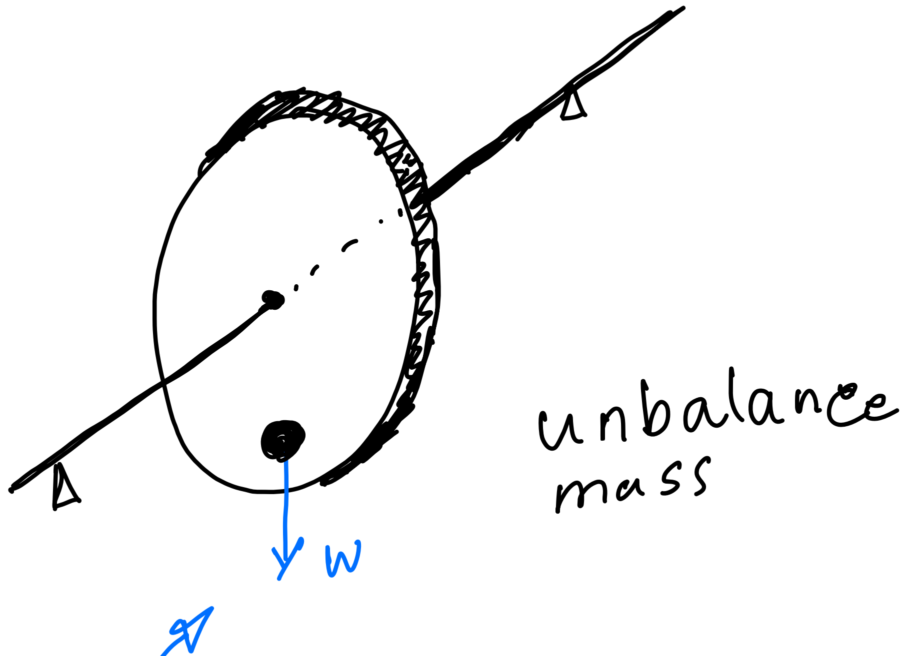
Rotor Unbalance



static:

Dynamic unbalance:

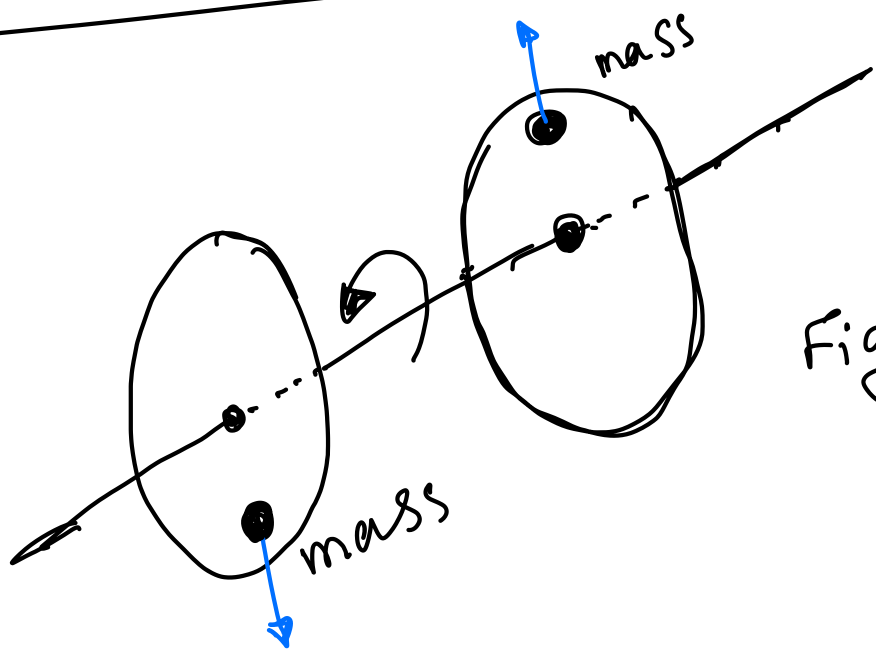
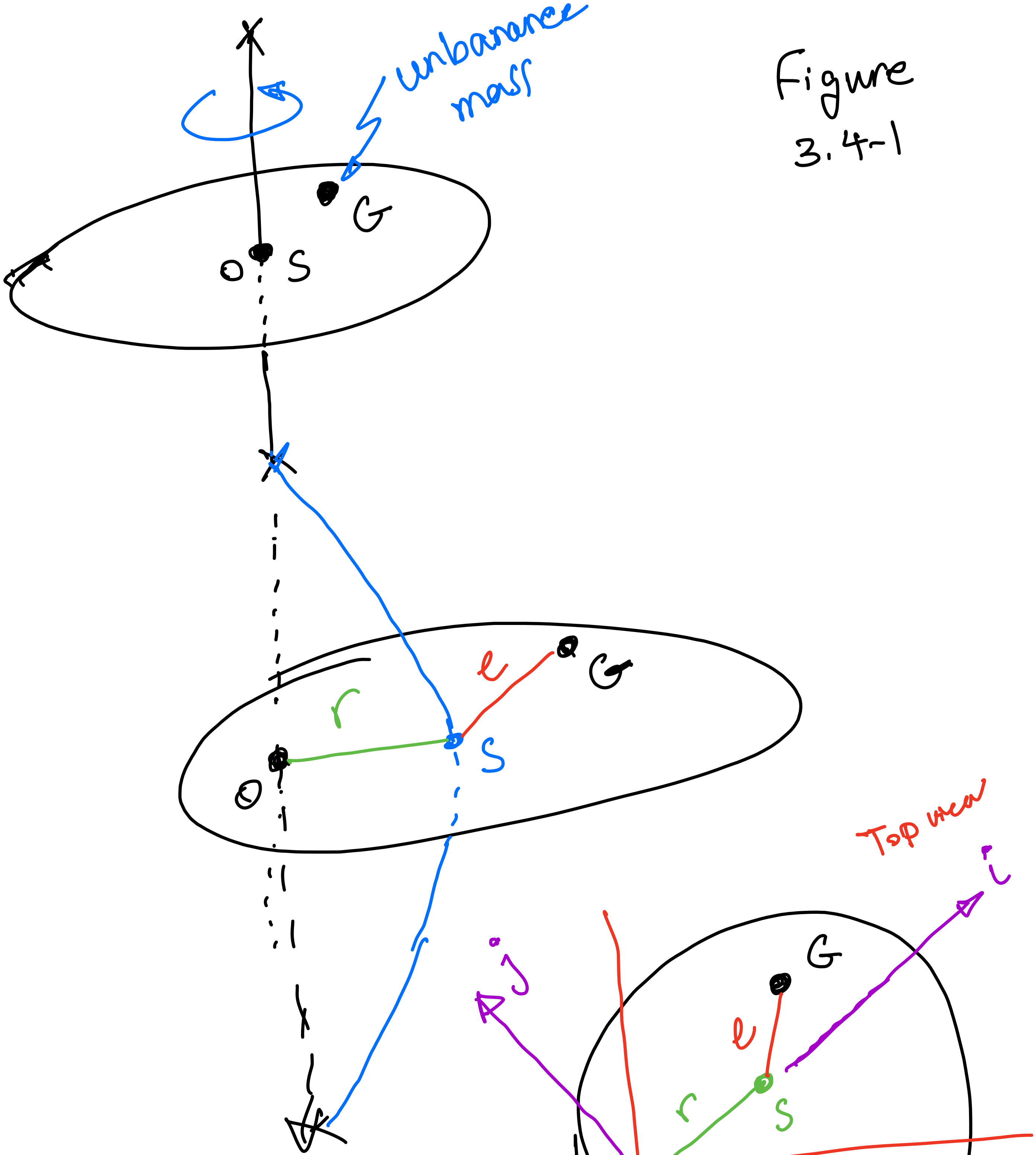


Fig. 3.3-2
Book

Figure 3.4-1



Acceleration:

$$\vec{a}_G = \vec{a}_S + \vec{a}_{G/S}$$

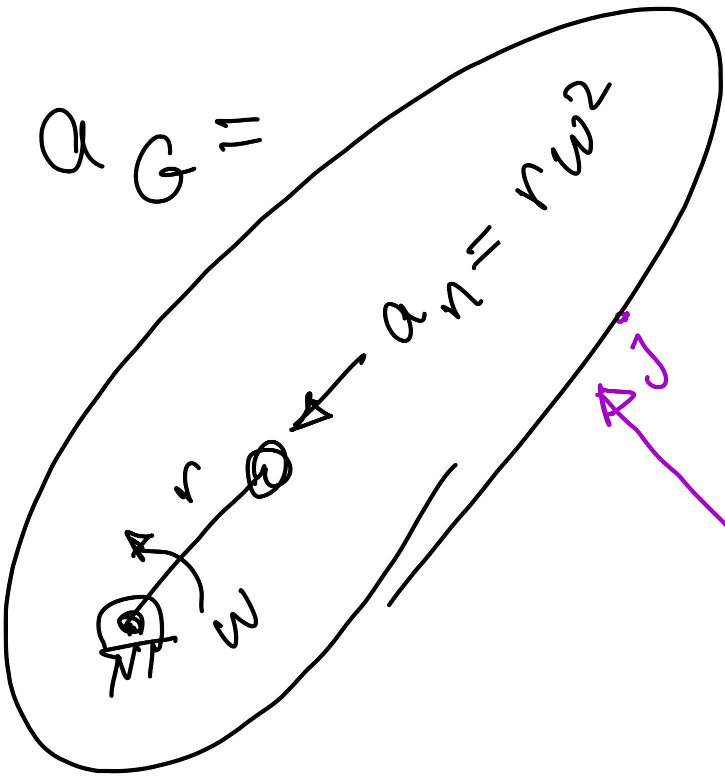
$$\begin{cases} e = SG \\ r = OS \end{cases}$$

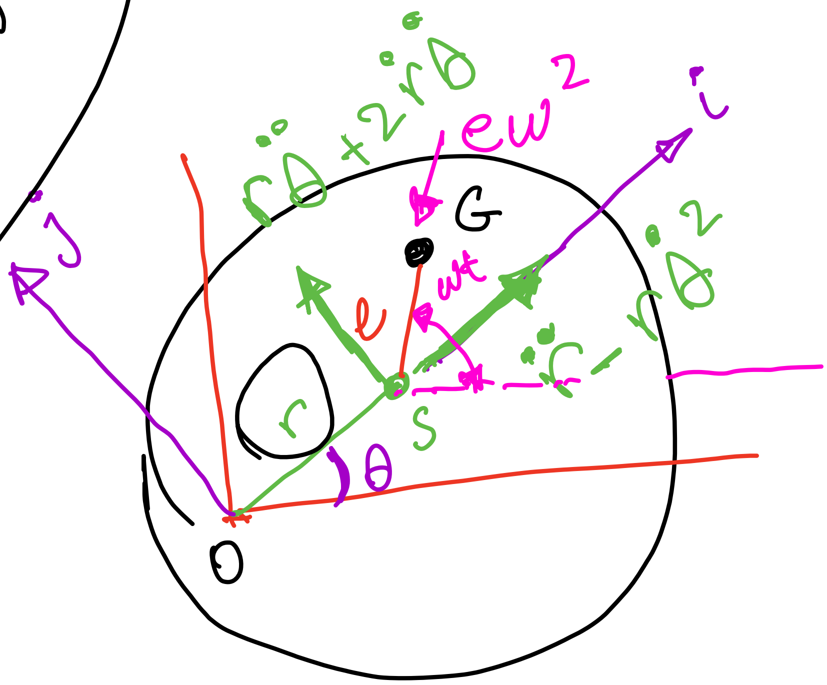
shaft is rotating at

a constant speed ω (the line $e = SG$)

The line $r = OS$ whirling at speed $\dot{\theta}$

$a_G =$





$$a_G = \left[(\ddot{r} - r\dot{\theta}^2) - e\omega^2 \cos(\omega t - \theta)\right] i$$

$$\left[(r\ddot{\theta} + 2\dot{r}\dot{\theta}) - e\omega^2 \sin(\omega t - \theta)\right] j$$

elasticity of the shaft is with

equivalent stiffness of K
and equivalent damping of c

$$\boxed{\Sigma F_i = m a_i}$$

$$-kr - c\dot{r} =$$

$$m\left[(\ddot{r} - r\dot{\theta}^2) - e\omega^2 \cos(\omega t - \theta)\right]$$

$$\Sigma F_j = m a_j$$

$$-cr\dot{\theta} = m\left[r\ddot{\theta} + 2\dot{r}\dot{\theta} - e\omega^2 \sin(\omega t - \theta)\right]$$

Rearrange:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = F$$

standard form

$$\ddot{r} + \frac{c}{m}\dot{r} + \left(\frac{K}{m} - \dot{\theta}^2\right)r = e\omega^2 \cos(\omega t - \theta)$$

$$r\ddot{\theta} + \left(\frac{c}{m}r + 2\dot{r}\right)\dot{\theta} = e\omega^2 \sin(\omega t - \theta)$$

steady state synchronous whirl:

$$\dot{\theta} = \omega \Rightarrow \ddot{\theta} = 0$$

$$\ddot{r} = \dot{r} = 0$$

$$\dot{\theta} = \omega \Rightarrow \theta = \omega t - \phi$$

$$\boxed{\begin{gathered} \frac{d\theta}{dt} = \omega \rightarrow \int d\theta = \int \omega \, dt \\ \theta = \omega t + \text{constant} \end{gathered}}$$

$$\begin{cases} \cancel{\ddot{r}} + \frac{c}{m} \cancel{\dot{r}} + \left(\frac{K}{m} - \dot{\theta}^2\right) r = e\omega^2 \cos(\omega t - \theta) \\ \cancel{r\ddot{\theta}} + \left(\frac{c}{m} r + \cancel{2\dot{r}}\right) \dot{\theta} = e\omega^2 \sin(\omega t - \theta) \end{cases}$$

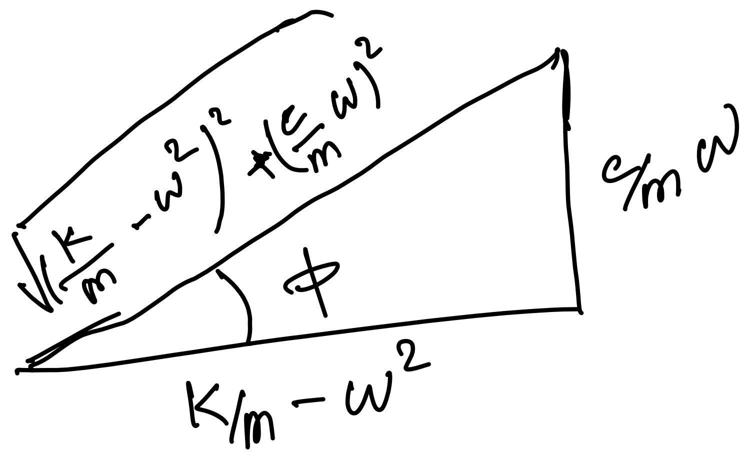
$$\begin{cases} \left(\frac{K}{m} - \omega^2\right) r = e\omega^2 \cos\phi \\ c/m \, \omega r = e\omega^2 \sin\phi \end{cases}$$

instead of m, c, k write the equations in terms of ξ and ω_n.

The phase angle ϕ:

$$\tan\phi = \frac{\frac{c}{m}\omega}{\frac{k}{m} - \omega^2} = \frac{2\xi\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$\omega_n = \sqrt{k/m}$ is the critical speed. $\xi = c/c_c$



$$r = \frac{me\omega^2}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$= \frac{e\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

(3.4-1 Book)

Example

Turbines operating above the critical speed must run through dangerous speed at resonance each time they are started or stopped. Assuming the critical speed ω_n to be reached with amplitude r_0 Determine the equation for the amplitude build up with time. Assume zero damping.

Salution: we will assume synchronous whirl $\rightarrow$ $\dot{\theta} = \omega = \text{constant}$

$\ddot{\theta} = 0$

$c = 0$

$$\begin{cases} \ddot{r} + \left(\frac{K}{m} - \omega^2\right) r = e\omega^2 \cos\phi & (1) \\ 2\dot{r}\omega = e\omega^2 \sin\phi & (2) \end{cases}$$

(2) $\Longrightarrow$ $\dot{r} = \frac{e}{2}\omega \sin\phi$

$\frac{dr}{dt} = \frac{e}{2}\omega \sin\phi \rightarrow \int dr = \int \frac{e}{2}\omega \sin\phi \, dt$

$r = \frac{e\omega}{2} t \sin\phi + r_0$ (3)

Dervative $\longrightarrow$ $\ddot{r} = \frac{d\dot{r}}{dt} = 0$ (4)

1 , 3 , 4 $\Longrightarrow$

$$\left(\frac{K}{m}-w^2\right)\left(\frac{ew}{2}t\sin\phi+r_0\right)=ew^2\cos\phi$$

constant

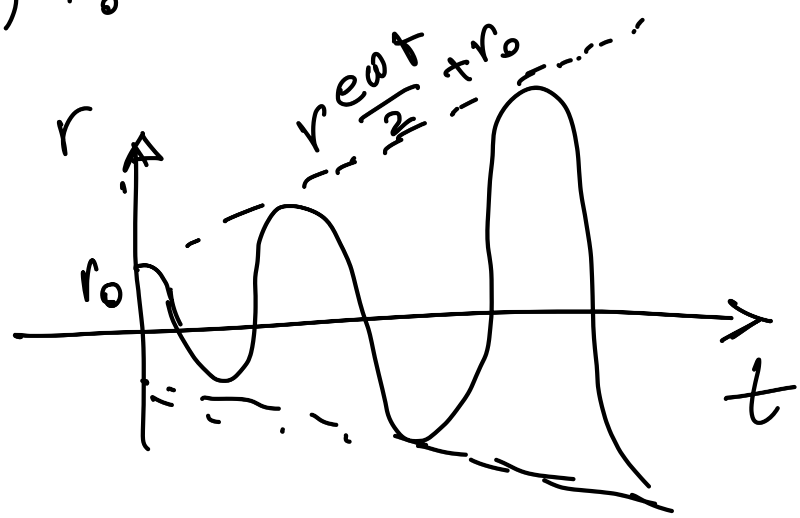
Because the right side is canstat then coefficient of t is zero

$$\left(\frac{K}{m}-w^2\right)\sin\phi=0$$

$\Longrightarrow$ Remaining

$$\left(\frac{K}{m}-w^2\right)r_0=ew^2\cos\phi$$

$$w=\sqrt{\frac{K}{m}}$$



unstable