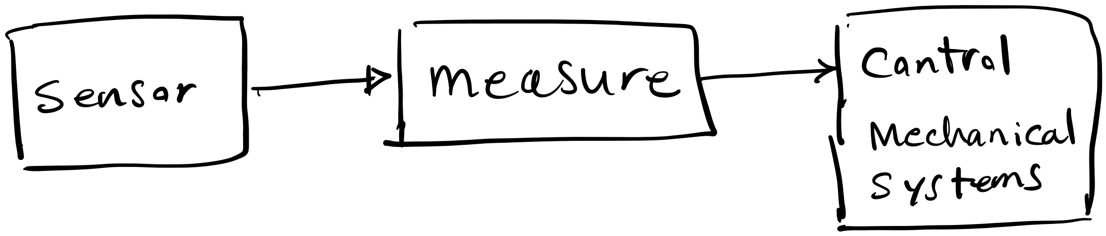
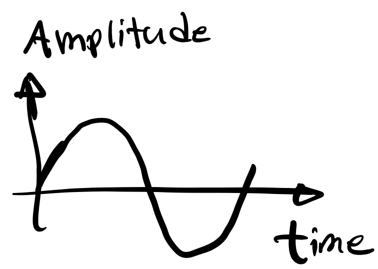
Filters and Amplifiers for Instrumentation in Mechanical systems (sensors)

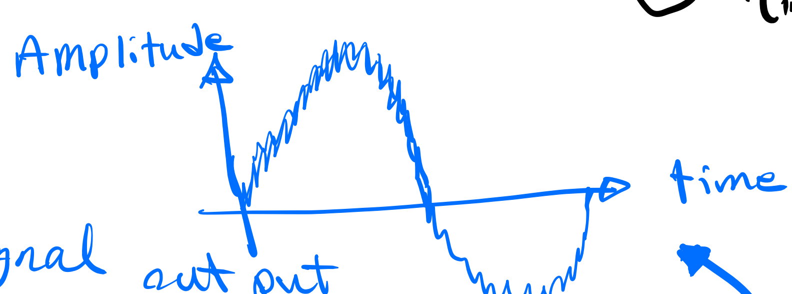


Sensor output { includes noise
has very small amplitudes (milli volts)

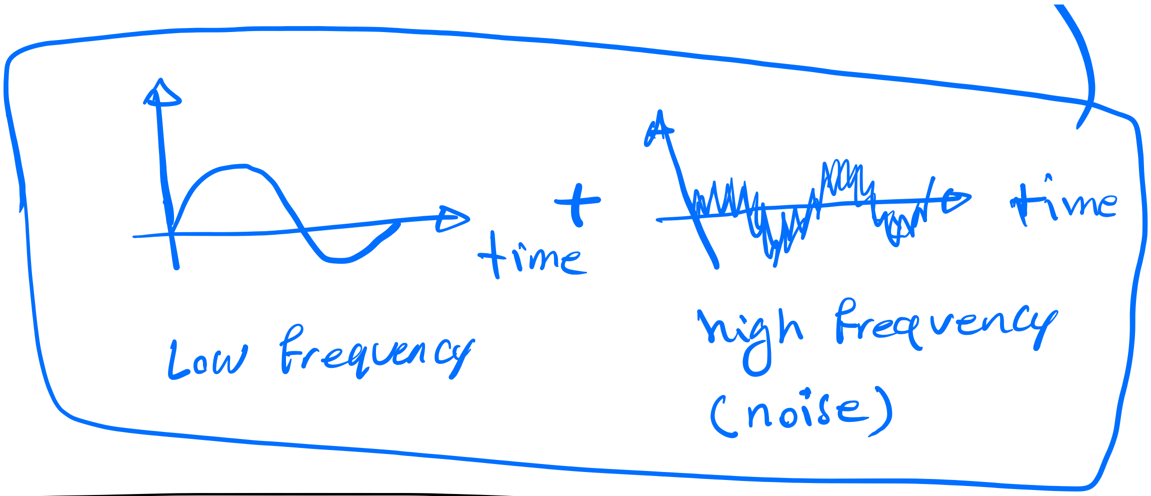
we need to remove noise from the sensor

sensor → signal





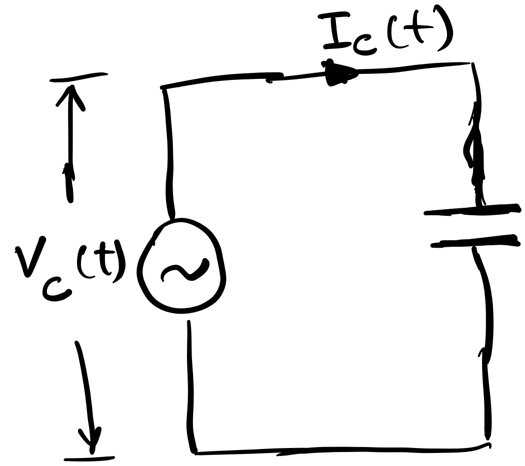
Actual Sensor signal output



- Capacitor circuits can be used as filters to remove noise from the sensor signal.

- operational amplifiers are used to amplify the small magnitude signal from the sensor to larger manitudes that can be measured.

Capacitors (Capacitive Reactance)



When a DC voltage is applied to a capacitor, the capacitor charges and no current flows in the circuit.

When AC voltage is applied the voltage is changing from a positive to a negative polarity at a rate determined by the frequency of the supply. The capacitor is charging and discharging.

Capacitive Reactance has the symbol X_c and has units measured in Ohms the same as resistance (R). It is calculated using the following formula.

$$X_c = \frac{1}{2\pi f C}$$

Capacitive Reactance formula

Where

X_c : Capacitive Reactance in ohms (Ω)

π (pi) : 3.142

f : Frequency in Hertz (Hz)

C : Capacitance in Farads (F)

Example

Calculate the capacitive reactance of a 220 nF capacitor at a frequency of 1 KHz, and at 20 KHz.

$$X_C = \frac{1}{2\pi f C}$$

At a frequency of 1 KHz :

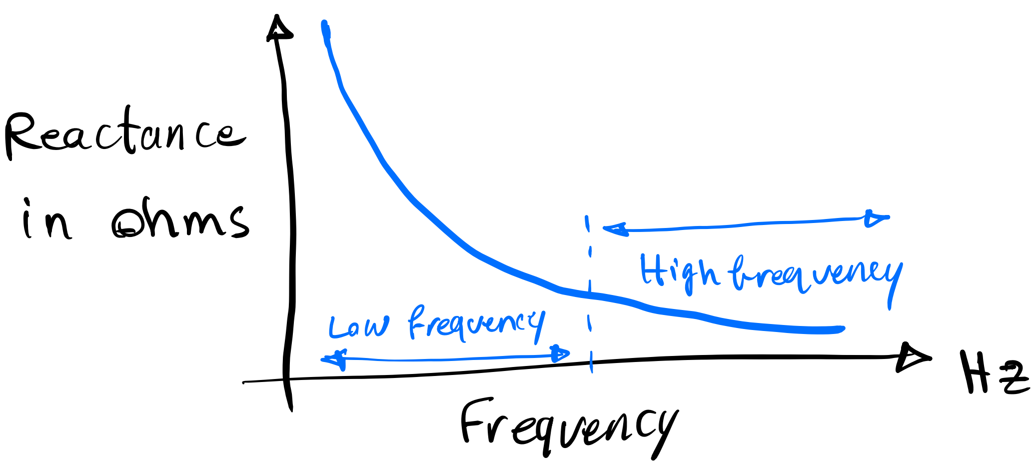
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 220 \times 10^{-9}}$$

$$X_C = 723.4\ \Omega$$

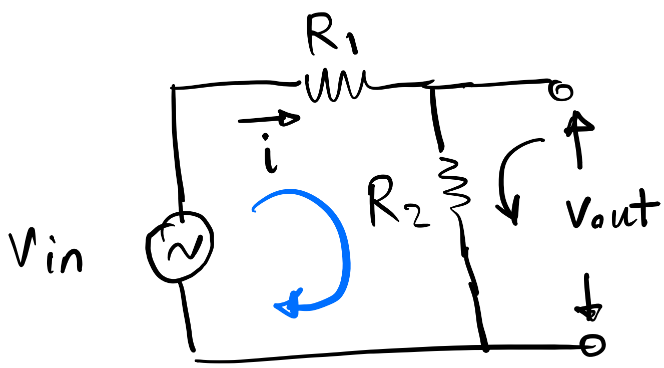
At a frequency of 20 KHz:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 20{,}000 \times 10^{-9}} = 36.2\ \Omega$$

Capacitive Reactance vs. Frequency.



Voltage Divider circuit:



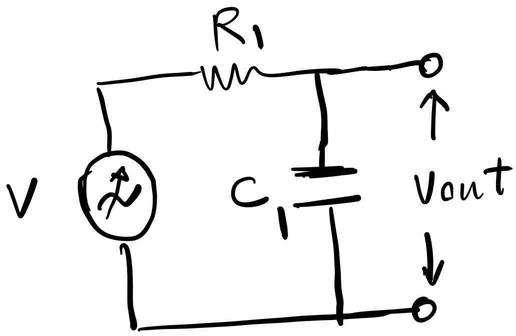
$$\begin{cases} V_{in} = R_1 i + R_2 i = (R_1 + R_2) i \\ V_{out} = R_2 i \Rightarrow i = \dfrac{V_{out}}{R_2} \end{cases}$$

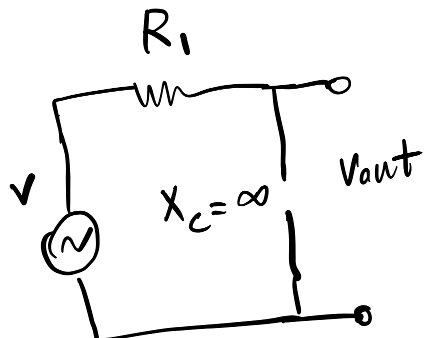
$$V_{in} = (R_1 + R_2) \frac{V_{out}}{R_2}$$

$$\boxed{V_{out} = V_{in} \frac{R_2}{R_1 + R_2}}$$

we have capacitor reactance X_c instead of resistors. which can be used instead of one of the resistors above in design of filters. This results in a frequvency-dependent RC voltage divider circuit. Low pass filters and high pass filters can be constructed by replacing one of the resistors in the voltage divider with a suitable capacitor as shown.

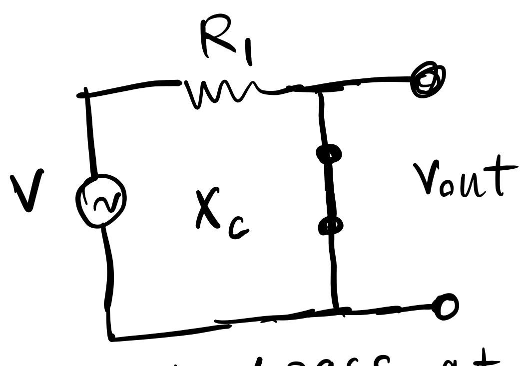
Low pass Filter





Low pass
at DC
Zero frequeny

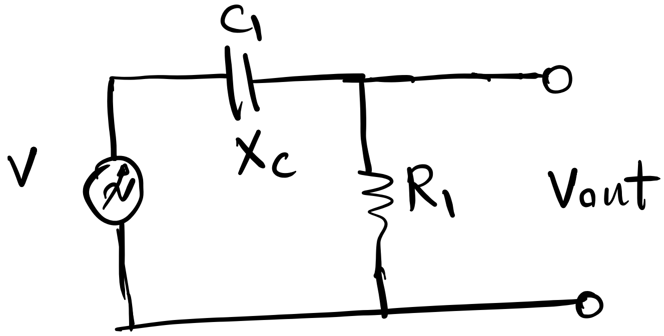
$$X_c = \frac{1}{2\pi f c} = \frac{1}{0} = \infty$$



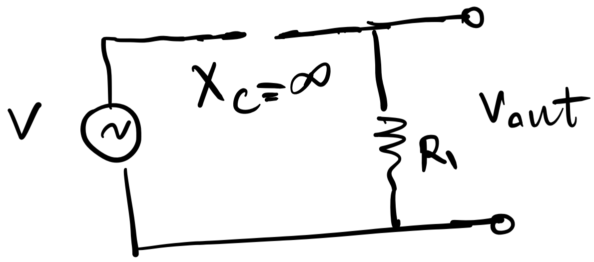
$$X_c = \frac{1}{2\pi f c} = \frac{1}{\infty} = 0$$

Low pass at high frequency

High pass Filter

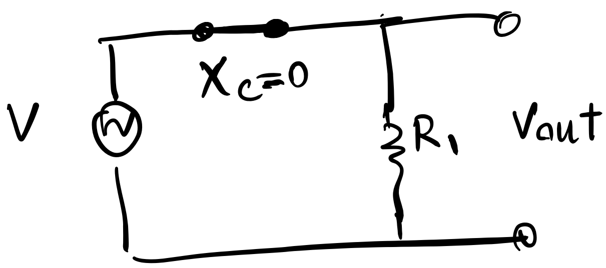


High pass filter



High pass at DC
zero frequency

$$X_c = \frac{1}{2\pi f C} = \frac{1}{0} = \infty$$

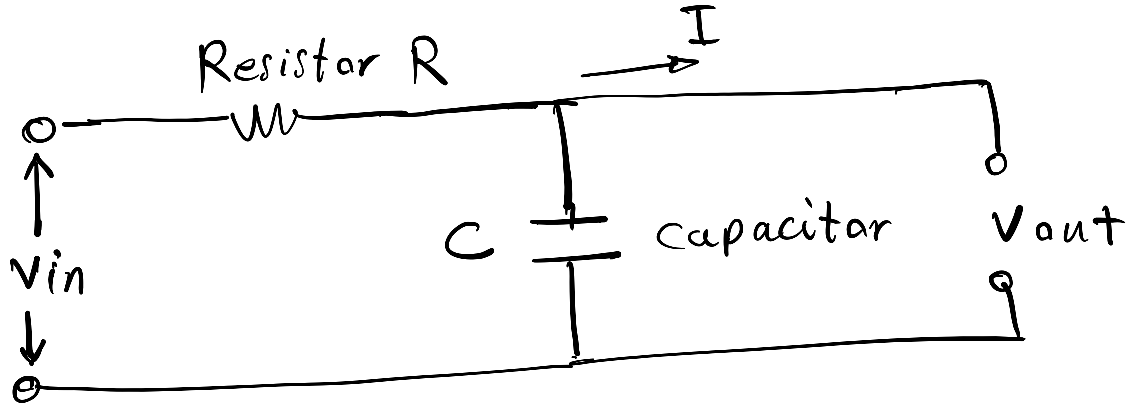


High pass at
high frequency

$$X_c = \frac{1}{2\pi f C} = \frac{1}{\infty} = 0$$

The property of Capacitive Reactance, makes the capacitor ideal for use in AC filter circuits.

Passive Low pass Filter



A low pass Filter is a circuit that can be designed to modify or reject unwanted high frequencies of an electrical signal and accept or pass only low frequency signals.

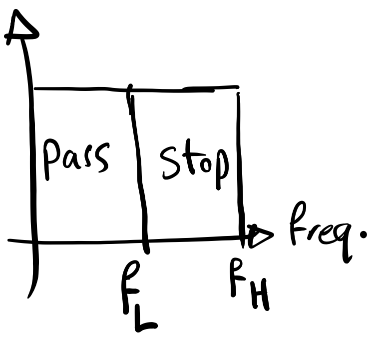
passive filters are generally constructed using simple RC (Resistor—capacitor) circuits, while higher frequency filters (above 100 KHz) are usually made from RLC (Resistor-Inductor-Capacitor) components.

Passive filters are made up of a passive components such as Resistors, capacitors and inductors, and have no amplifying elements (transistors, op-amp, etc) so have no signal gain, therefor their output level is always less than the input.

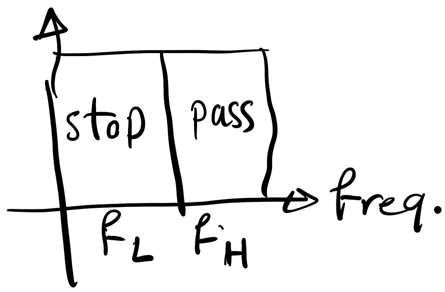
- The law pass Filter: only allows low frequency signal from 0 Hz to its cut-off frequency f_c point to pass while blocking any higher frequency

- The high pass Filter allows high frequency signals from its cut-off frequency, f_c point and higer to infinity to pass through while blocking those any lower.
- The band pass Filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies of this frequency band.

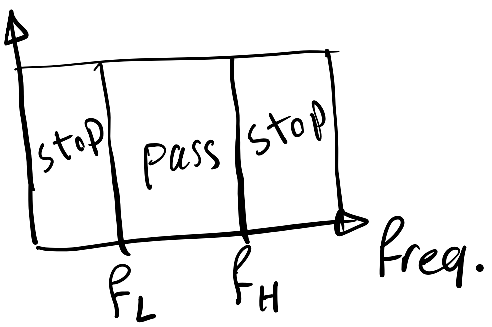
Indeal Filter response:



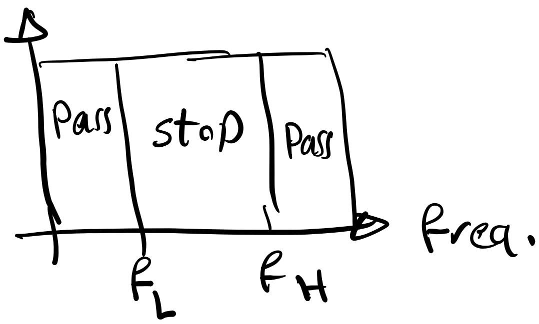
Low pass Filter



High pass Filter



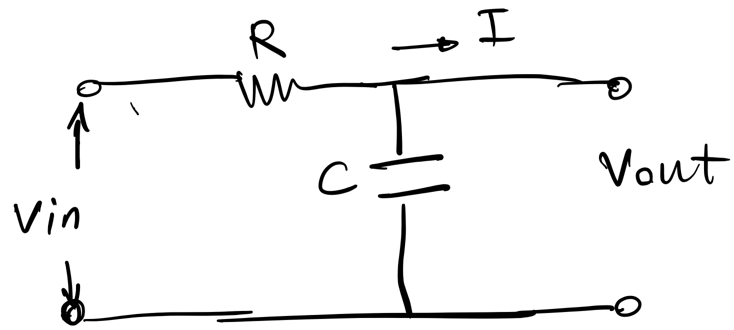
Band pass Filter



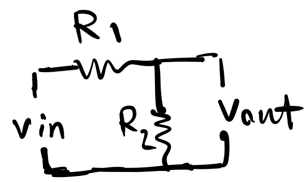
Band stop Filter

The frequency at which the transition occurs is called the "cut-off" or "corner" frequency.

RC Low pass Filter circuit



Voltage divider circuit



$$V_{out} = V_{in} \frac{R_2}{R_1+R_2}$$

$R_1 + R_2$ is the total resistance.

in AC circuit the capacitive reactance is $X_c = \frac{1}{2\pi f C}$ ohms

How do we add X_C to R ?

Opposition to current flow in an AC circuit is called impedance, symbol Z is used.

The impedance is calculated as:

$$Z = \sqrt{R^2 + X_c^2}$$

RC Divider Equation:

$$V_{out} = V_{in} \frac{X_c}{\sqrt{R^2 + X_c^2}} = V_{in} \frac{X_c}{Z}$$

So, by using the divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

Example

A Low pass Filter circuit consisting of a resistor of 4.7 KΩ in series with capacitor of 47 nF is connected across a 10 v sinusoidal supply. calculate the output voltage (V_{out}) at a frequency of 100 Hz, and 10 KHZ.

voltage output at 100 Hz:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 47 \times 10^{-9}} = 33\,863\ \Omega$$

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{33\,863}{\sqrt{4700^2 + 33863^2}}$$

$$V_{out} = 9.9\ \text{v}$$

Voltage output at a frequency of 10,000 Hz (10 KHz):

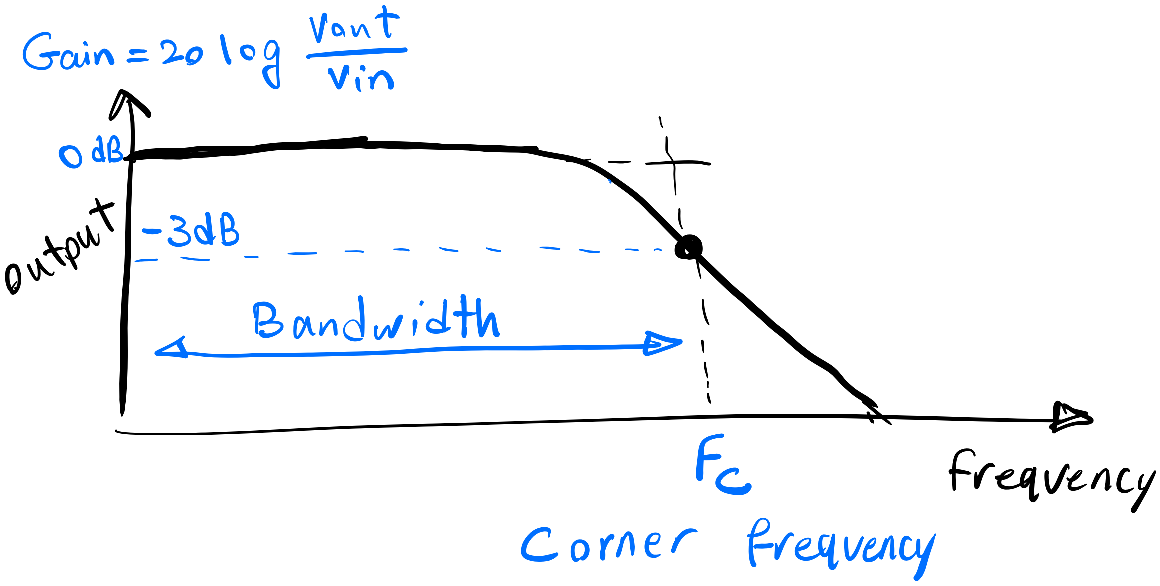
$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10000 \times 47 \times 10^{-9}} = 338.6\,\Omega$$

$$V_{out} = V_{In} \frac{X_C}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6^2}}$$

$$= 0.718\ V$$

From the results, we can see that as frequency applied to the RC network increases from 100 Hz to 10 KHz, the voltage dropped across the capacitor and therefore the output voltage (V_{out}) from the circuit decreases from 9.9 v to 0.718 v.

Frequency response of a 1st-order Low pass Filter :



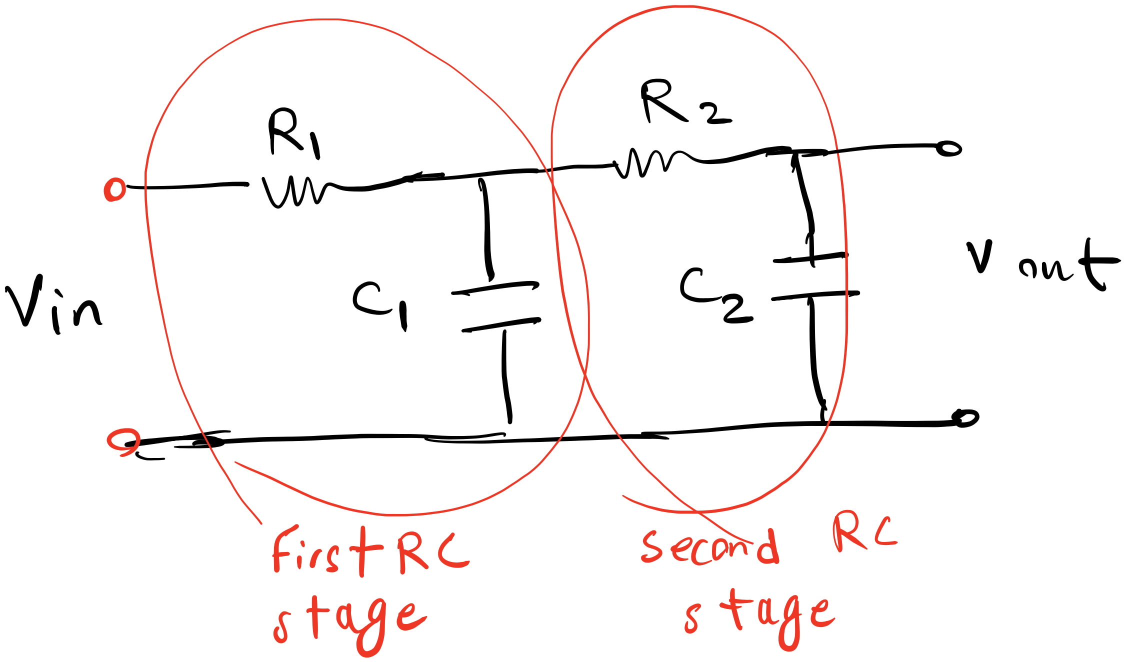
The frequency response of the filter is nearly flat for low frequencies and all of the input signal is passed directly to the output, resulting in a gain of nearly 1 ($\frac{V_{out}}{V_{in}} = 1$ $\Rightarrow$ $V_{out} = V_{in}$), called unity, until it reaches its cut-off frequency point (f_c). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor.

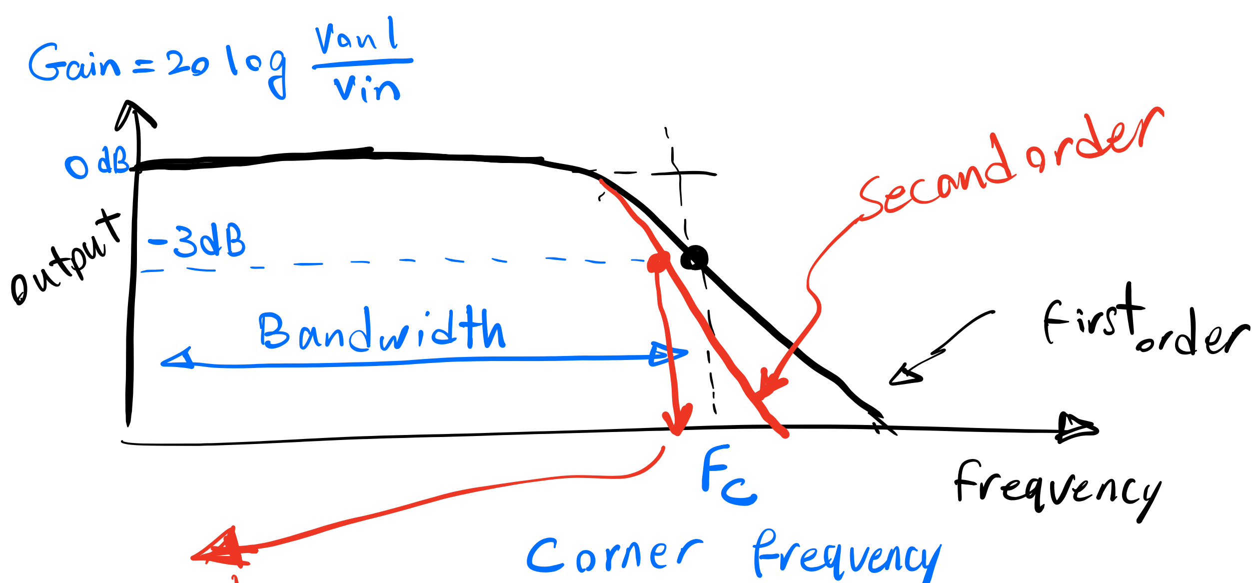
cut-off frequency:

$$f_c = \frac{1}{2\pi RC} \Rightarrow C = \frac{1}{2\pi f_c R}$$

Design the law pass Fiter by calculating a suitable capacitance C for the desired frequency cut-off f_c and an available Resistor (for example 10 KΩ)

Second-order Low pass Filter

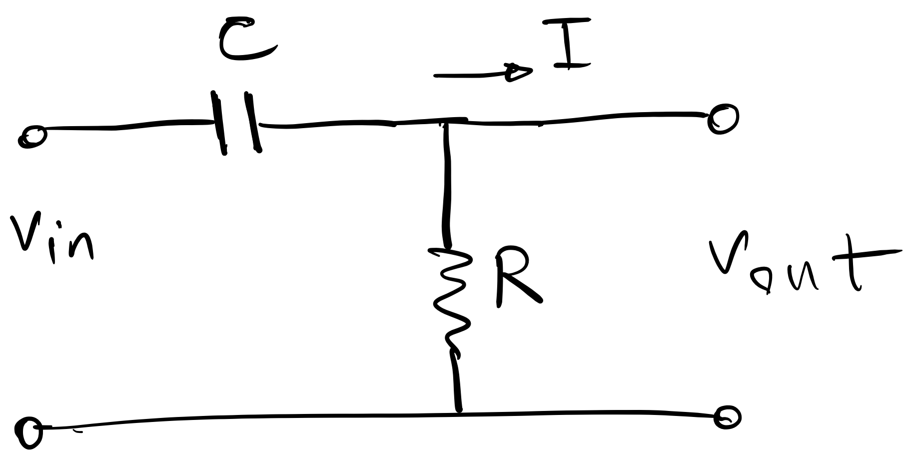




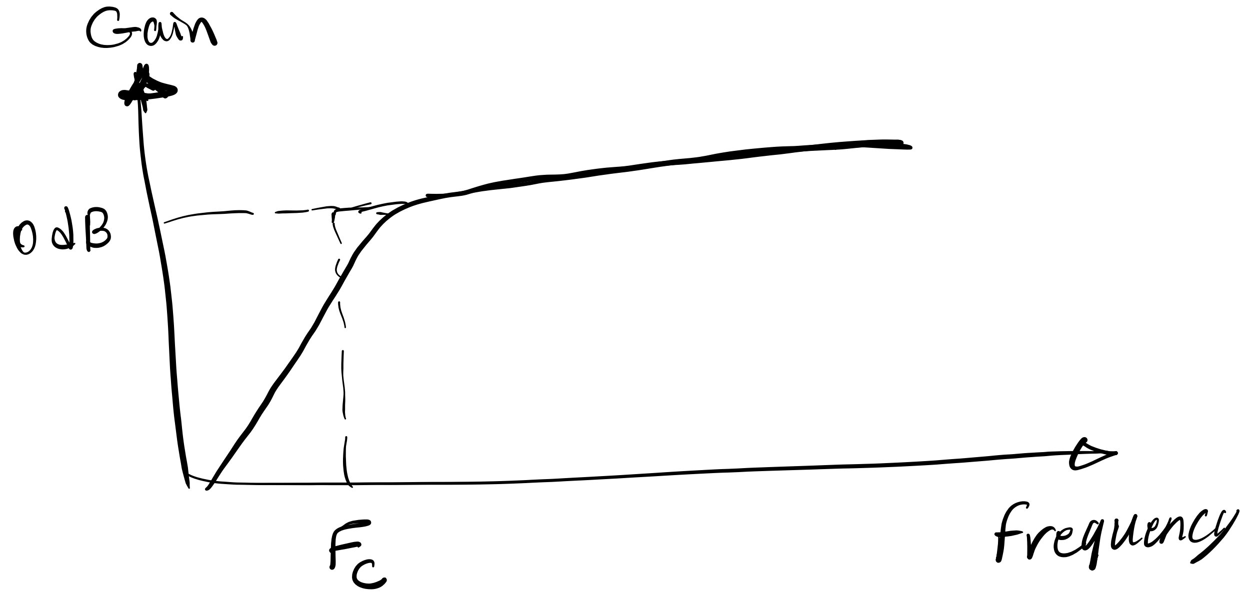
Second order corner frequency

$$\boxed{F_C} = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \text{ Hz}$$

Passive High pass filter



Frequency response of a 1st order High pass filter



cut-off frequency:

$$f_c = \frac{1}{2\pi RC}$$

$$\frac{V_{out}}{V_{in}} = A_V = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{Z}$$

at low freq.: $X_C \to \infty$, $V_{out} = 0$

at high freq: $X_C \to 0$, $V_{out} = V_{in}$

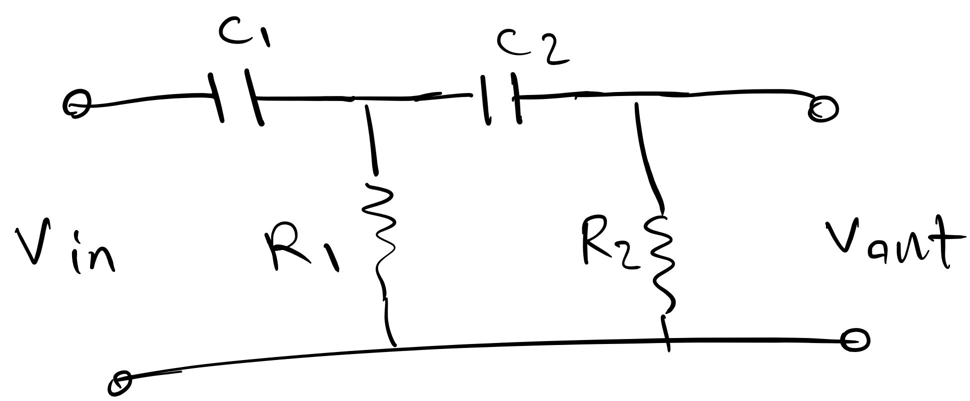
Example

calculate the cut-off or "breakpoint" frequency (f_c) for a passive high pass Filter considering 82 pF capacitor in series with 240 KΩ resistor

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 240000 \times 82 \times 10^{-12}}$$

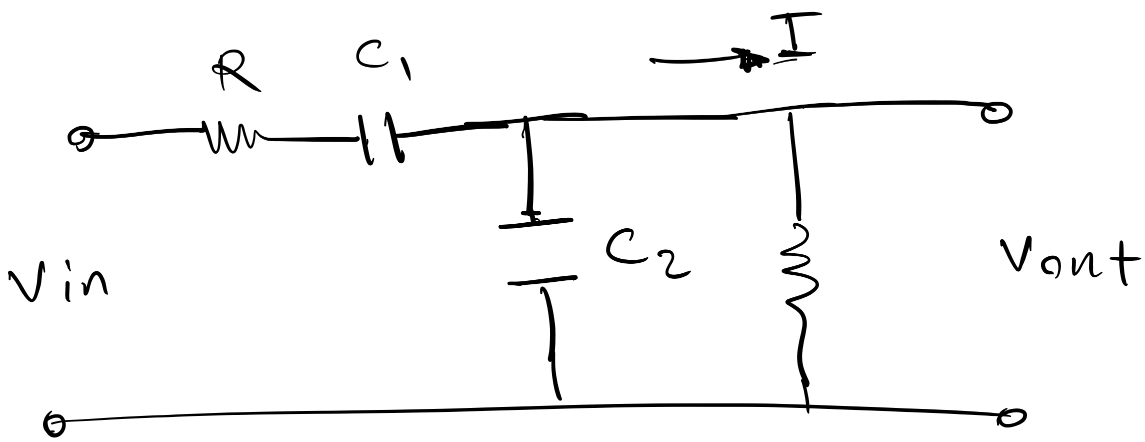
$f_c = 8087$ Hz or 8 KHZ

second-order Hig pass Filter



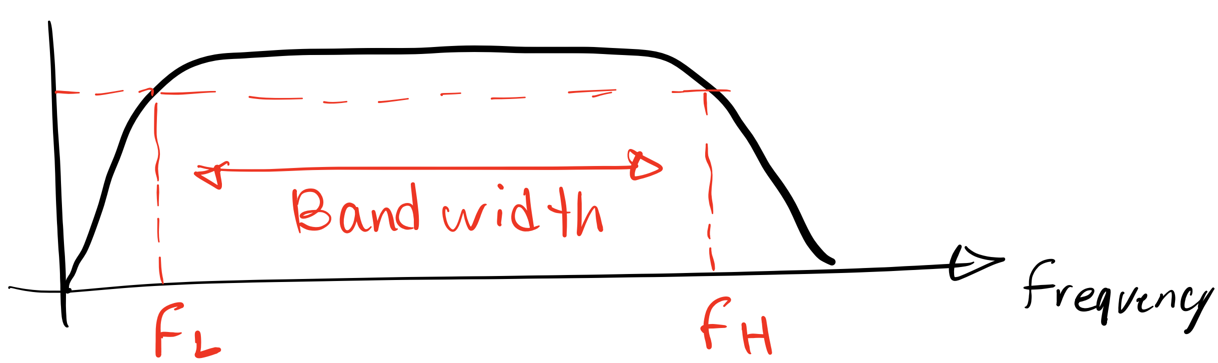
$$f_c = \frac{1}{2\pi\sqrt{R_1C_1R_2C_2}} \text{ Hz}$$

Passive Band pass filter



passive Band pass Filters can be made by connecting together a low pass filter with a high pass filter.

Frequency response



The upper and lower cut-off frequency points for a band pass filter can be found using the same formula as that for both the low and high pass filters:

$$f_c = \frac{1}{2\pi RC} \text{ Hz}$$

Example

A second-order band pass filter is to be constructed using RC components that will only allow a range of frequencies to pass above 1 KHz and below 30 KHz use both resistors to be 10 kΩ.

Calculate the two capacitor values.

The high pass stage:

cut-off frequency is $f_L = 1$ KH

$$C_1 = \frac{1}{2\pi f_L R} = \frac{1}{2\pi \times 1000 \times 10{,}000} = 15.9 \text{ nF}$$

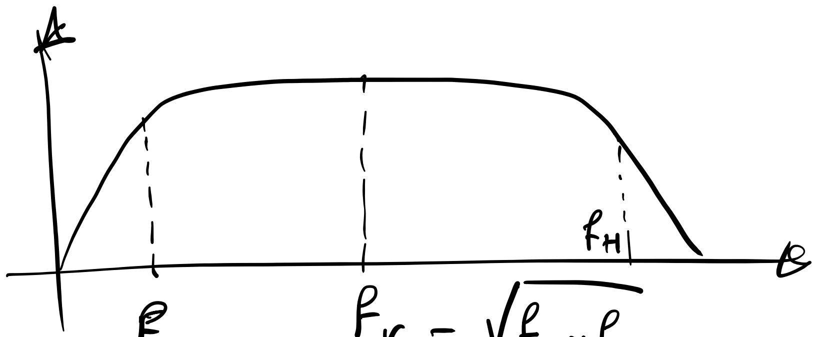
The nearest preffered value $C_1 = 15$ nF

The Low pass Filter stage

$F_H = 30$ KHz

$$C_2 = \frac{1}{2\pi f_H R} = \frac{1}{2\pi \times 30000 \times 10000} = 530 \text{ pF}$$

Center Frequency

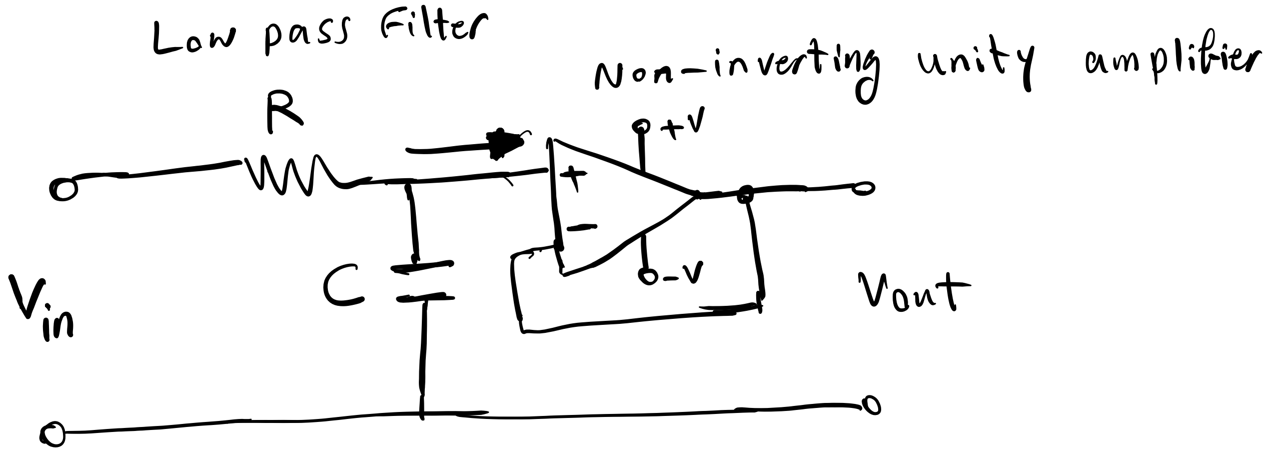


$f_r = \sqrt{f_L \times f_H}$

Buffering individual Filter stages

one way of combining amplification and filtering into the same circuit would be to use an operational Amplifier (op-amp). The op-amp not only introduce gain but provide isolation between stages, which known as Active Filters.

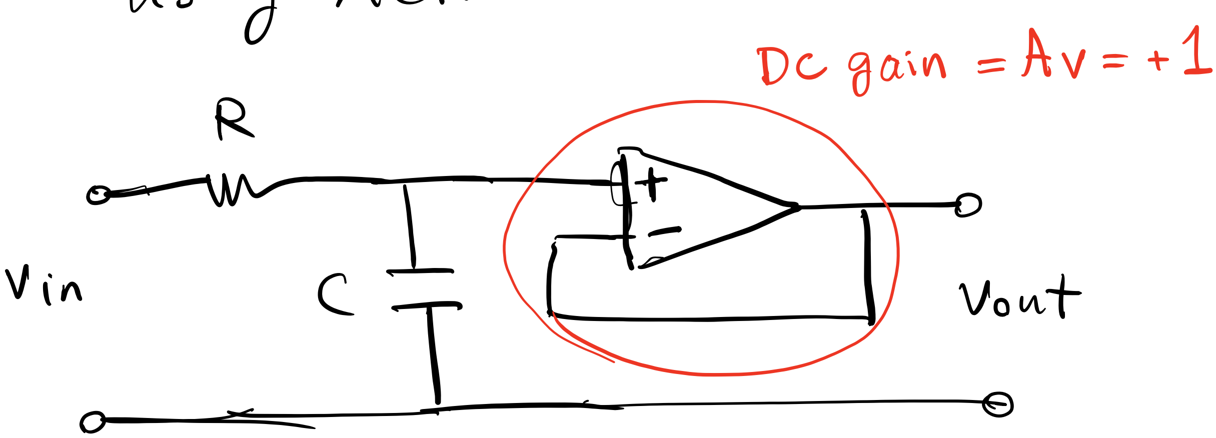
Active Low pass Filter



By cambining a basic RC Low Pass Filter circuit with an operational amplifier we can create an Active Low pass Filter circuit complete with amplification.

The main disadvatage of passive filters is that the amplitude of the output signal is less than the input (the gain is never greater than 1 because the load impedance affects the fiters output)

with passive filter circuits containing multiple stage, this loss in signal called "Attenuation" and can become severe. one way of restoring or contralling this loss of signal is by using amplification by using Active Filters.

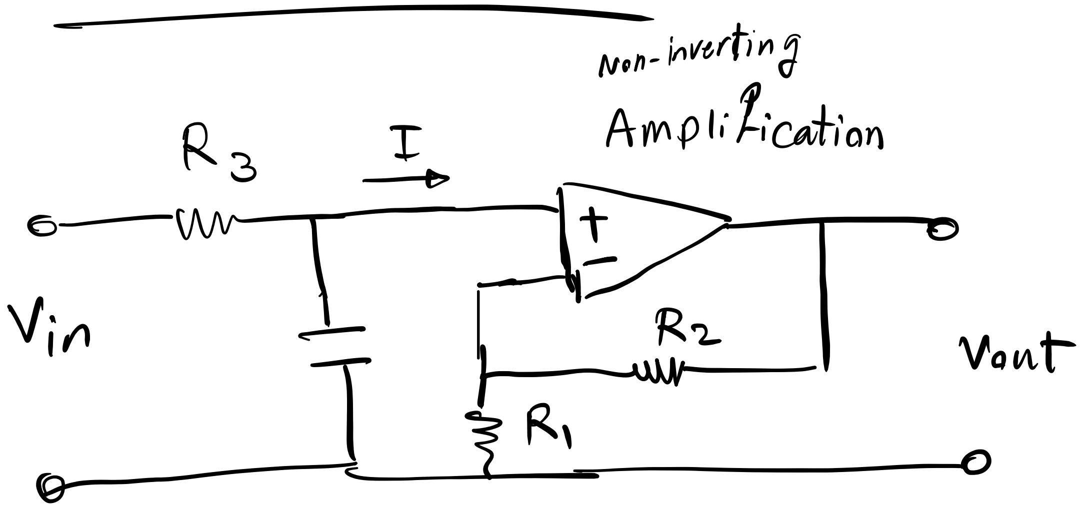


$$f_c = \frac{1}{2\pi RC}$$

The advantage of this canfiguration

is that the op-amp prevents excessive loading on the filters output, while prevents the filter cut-of frequency paint from being affected.

Active Low pass Filter with Amplification



Amplification factor = DC gain

$$= \left[1 + \frac{R_2}{R_1}\right]$$

For a non-inverting amplifier cicuit the magnitude of the voltage gain for the filter is given as a function of the feedback resistor (R_2) and input resistor (R_1) :

$$DC\ gain = \left[1 + \frac{R_2}{R_1}\right] \quad A_F$$

Gain of a first-order low pass filter

$$Voltage\ Gain\ (A_V) = \frac{V_{out}}{V_{int}} = \frac{A_F}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$$

where

A_F = the pass band of the filter $\left(1+\frac{R_2}{R_1}\right)$

f = the frequency of the input

signal in Hertz (Hz)

f_c = the cut-off frequency in Hertz

Example

Design a non-inverting active low pass filter circuit that has a gain of ten at low frequencies, high frequency cut-off or conner frequency of 159 Hz and an input impedance of 10 KΩ.

The voltage gain of a non-inverting operational amplifier is given as:

$$A_F = 1 + \frac{R_2}{R_1} = 10$$

Assume a value for resistor R_1 of 1 KΩ rearranging the

Formula above gives a value of R_2 of :

$$1 + \frac{R_2}{R_1} = 10$$

$$\frac{R_2}{R_1} = 10 - 1$$

$$R_2 = (10-1) R_1$$

$$R_2 = (10-1)\ 1\,K\Omega = 9\,K\Omega$$

However, a $9\,K\Omega$ resistor does not exist so the next preffered value of $9.1\,K\Omega$ is used instead. Converting this voltage gain to an equivalent decibel dB

Value give:

Gain in dB = 20 log A

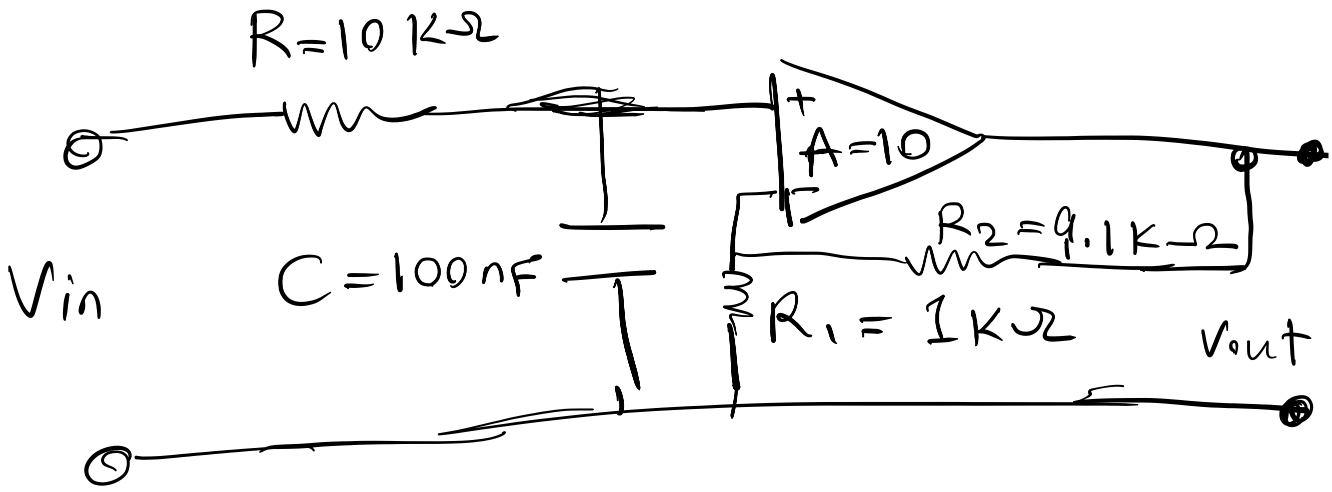
$= 20 \log 10 = 20$ dB

$f_c = 159$ Hz Low pass filter

$$f_c = \frac{1}{2\pi RC}$$

$$C = \frac{1}{2\pi f_c R} = \frac{1}{2\pi \times 159 \times 10000} = 100 \text{ nF}$$

C = 100 nF



R=10 KΩ
+
A=10
−
R_2=9.1 K-Ω
Vin
C=100 nF
R_1= 1 KΩ
Vout