Instrumentation and Controls

ETM 3301

Lecture 10

Instructor

Dr. Farbod Khoshnoud

Chapter 4: Block Diagram Representation and Simplification

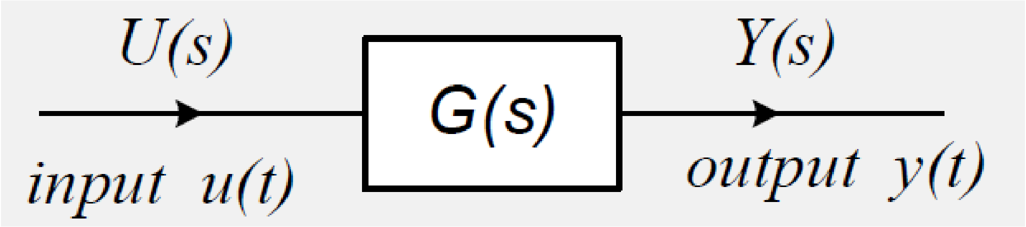
- What is a block diagram?
- Block diagram simplification rules.
- Use block diagram simplification rules to determine the overall transfer function of a system illustrated by a block diagram.

Transfer Function and Block Diagram

- A simple block diagram illustrates a system's input, output and transfer function. The signal flows from input, through the transfer function, to output.

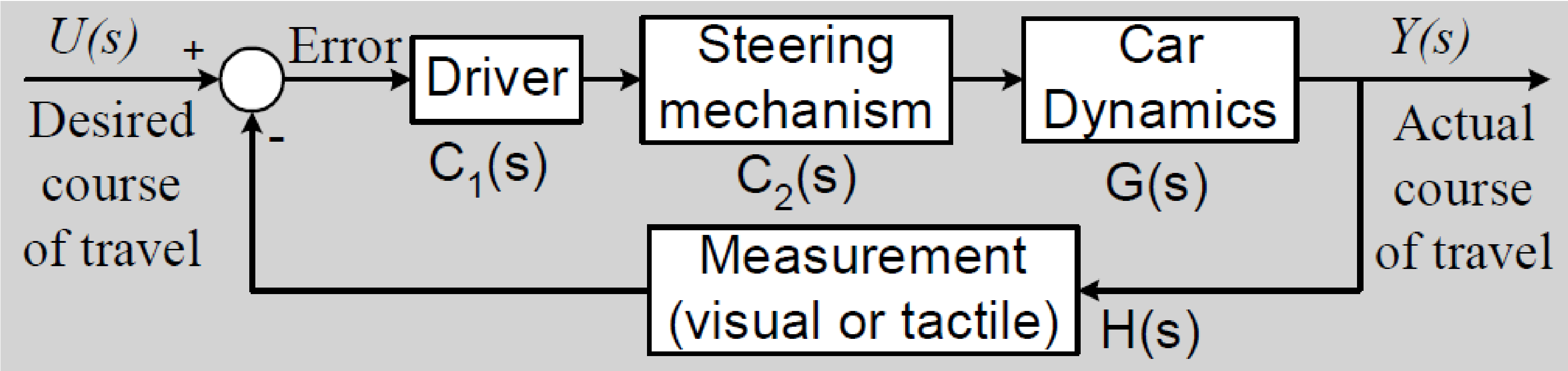
$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]}$$

Input	Output
Action	Consequence
Cause	Effect
Command	Response



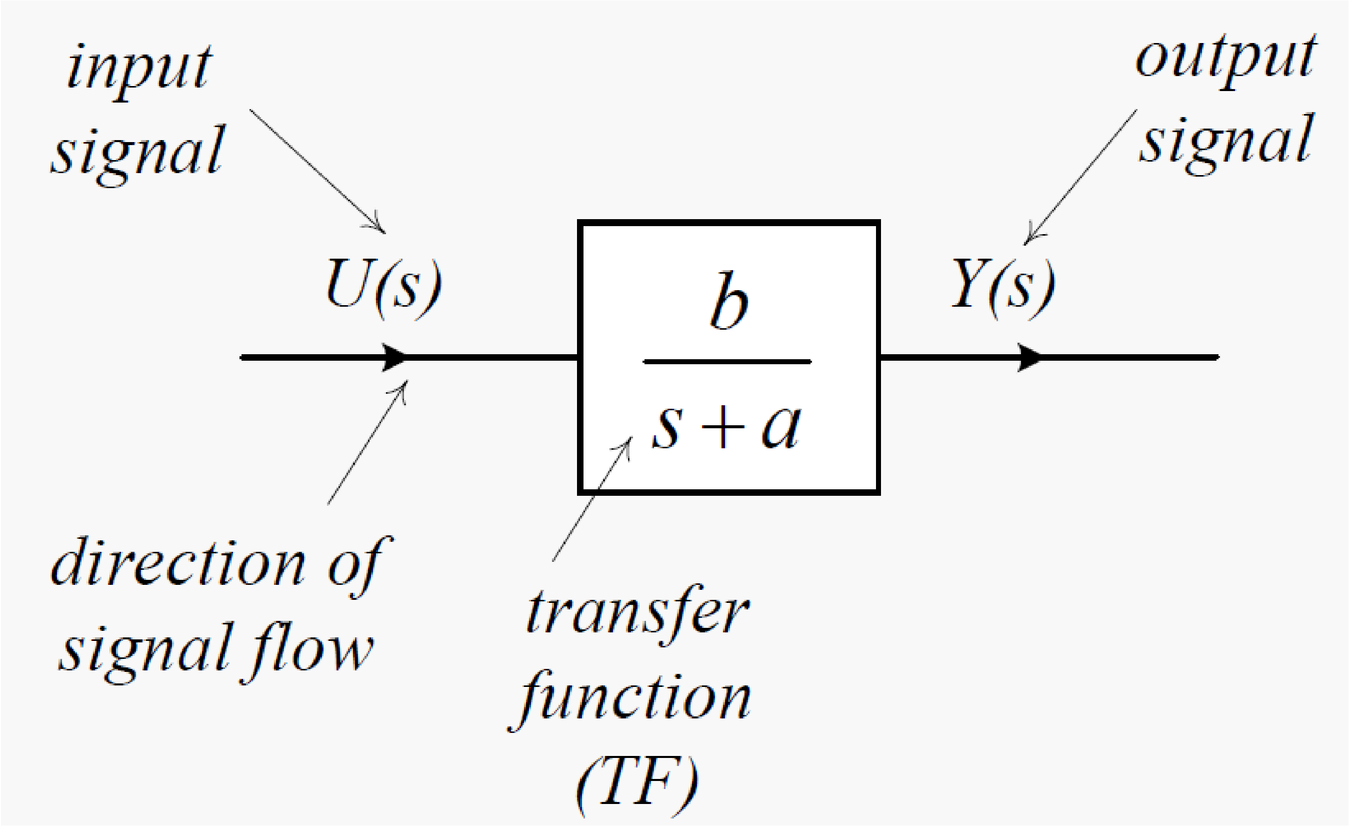
Block Diagram Representation

- A control system may consist of a number of components.
  - A block diagram is used to show the functions performed by each component.



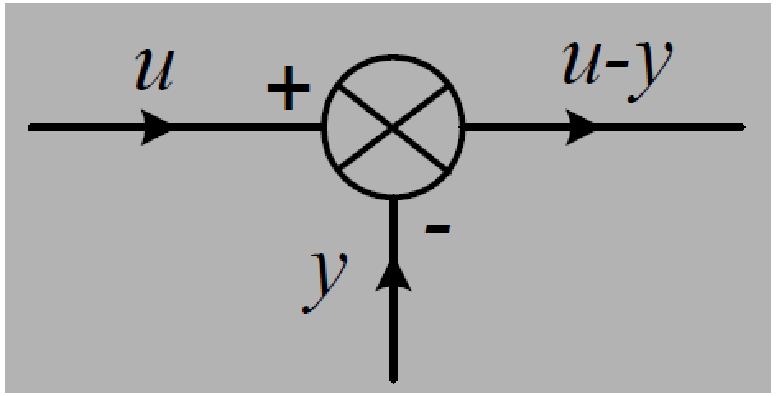
Car steering control system

Block Diagram Basic Building Block

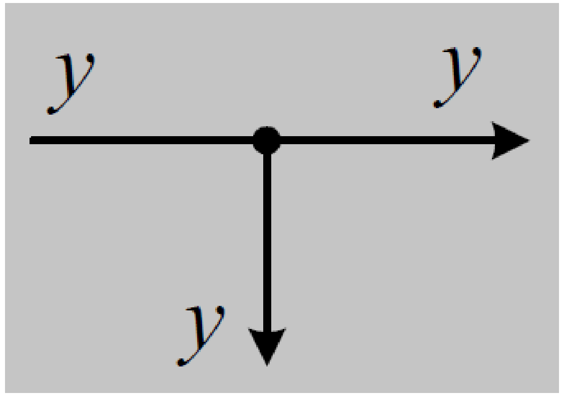


Block Diagram Connection Points

- Summing point

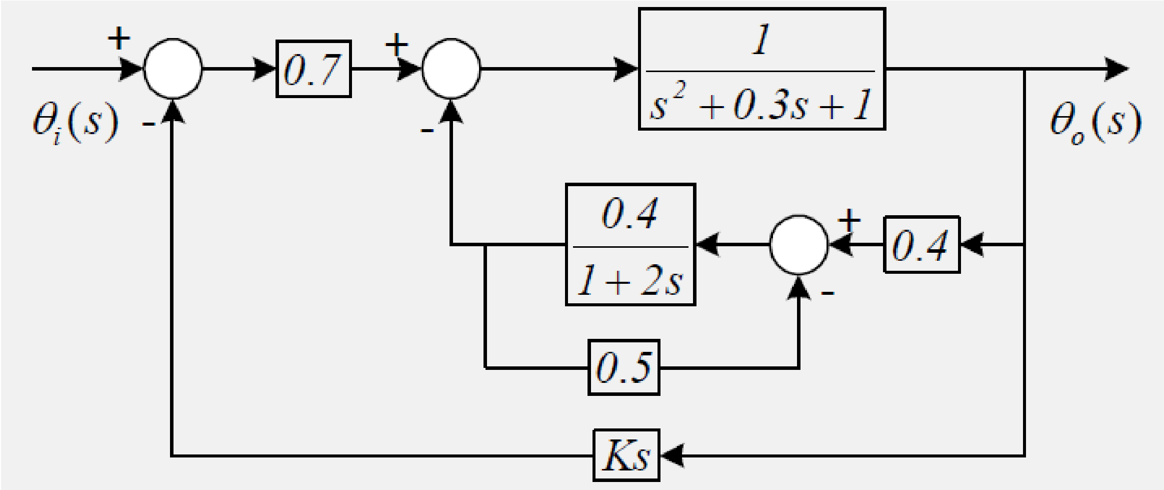


- Branch (pick up) point

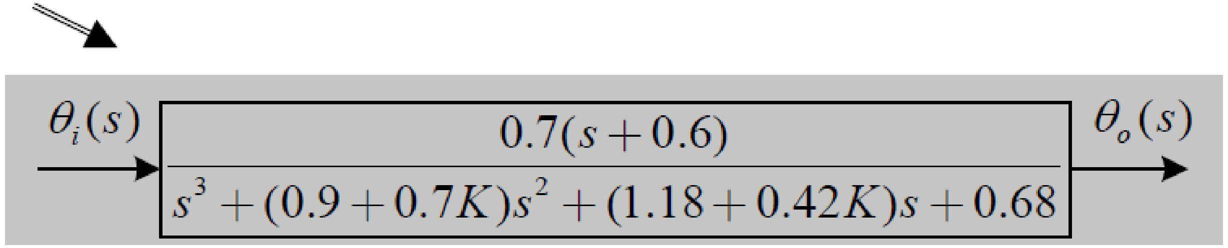


Block Diagram Simplification

- The block diagram representation of a given system often can be reduced by block diagram reduction techniques to a simplified block diagram with fewer blocks than the original diagram.



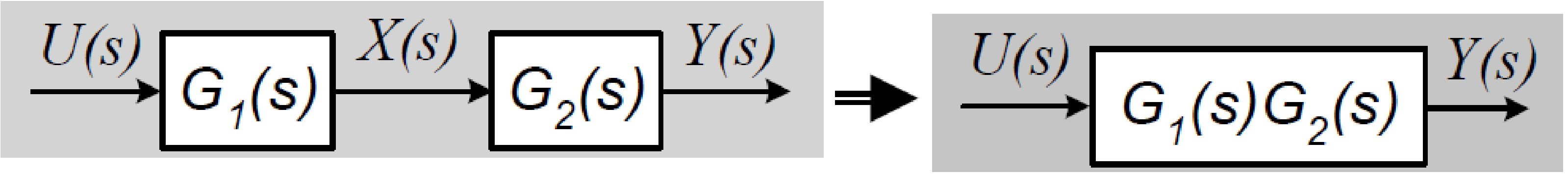
Aircraft pitch control system



Block Diagram Simplification Principle and Methods

- **Principle:**
  - Preserve the relationship between different signals.
- **Methods:**
  - Rearranging blocks.
  - Rewriting the same equation in a different way.

Combining Cascade (Series) Blocks



- $G_1(s)$, transfer function of first component
- $G_2(s)$, transfer function of second component

First component: $X(s) = G_1(s)\, U(s)$

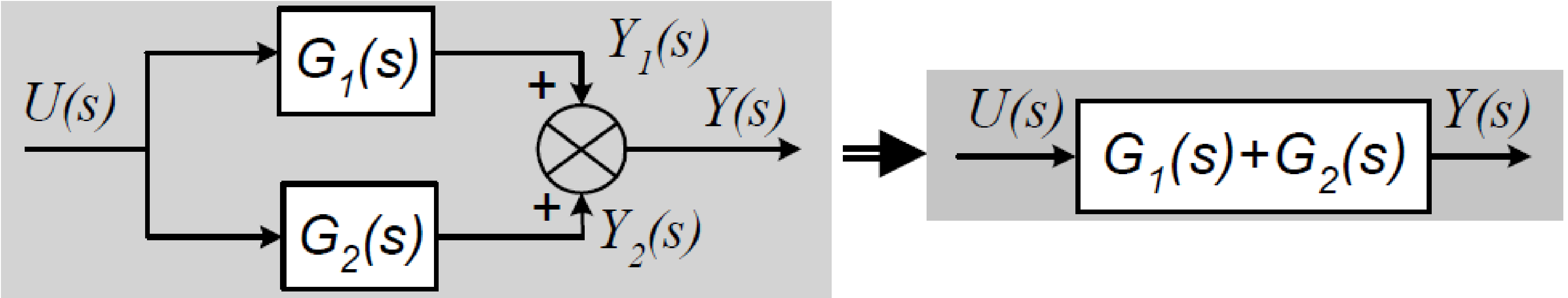
Second component: $Y(s) = G_2(s)\, X(s)$

Output: $Y(s) = G_2(s)\, G_1(s)\, U(s)$

- System transfer function

$$G(s) = \frac{Y(s)}{U(s)} = G_2(s)G_1(s) = G_1(s)G_2(s)$$

Combining Parallel Blocks



- $G_1(s)$, transfer function of first component
- $G_2(s)$, transfer function of second component

$$Y_1(s) = G_1(s)\ U(s) \qquad\qquad Y_2(s) = G_2(s)\ U(s)$$

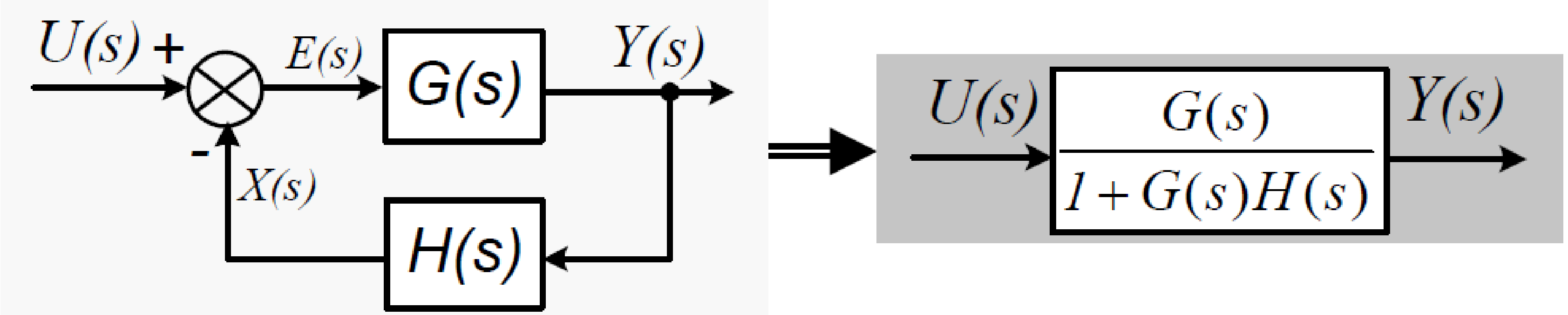
Output: $Y(s) = Y_1(s) + Y_2(s) = G_1(s)U(s) + G_2(s)\ U(s)$

$$= [G_1(s) + G_2(s)]\ U(s)$$

- System transfer function

$$G(s) = \frac{Y(s)}{U(s)} = G_1(s) + G_2(s)$$

Negative Feedback Connection, 1



- $G(s)$, forward path transfer function
- $H(s)$, feedback path transfer function

$$Y(s) = G(s)E(s) \qquad E(s) = U(s) - X(s) = U(s) - H(s)Y(s)$$

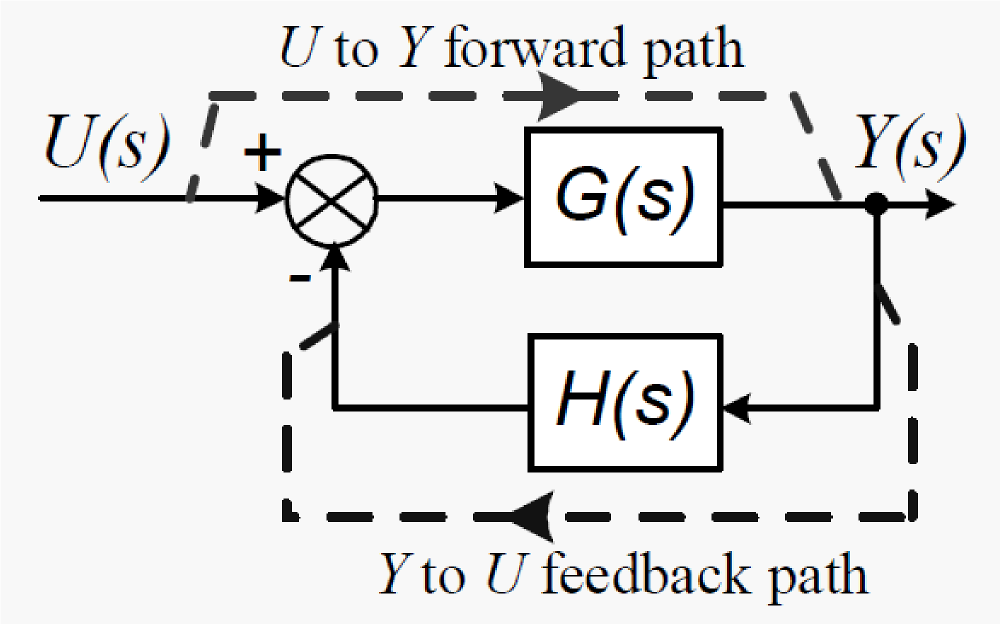
$$Y(s) = G(s)[U(s) - H(s)Y(s)] = G(s)U(s) - G(s)H(s)Y(s)$$

$$Y(s) + G(s)H(s)Y(s) = G(s)U(s)$$

$$Y(s) = \frac{G(s)}{1+G(s)H(s)}U(s)$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Negative Feedback Connection, 2

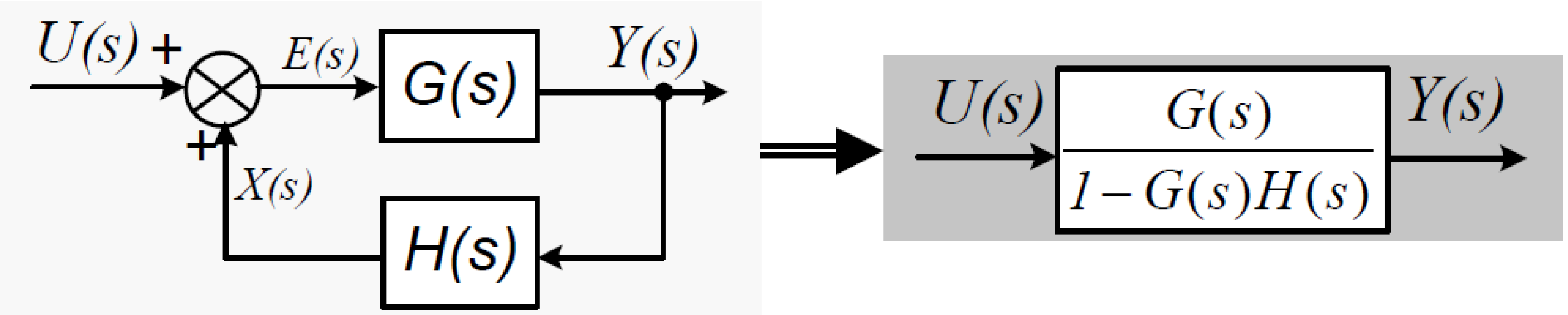


- $G(s)$, forward path transfer function
- $H(s)$, feedback path transfer function

$$CL\ TF = \frac{(forward\ \ path\ \ TF)}{1+(forward\ \ path\ \ TF)\times(feedback\ \ path\ \ TF)}$$

$$T(s) = \frac{G(s)}{1+G(s)H(s)}$$

Positive Feedback Connection



- $G(s)$, forward path transfer function
- $H(s)$, feedback path transfer function

$$Y(s) = G(s)E(s) \qquad E(s) = U(s) + X(s) = U(s) + H(s)Y(s)$$

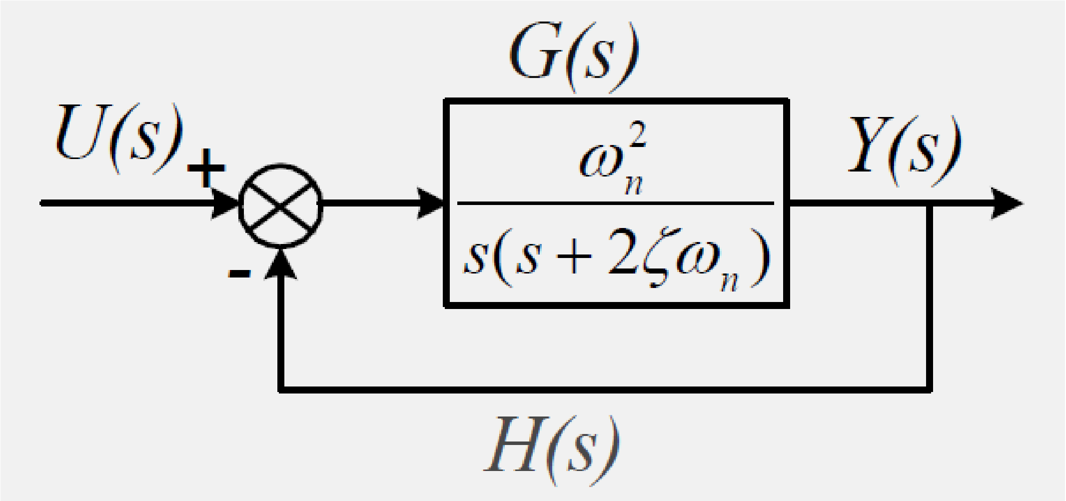
$$Y(s) = G(s)[U(s) + H(s)Y(s)] = G(s)U(s) + G(s)H(s)Y(s)$$

$$Y(s) - G(s)H(s)Y(s) = G(s)U(s)$$

$$Y(s) = \frac{G(s)}{1 - G(s)H(s)}U(s)$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

Simple Negative Feedback Example, 1



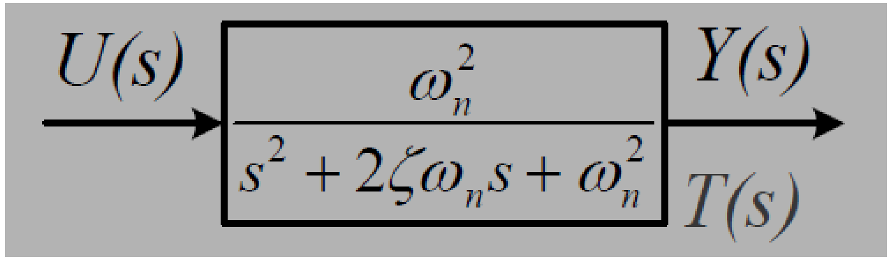
- Forward path TF:

$$G(s)=\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

- Feedback path TF:

$$H(s)=1$$

$$CL\ TF=\frac{(forward\ \ path\ \ TF)}{1+(forward\ \ path\ \ TF)\times(feedback\ \ path\ \ TF)}$$



Simple Negative Feedback Example, 2

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{\dfrac{\omega_n^2}{s(s + 2\zeta\omega_n)}}{1 + \dfrac{\omega_n^2}{s(s + 2\zeta\omega_n)} \times 1}$$

$$= \frac{\omega_n^2}{s(s + 2\zeta\omega_n)\left(1 + \dfrac{\omega_n^2}{s(s + 2\zeta\omega_n)}\right)}$$

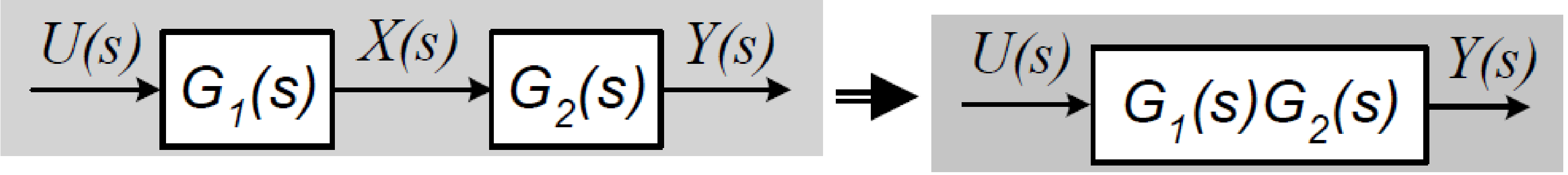
$$= \frac{\omega_n^2}{s(s + 2\zeta\omega_n) + \omega_n^2} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{\dfrac{A}{B}}{C + D} = \frac{A}{B(C + D)}$$

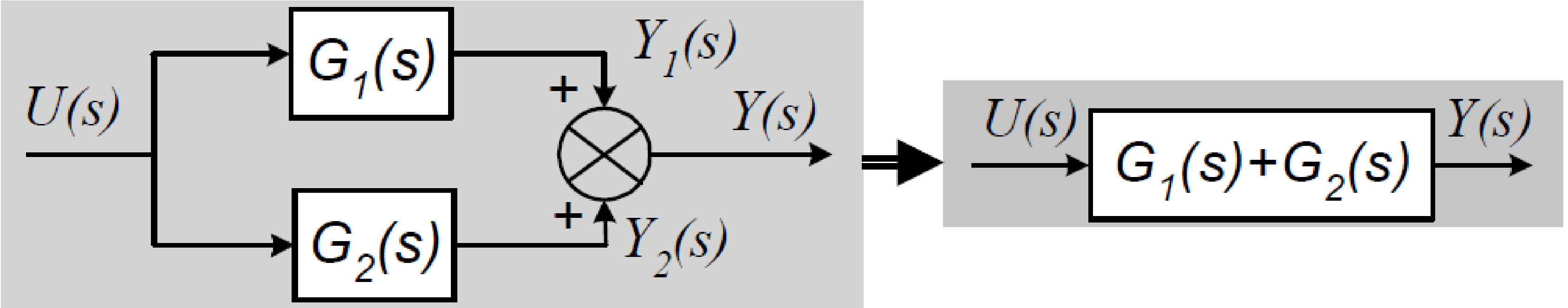
$$\frac{\dfrac{1}{2}}{2} = \frac{1}{2 \times 2} = \frac{1}{4}$$

Three Typical Cases

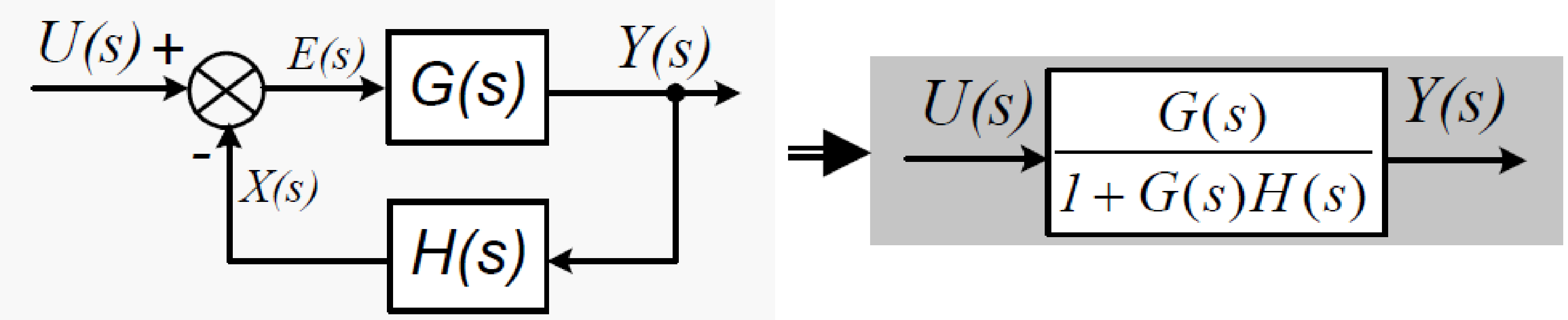
- Series Connection



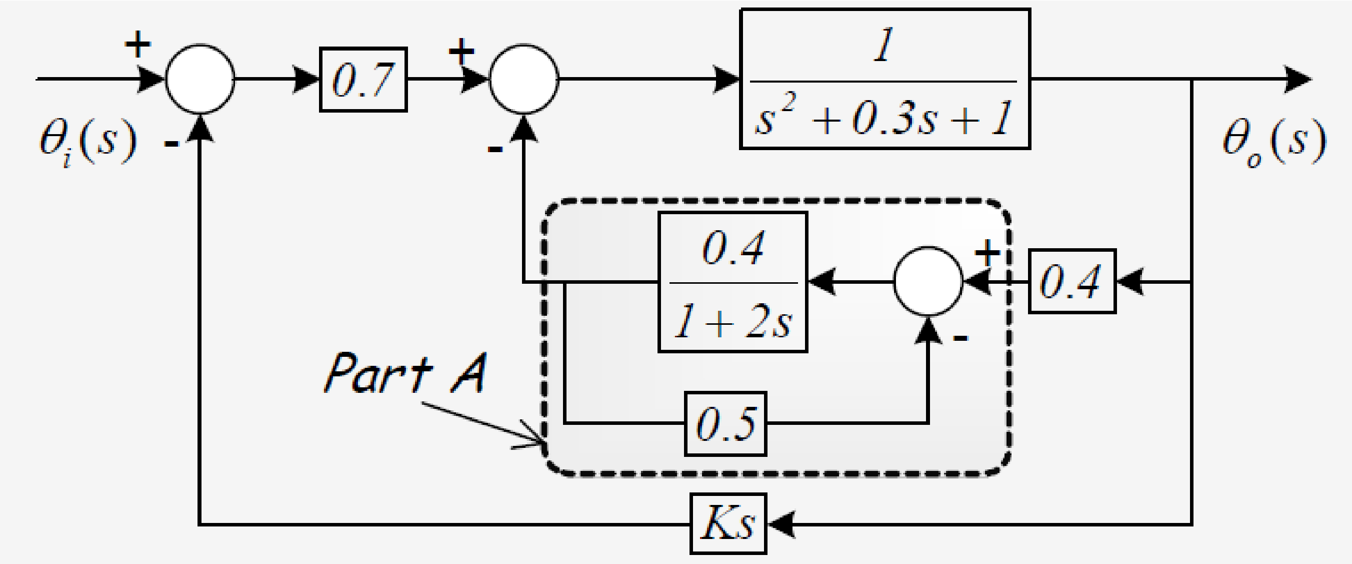
- Parallel Connection



- Negative Feedback Connection



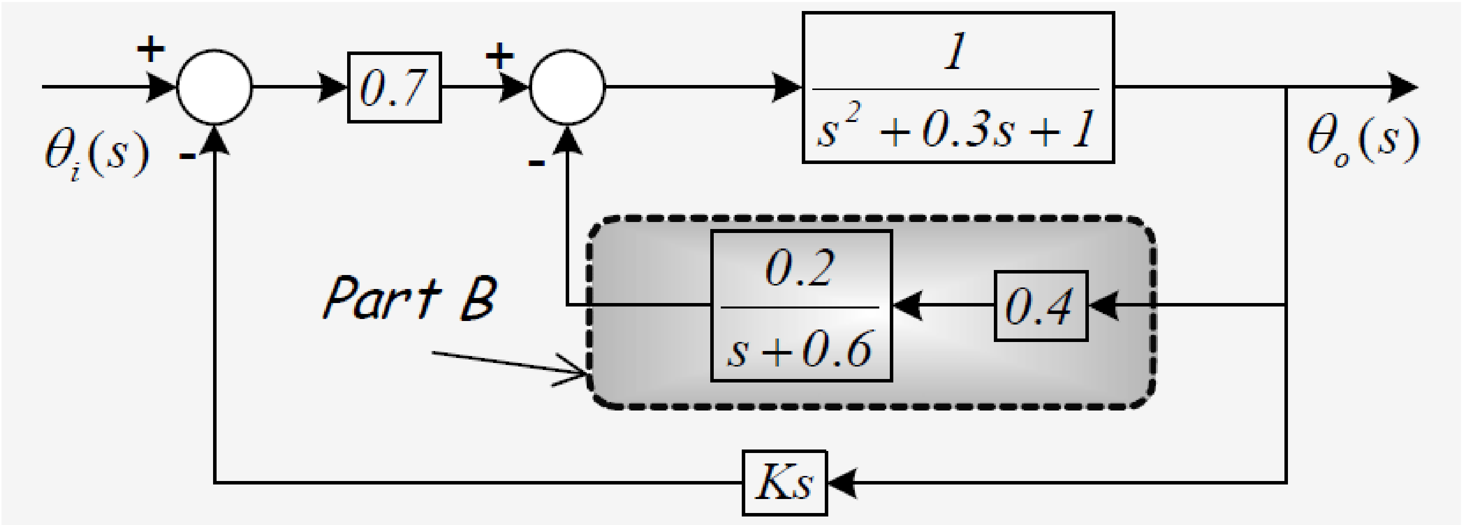
Block Diagram Simplification Example



Transfer function of Part A

$$\frac{\dfrac{0.4}{1+2s}}{1+\dfrac{0.4}{1+2s}\times 0.5}=\frac{0.4}{1+2s+0.2}=\frac{0.4}{2s+1.2}=\frac{0.2}{s+0.6}$$

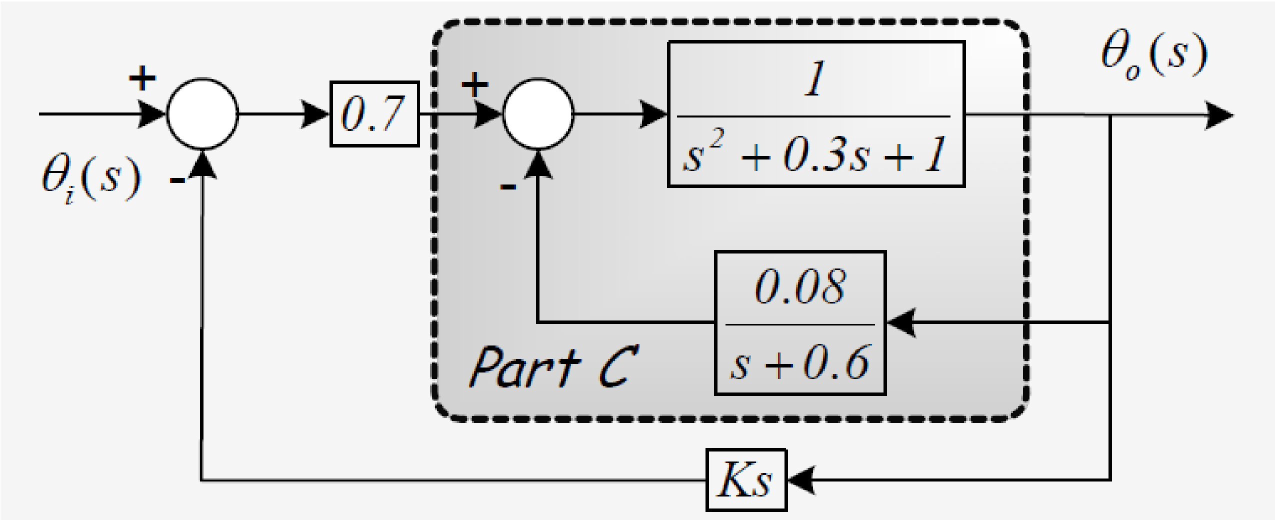
Block Diagram Simplification Example



Transfer function of Part B

$$\frac{0.2}{s+0.6} \times 0.4 = \frac{0.08}{s+0.6}$$

Block Diagram Simplification Example

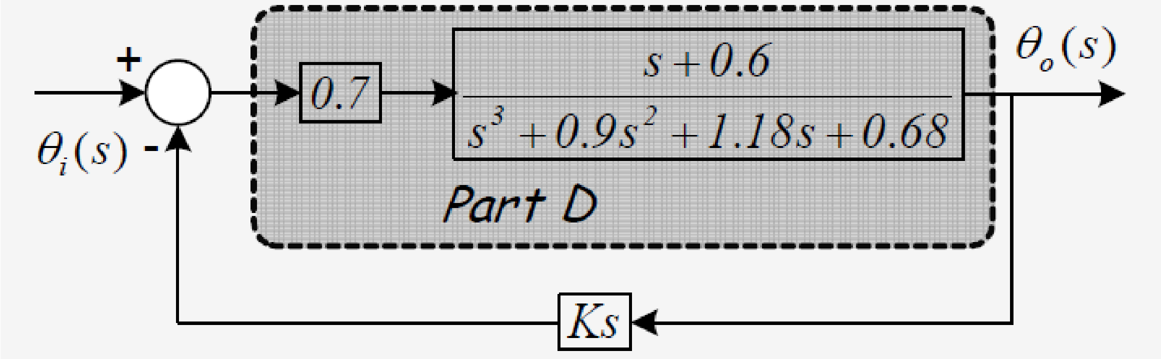


Transfer function of Part C

$$\frac{\dfrac{1}{s^2+0.3s+1}}{1+\dfrac{1}{s^2+0.3s+1}\times\dfrac{0.08}{s+0.6}}=\frac{s+0.6}{(s^2+0.3s+1)(s+0.6)+0.08}$$

$$=\frac{s+0.6}{s^3+0.9s^2+1.18s+0.68}$$

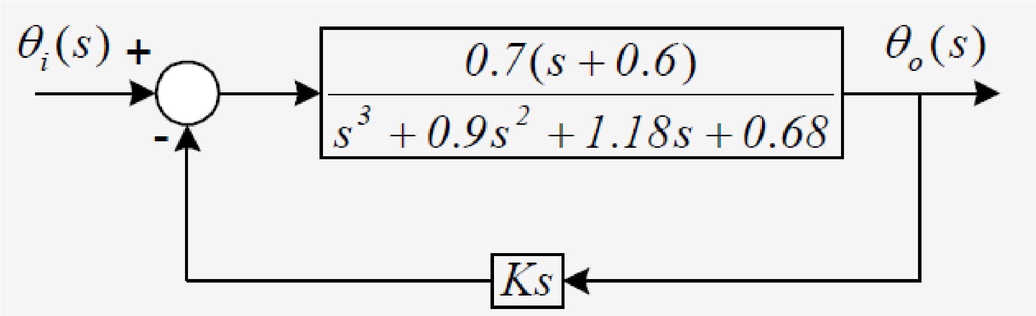
Block Diagram Simplification Example



Transfer function of Part D

$$0.7 \times \frac{s+0.6}{s^3+0.9s^2+1.18s+0.68} = \frac{0.7(s+0.6)}{s^3+0.9s^2+1.18s+0.68}$$

Block Diagram Simplification Example



Final Transfer Function

$$T(s)=\frac{\theta_o(s)}{\theta_i(s)}=\frac{\dfrac{0.7(s+0.6)}{s^3+0.9s^2+1.18s+0.68}}{1+\dfrac{0.7(s+0.6)}{s^3+0.9s^2+1.18s+0.68}\times Ks}$$

$$=\frac{0.7(s+0.6)}{s^3+0.9s^2+1.18s+0.68+0.7Ks(s+0.6)}$$

$$=\frac{0.7(s+0.6)}{s^3+(0.9+0.7K)s^2+(1.18+0.42K)s+0.68}$$