Thevenin's Theorem

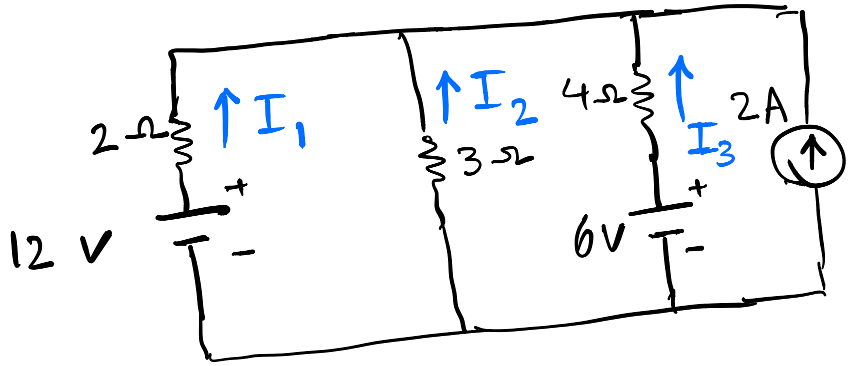
Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load. Thevenin's theorem is especially useful in analysing power systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it.

To find the Thevenin equivalent circuit you need to disconnect the load resistor and then calculate R_{Th} and V_{Th}. To find R_{Th} you need to short circuit the voltage sources (replace them with wires) and open circuit the current sources.

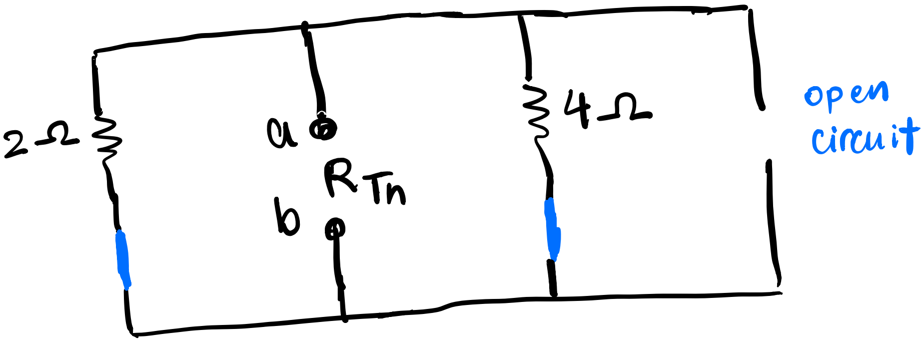
To find V_{Th} you need to put back the voltage and current sources and find out the voltage across the removed load resistor.

Example

Find the power dissipated in the $3\,\Omega$ resistor in the circuit shown below using Thevenin's Theorem.



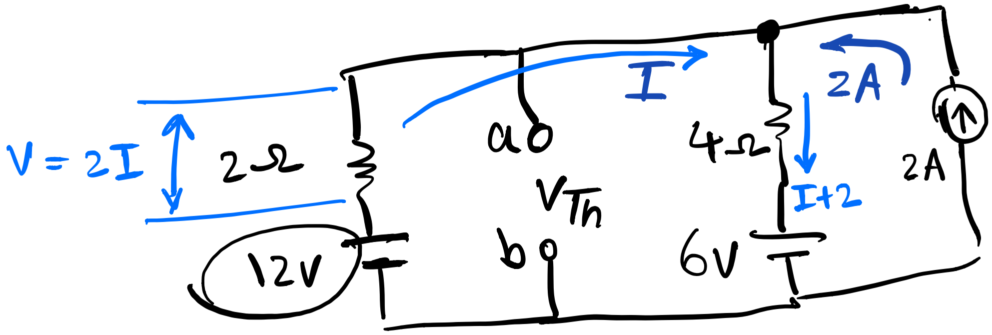
To Find R_{Th}, disconnect the $3\,\Omega$, short circuit voltage sources and open circuit current sources as shown.



R_{Th} is the total resistance of the circuit between a and b. In this case the 2Ω and 4Ω resistors are connected in parallel and

$$R_{th} = \frac{4 \times 2}{4+2} = 1.33\ \Omega$$

To find V_{Th}, put back the voltage sources and the current source and Find the voltage between a and b

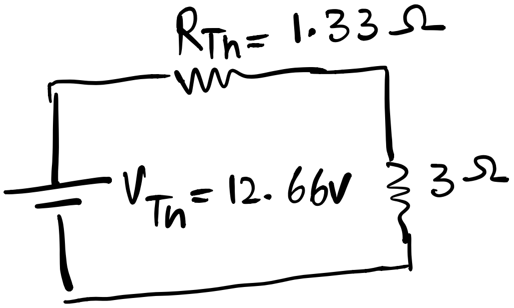


$$12\ V = 2I + 4(I+2) + 6$$

$$12 = 6I + 14 \Rightarrow I = \frac{-2}{6} = -0.33\ A$$

$$V_{Th} = 12 + (0.33 \times 2) = 12.66V$$

Draw the equivalent Thevenin circuit and connect back the 3 Ω resistor and find the power dissipated in this resistor.



power in the 3 Ω resistor = $V \times I$

$V = RI$ power $= V \times \left(\frac{V}{R}\right) = \frac{V^2}{R} = RI^2$

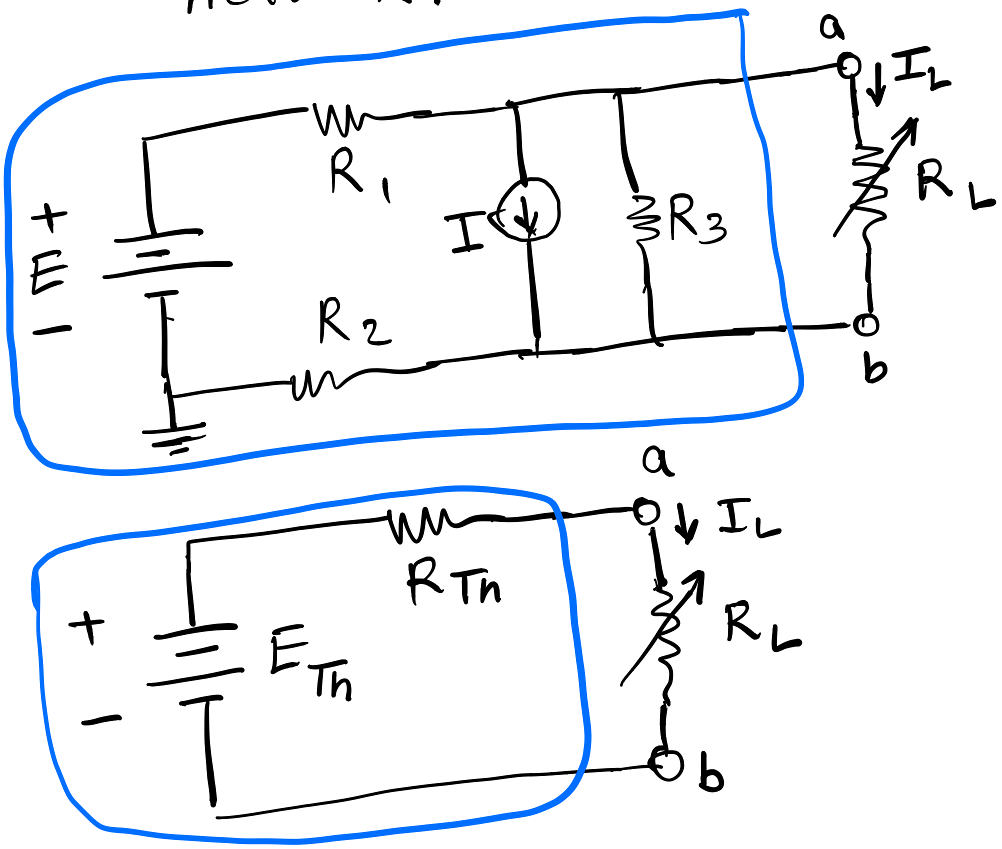
power $= \left(\frac{12.66}{1.33+3}\right)^2 \times 3 = 25.6$ Watts

---

Thevenin's Theorem procedure

1. Remove that portion of the network where the Thevenin equivalent circuit is foud. This requires the the load resistor R_L be

Temporarily removed from the network.

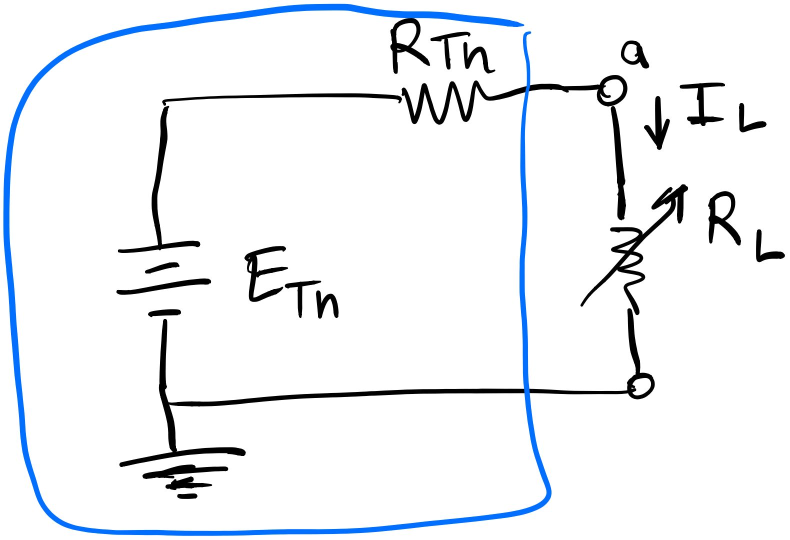


- Mark the terminals of the remaining two-terminal network (a and b).

- Calculate R_{Th} by first setting all sources to zero (Voltag sources are replaced by short circuits, and current sources by open circuits). and finding the resultant resistance between the two marked terminals (If there is internal resistance for current source or voltage source, it should be included.)

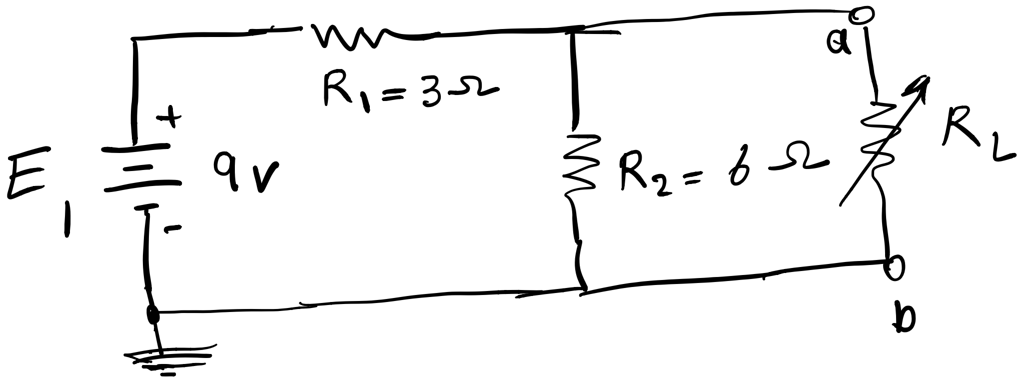
- Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.
(It is the open-circuit potential between the two terminals (a and b).

- Draw the Thevenin equivalent circuit.

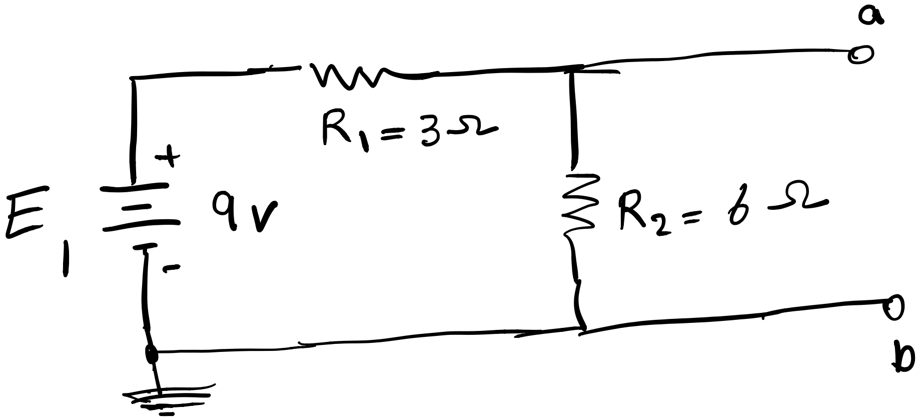


Example:

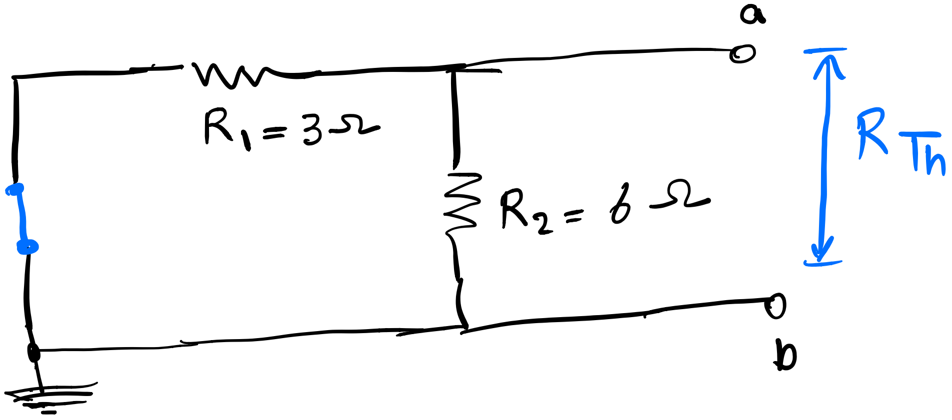
Find the Thevenin equivalent circuit for the network. Then find the current through R_L for values of $2\,\Omega$, $10\,\Omega$, and $100\,\Omega$.

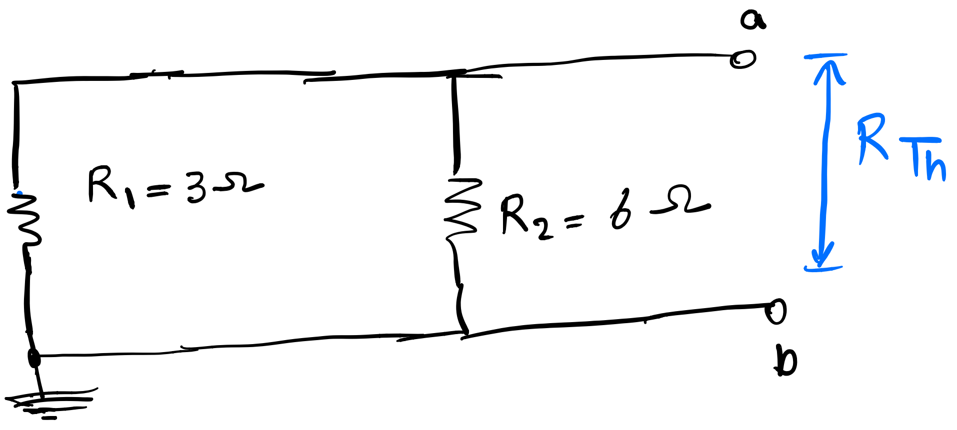


- Remove R_L



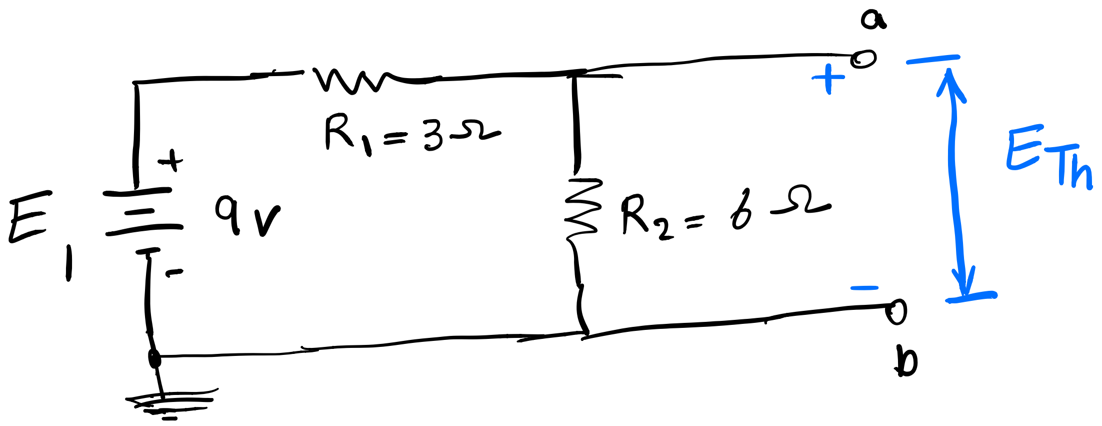
- Replace voltage source E_1 with short circuit.

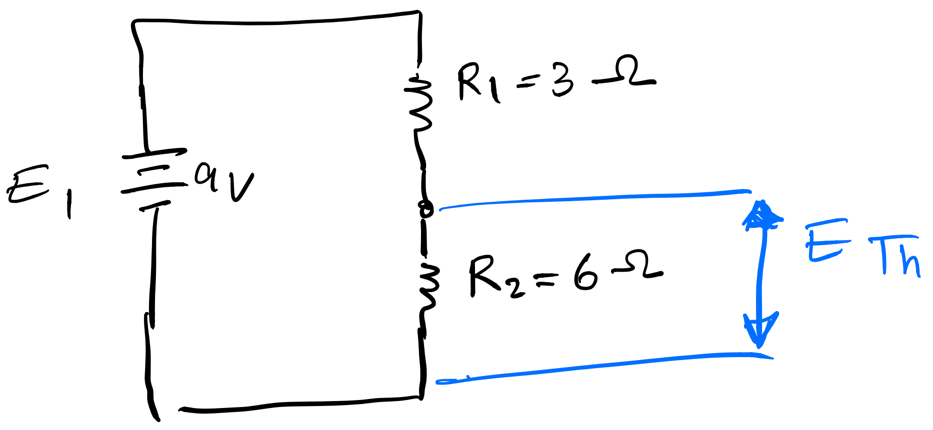




$$R_{Th} = R_1 \parallel R_2 = \frac{(3\,\Omega)(6\,\Omega)}{(3\,\Omega)+(6\,\Omega)} = 2\,\Omega$$

- place back the voltage source:

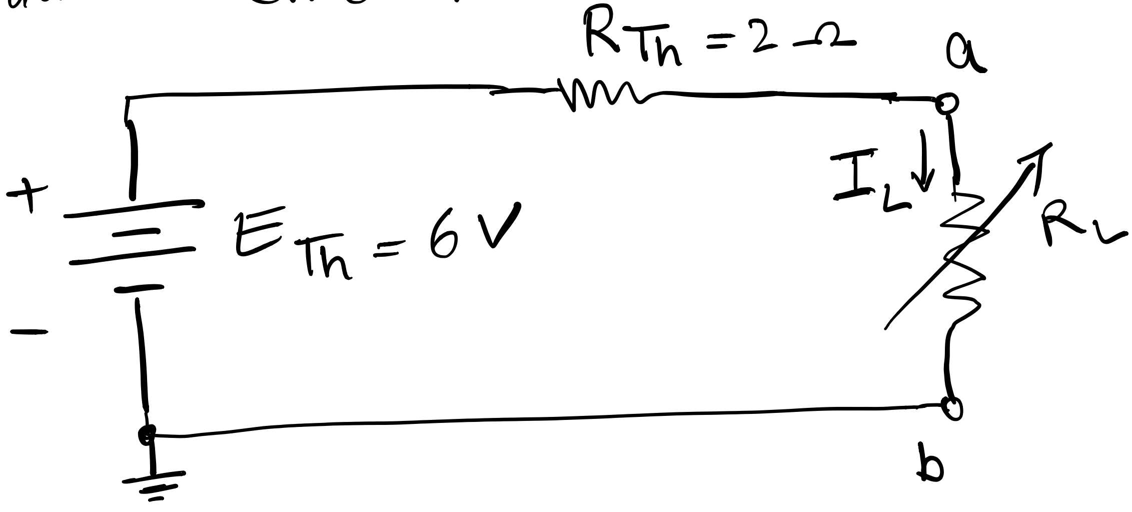




Use voltage divider rule (E_{Th} is the same voltag across R_2).

$$E_{Th} = \frac{R_2 E_1}{R_1 + R_2} = \frac{(6\Omega)(9V)}{(3\Omega) + (6\Omega)} = 6V$$

Draw the circuit:



$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$$R_L = 2\Omega \Rightarrow I_L = \frac{6V}{2\Omega + 2\Omega} = 1,5A$$

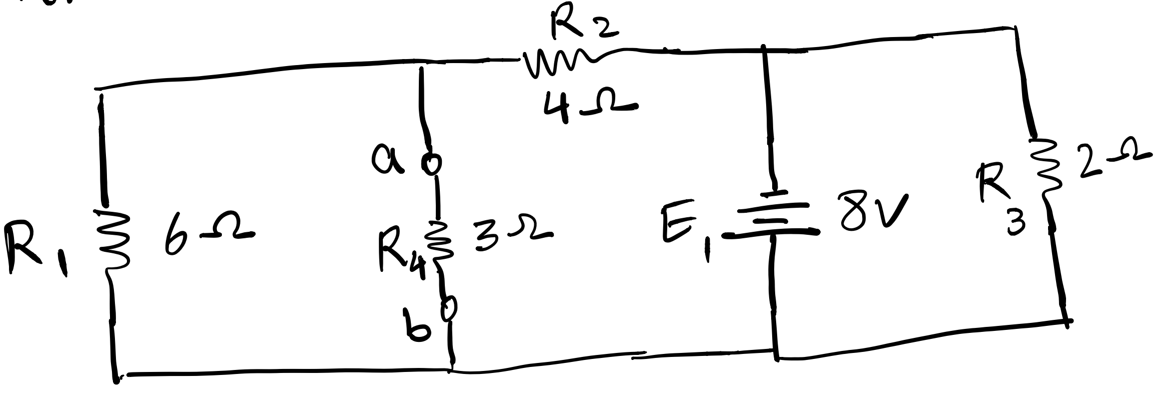
$$R_L = 10\Omega \Rightarrow I_L = \frac{6V}{2\Omega + 10\Omega} = 0.5A$$

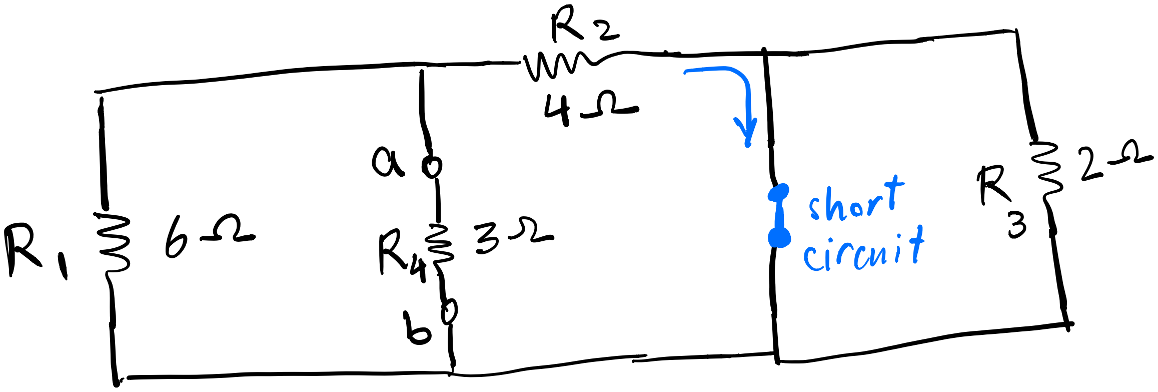
$$R_L = 100\,\Omega \Rightarrow I_L = \frac{6\,V}{2\,\Omega + 100\,\Omega} = 0.06\,A$$

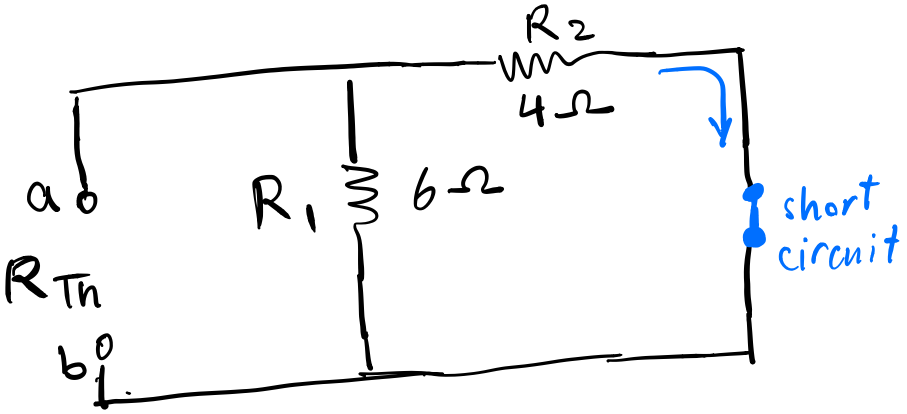
IF Thevenin's Theorem were unavailable, each change in R_L would require that the entire network to be rexamined to find the new value of I_L

Example

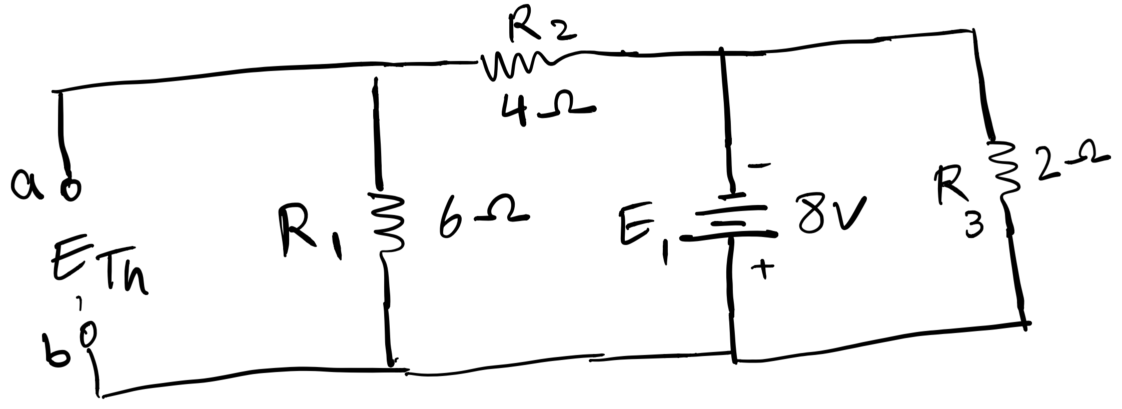
Find the Thevenin equivalent circuit for the network.







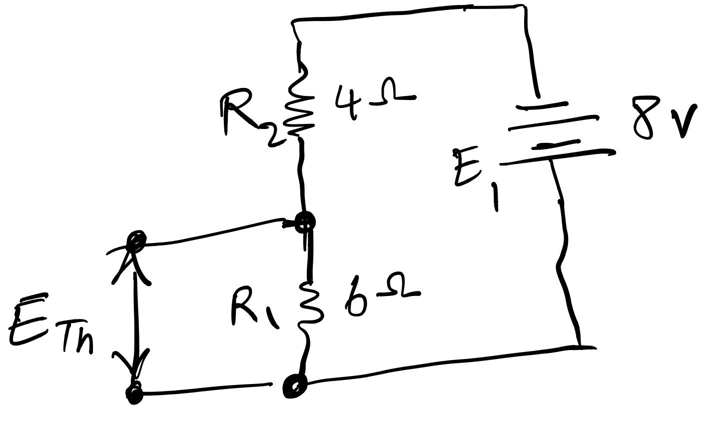
$$R_{Th} = R_1 \| R_2 = \frac{(6\Omega)(4\Omega)}{6\Omega + 4\Omega} = \frac{24\Omega}{10} = 2.4\Omega$$



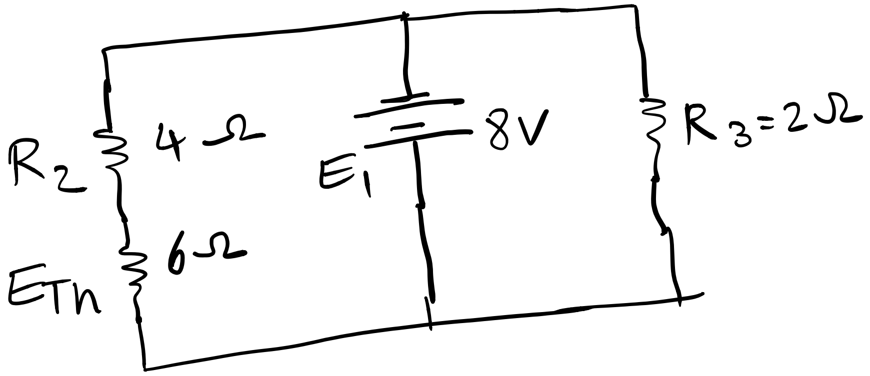
The voltage is the same across parallel elements. Therfore, the voltag across R_1 is E_{Th} and R_3 is 8V.

Applying the voltag divider rule:

$$E_{Th} = \frac{R_1 E_1}{R_2 + R_1}$$



$$E_{Th} = \frac{(6\,\Omega)(8\,V)}{6\,\Omega + 4\,\Omega} = \frac{48V}{10} = 4.8\,V$$



The Thevenin equivalent circuit for the network