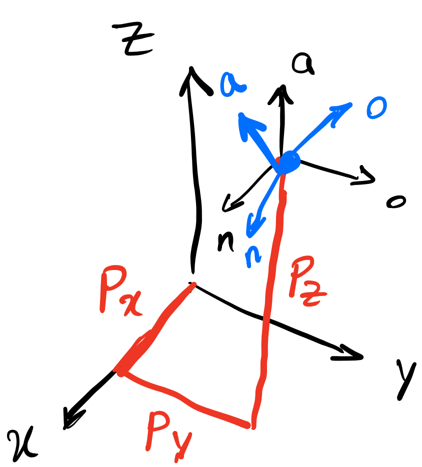
Example

(Example 2.21 in the third edition, example 2.20 in the second edition)
Note: Values used in this example are from the second edition of the book)

The desired final position and orientation of the hand of a catesian-RPY Robot is given below. Find the necessary RPY angles and displacements.

$$
{}^{R}T_{P} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.354 & -0.674 & 0.649 & 4.33 \\ 0.505 & 0.722 & 0.475 & 2.50 \\ -0.788 & 0.160 & 0.595 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



Using the inverse Kinematic equations (page 68 in the Book) we have:

$$\phi_a = ATAN2(n_y, n_x) = ATAN2(0.505, 0.354)$$

$$= 55^\circ \text{ or } 235^\circ$$

$$\phi_o = ATAN2(-n_z, \underbrace{(n_x C\phi_a + n_y S\phi_a)}))$$

$$= ATAN2(0.788, 0.616) = 52^\circ \text{ or } 128^\circ$$

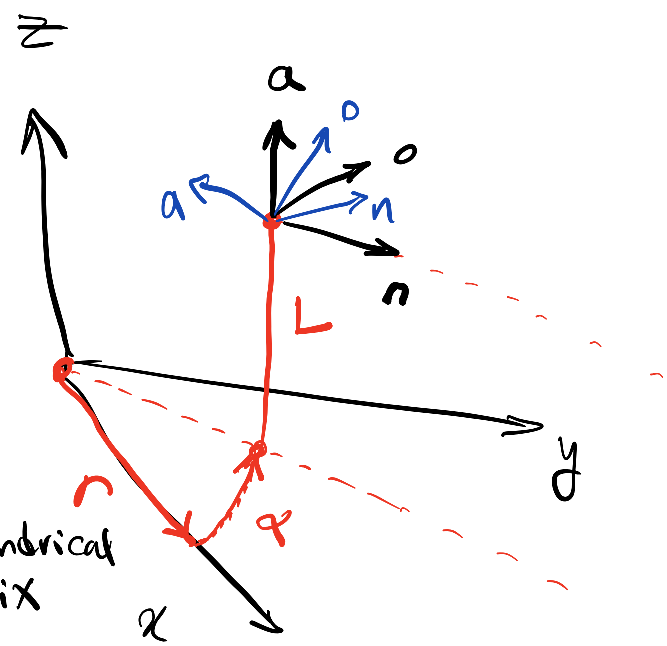
$$\phi_n = ATAN2((-a_y C\phi_a + a_x S\phi_a), (o_y C\phi_a - o_x S\phi_a))$$

$$= ATAN2(0.259, 0.966) = 15^\circ \text{ or } 195^\circ$$

$P_x = 4.33 \quad P_y = 2.5 \quad P_z = 8$ units

Example (2.21 Book)

For the same position and orientation as in the previous example, find all necessary joint variables if the robot is cylindrical-RPY.



Last column of the cylindrical matrix

$$\left.\begin{aligned} r\,c\alpha &= 4.33 \\ r\,s\alpha &= 2.5 \end{aligned}\right\} \Rightarrow \quad \alpha = 30^\circ$$

$$L = P_z = 8$$

$\Phi_a + \alpha = 55^\circ \Rightarrow \Phi_a = 25^\circ$

$S\alpha = 0.5 \Rightarrow r = 5$

$\Phi_o = 52^\circ$ (same as the previous example)

$\Phi_n = 15^\circ$

<u>Euler Angles</u> (sec. 2.10.2 Book)

Euler angles are very similar to RPY, except that the last rotation is also about the current a-axis.

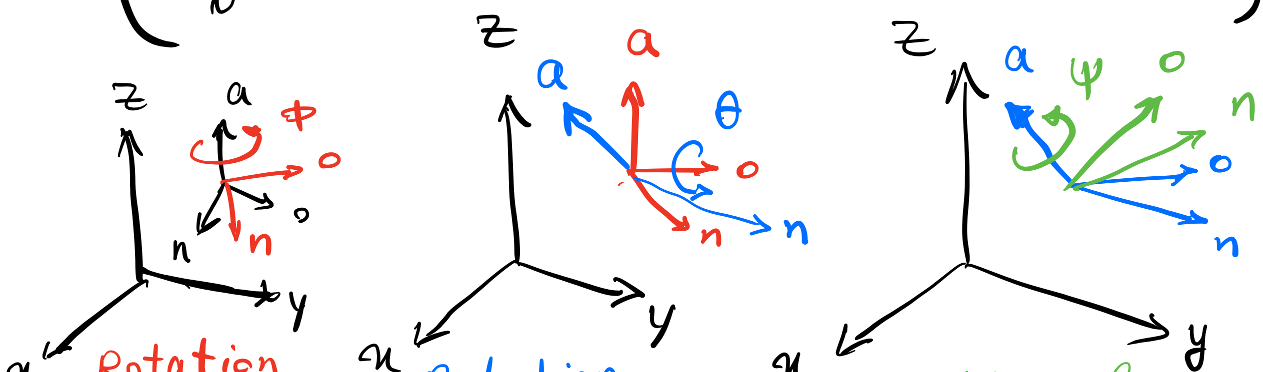
The rotations rpresenting the Euler angles will be:

- Rotation of ϕ about the a-axis, followed by
- Rotation of θ about the o-axis, followed by
- Rotation of ψ about the a-axis

The matrix representing the Euler angles orientation change will be:

$$Euler(\phi, \theta, \psi) = Rot(a, \phi)\, Rot(o, \theta)\, Rot(a, \psi)$$

$$= \begin{bmatrix} C\phi C\theta C\psi - S\phi S\psi & -C\phi C\theta S\psi - S\phi C\psi & C\phi S\theta & 0 \\ S\phi C\theta C\psi + C\phi S\psi & -S\phi C\theta S\psi + C\phi C\psi & S\phi S\theta & 0 \\ -S\theta C\psi & S\theta S\psi & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation of ϕ about the a-axis

Rotation of θ about the o-axis

Rotation of ψ about the a-axis

The inverse kinematic solution for the Euler angles can be found in a manner very similar to RPY.

$$Rot^{-1}(a,\phi) \times \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta C\psi & -C\theta S\psi & S\theta & 0 \\ S\psi & C\psi & 0 & 0 \\ -S\theta C\psi & S\theta S\psi & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
\begin{bmatrix}
n_x C\phi + n_y S\phi & o_x C\phi + o_y S\phi & a_x C\phi + a_y S\phi & 0 \\
-n_x S\phi + n_y C\phi & -o_x S\phi + o_y C\phi & -a_x S\phi + a_y C\phi & 0 \\
n_z & o_z & a_z & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
= \begin{bmatrix}
C\theta C\psi & -C\theta S\psi & S\theta & 0 \\
S\psi & C\psi & 0 & 0 \\
-S\theta C\psi & S\theta S\psi & C\theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

From the 2,3 elements:

$$-a_x S\phi + a_y C\phi = 0 \Rightarrow \phi = \text{ATAN2}(a_y, a_x)$$

$$\text{or} \quad \phi = \text{ATAN2}(-a_y, -a_x)$$

From the 2,1 and 2,2 elements we get:

$$S\psi = -n_x S\phi + n_y C\phi$$

$$C\psi = -o_x S\phi + o_y C\phi$$

$$\Psi = ATAN2\left[(-n_x S\phi + n_y C\phi), (-o_x S\phi + o_y C\phi)\right]$$

From the 1,3 and 3,3 elements we get:

$$S\theta = a_x C\phi + a_y S\phi$$
$$C\theta = a_z \longrightarrow \theta = ATAN2\left[(a_x C\phi + a_y S\phi), a_z\right]$$

<u>Example</u> (2.22 Book)

The desired final orientation of the hand of a Cartesian-Euler robot is given. Find the necessary Euler angles.

similar to the previous problem but using Euler angles.

For next time see

section 2.12

Denavit-Hartenberg representation

(D-H)