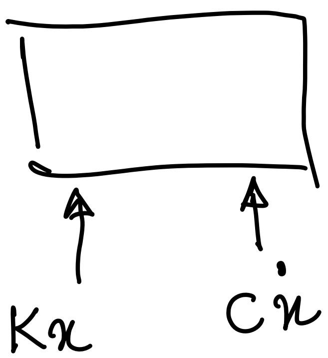
Figure 3.5-2 Book :



Vibration Isolation (section 3.6)



Force transmitted through the spring and damper is:



$$x = \underline{X} e^{i\omega t} \longrightarrow \dot{x} = \underline{X} i\omega e^{i\omega t}$$

$$F_T \text{ (transmitted force)} = Kx + c\dot{x}$$

$$F_T = KXe^{i\omega t} + cXi\omega e^{i\omega t}$$

Amplitude of (Max) $F_T = \underbrace{KX}_{\text{Real}} + \underbrace{cX\omega i}_{\text{Imag.}}$

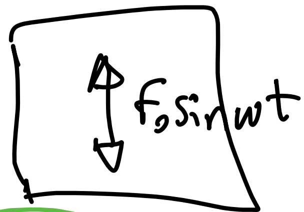
Magnitude $F_T = \sqrt{(Kx)^2 + (c\omega x)^2}$ Eq. 1

$$= Kx\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}$$

$\xi = \frac{c}{c_c}$ $c_c = 2\sqrt{Km}$ $\omega_n = \sqrt{\frac{K}{m}}$ Single DoF

Reffering to the forced vibration ($f_0 \sin\omega t$) Equation in the beginning of the chapter-

$$X = \frac{F_0/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$



Eq. 2

The transmissibility TR :

Defined as the ratio of the transmitted force to that of the disturbing force, is then $\frac{\text{Eq. 1}}{\text{Eq. 2}} =$

$$TR = \left| \frac{F_T}{F_0} \right| = \sqrt{\frac{1 + (2\xi \omega/\omega_n)^2}{[1-(\omega/\omega_n)^2]^2 + [2\xi \omega/\omega_n]^2}}$$

$\left| \frac{X}{Y} \right|$ shows that the motion transmitted from the supporting structure to the mass m is less than 1 when ratio $\frac{\omega}{\omega_n}$ is greater than $\sqrt{2}$.

This indicates that the natural frequency ω_n of the supported system (mass-spring-damper) must be small compared to that of the disturbing frequency ω.

This requirement can be met by using a soft spring!

$$TR = \left|\frac{F_T}{F_0}\right| = \left|\frac{X}{Y}\right|$$

when damping is negligible, the transmissibility equation reduces to:

$$TR = \left|\frac{F_T}{F_0}\right| = \sqrt{\frac{1+\cancel{(2\xi\omega/\omega_n)^2}}{\left[1-(\omega/\omega_n)^2\right]^2+\cancel{\left[2\xi\omega/\omega_n\right]^2}}}$$

ξ small

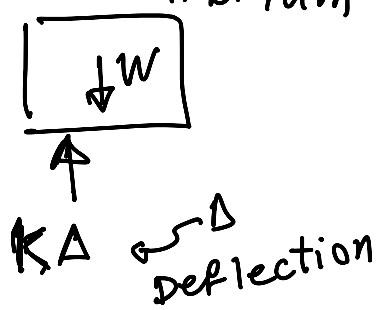
ξ small

negligible damping:

$$TR = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\omega_n^2 = \frac{g}{\Delta}$$

Static equilibrium



$$\Sigma F = 0$$

$$K\Delta = W$$

$$K\Delta = mg$$

$$TR = \frac{1}{\omega^2 \frac{\Delta}{g} - 1}$$

$(\omega = 2\pi f)$

$$TR = \frac{1}{(2\pi f)^2 \frac{\Delta}{g} - 1}$$

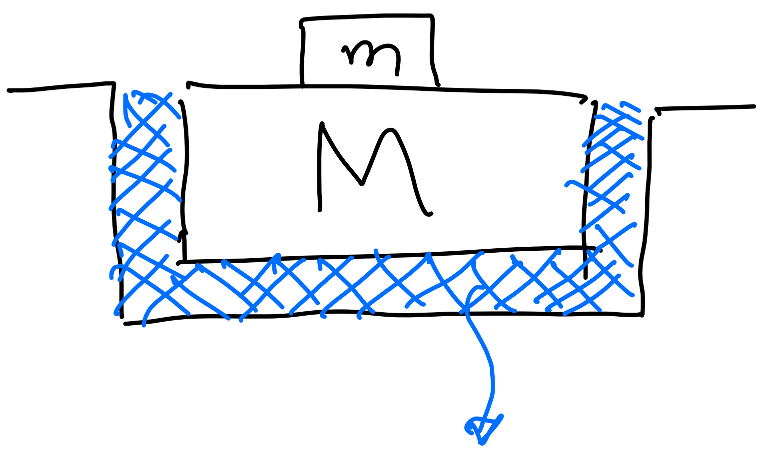
$$\frac{K}{m} = \frac{g}{\Delta}$$

$$\omega_n^2 = g/\Delta$$

To reduce the amplitude X of the isolated mass m without changing TR (because we can not change external (disturbing/excitation) force)

mass, m, is often mounted on a large mass M, as shown below.

The stiffness K must then be increased to keep the ratio $\frac{K}{(m+M)}$ constant.



large K

The amplitude X is reduced because K appears in the denominator

$$X = \frac{F_o/K}{\sqrt{\left(1-\left(\frac{w}{w_n}\right)^2\right)^2 + \left(2\xi \frac{w}{w_n}\right)^2}}$$

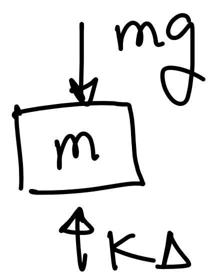
Example

A machine of 100 kg is supported on springs of total stiffness 700 kN/m and has an unbalanced rotating element, which results in a disturbing force of 350 N at a speed of 3000 rpm. Assuming a damping factor of $\xi = 0.02$, determine:

a) its amplitude of motion due to the unbalance

b) the transmissibility

c) the transmited force



static Deflection :

$$mg = K\Delta \qquad \Delta = \frac{mg}{K}$$

$$\frac{100 \times 9.81}{700 \times 10^{3}} = 1.401 \times 10^{-3} \ m = 1.401 \ mm$$

Natural frequency :

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{1.401 \times 10^{-3}}}$$

$$f_n = \frac{\omega_n}{2\pi} = 13.32 \ Hz \ ??$$

a)

$$\boxed{X = \frac{F_0/K}{\sqrt{\left(1-\left(\frac{w}{w_n}\right)^2\right)^2 + \left(2\xi \frac{w}{w_n}\right)^2}}}$$

$$X = \frac{\frac{350}{700\times10^3}}{\sqrt{\left[1-\left(\frac{50}{13.32}\right)^2\right]^2 + \left[2\times0.2\times\frac{50}{13.32}\right]^2}}$$

$$3000 \text{ rpm} = 3000 \frac{\text{Rev.}}{\text{min}} = 3000 \frac{\text{Rev.}}{\text{min}} \frac{\text{min}}{60\text{S}}$$

$$3000 \text{ rpm} = 50 \frac{\text{rev.}}{\text{sec.}} \; \text{Hz}$$

$$X = 3.79\times10^{-5} \text{ m}$$

b)

$$TR = \left|\frac{F_T}{F_o}\right| = \sqrt{\frac{1+(2\xi\omega/\omega_n)^2}{\left[1-(\omega/\omega_n)^2\right]^2+\left[2\xi\omega/\omega_n\right]^2}}$$

$$TR = \frac{\sqrt{1+\left(2\times 0.2\times\frac{50}{13.32}\right)^2}}{\sqrt{\left[1-\left(\frac{50}{13.32}\right)^2\right]^2+\left(2\times 0.2\times\frac{50}{13.32}\right)^2}}$$

$$TR = 0.137$$

c) The transmitted force is the disturbing force multiplied by

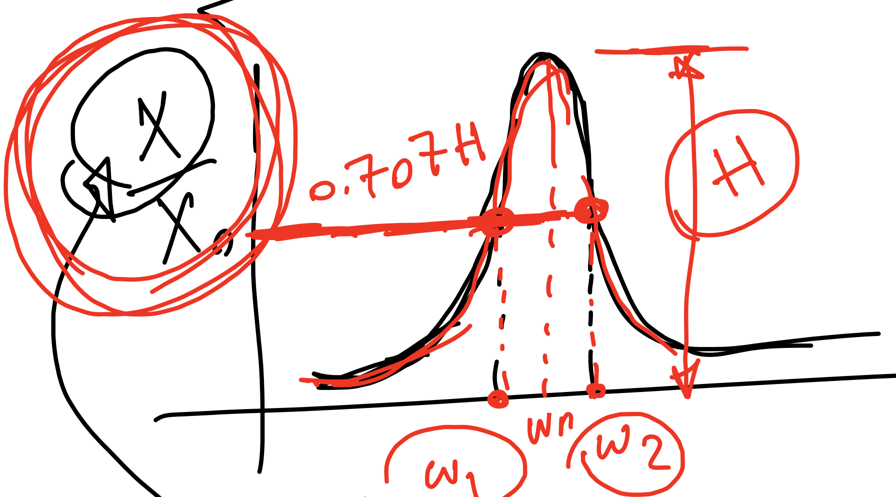
Transmissibility

$$F_{TR} = 350 \times 0.137 = 47.89 \text{ N}$$

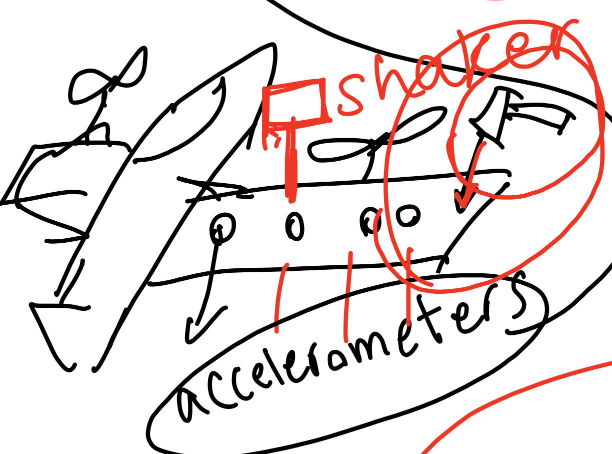
$$TR = \left| \frac{F_{TR}}{F_0} \right| \Longrightarrow F_{TR} = (TR)(F_0)$$

Next lecture

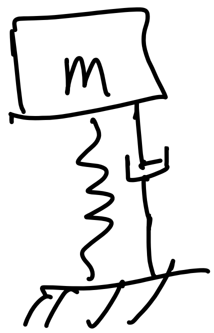
sharpness of resonance



Modal Testing



Aircraft
Car
Machinery



$$\xi = \frac{\omega_2 - \omega_1}{\;}$$


ω_n